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Foundations of special relativity: kinematic axioms for Minkowski space-time

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Minkowski space-time is developed in terms of undefined elements called "particles" and a single undefined relation, the "signal relation". Particles correspond physically to "inertial particles" and the signal relation corresponds to "light signals". The undefined basis is similar to that of Walker [*1*, *2*].

Altogether there are eleven axioms. The first five are similar in content to those of Walker [*1*, *2*]. Of the remaining axioms, four concern sets of particles which coincide at any one given event and which are called SPRAYs . We postulate:

(i) between any two distinct particles of a SPRAY , there is a particle which is distinct from both,

(ii) each SPRAY is isotropic,

(iii) there is a SPRAY which has a maximal symmetric sub-SPRAY of four distinct particles, and

(iv) each bounded infinite sub-SPRAY is compact.

The essential content of the remaining two axioms is that: space-time can be "connected" by particles; and that, given any two distinct particles which coincide at some event, there is a third distinct particle which forms the third side of a "triangle".

The ensuing discussion falls naturally into two parts: the

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development of rectilinear kinematics, which is in many ways similar to the geometry of coplanar subsets in absolute geometry; and the extension to three-dimensional kinematics which is established by first showing that each SPRAY is a three-dimensional hyperbolic space and then extending a correspondence between homogeneous coordinates and space-time coordinates. These ideas will now be described in more detail.

Collinear sub-SPRAYs are shown to exist and their properties are discussed. Then the existence of maximal collinear sets of particles is demonstrated and it is found that they have many properties which are analogous to properties of coplanar subsets in the theory of absolute geometry. The concept of parallelism is applied to particles and, as in absolute geometry, we are faced with the possibilities of there being none, one, or two distinct particles which are parallel to a given particle through a given event. It is shown that there are two types of parallels, which may or may not be distinct, and that both types of parallels lead to equivalence relations of parallelism. The set of all events in a maximal collinear set can then be "coordinatised" with respect to any equivalence class of parallels. Both relations of parallelism turn out to be invariant with respect to reflection mappings. By composing several reflection mappings, it is possible to generate "pseudo-rotations", space translations and time translations. It transpires that the uniqueness of parallelism is a theorem, which is a marked contrast with the theory of absolute geometry! This remarkable finding implies that each particle moves with uniform velocity.

It is shown that each SPRAY is a three-dimensional hyperbolic space, with particles corresponding to "points" and with relative velocity as a metric function. Homogeneous coordinates in three-dimensional hyperbolic space correspond to space-time coordinates "within a light cone". The extension of this correspondence to all events gives rise to the concept of a coordinate frame. Associated with each coordinate frame is a position-space which is shown to be a three-dimensional euclidean space. Transformations of homogeneous coordinate systems then correspond to "homogeneous Lorentz transformations" from which the "inhomogeneous Lorentz transformations" are derived.

References

[1] A.G. Walker, "Foundations of relativity. I, II", *Proc. Roy. Soc. Edinburgh Sect. A* 62 (1948), 319-335.

[2] A.G. Walker, "Axioms for cosmology", *The axiomatic method, with special reference to geometry and physics*, 308-321 (*Proc. International Symposium, University of California, Berkeley, December 26, 1957 - January 4, 1958.* Studies in Logic and the Foundations of Mathematics. North-Holland, Amsterdam, 1959).

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