# BRANCHING AND APPLIED PROBABILITY

Papers in honour of PETER JAGERS

Edited by S. ASMUSSEN, F. KLEBANER, O. NERMAN and V. VATUTIN



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Frontispiece: Peter Jagers

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### PREFACE

It gives us great pleasure to present this volume of papers honouring Peter Jagers. Peter turned 75 in 2016 and has been an instrumental figure in the community in branching, applied probability and further aspects of stochastics. His contributions to branching processes are known to all, but one should also mention the role he has had in conveying communication and interaction between scientists in the West and in the East, in professional societies and in the Scandinavian stochastics environment. Further, from the point of view of the Applied Probability journals, he has been extremely useful via the long period of time in which he has been one of the Editors or Co-ordinating Editors, standing behind four Editors-in-Chief to advise on extraordinary matters.

The present volume contains a number of invited scientific papers by authors who all in some sense have interacted with Peter or otherwise have been close to him. In addition, there are some personal memories by Peter, some recollections by Eugene Seneta and a sketch of Peter as an academic leader and colleague by Olle Nerman.

We are grateful to all contributors and to Daryl Daley and Emma Talib for their assistance in producing this volume. But above all, our gratitude goes to Peter for all he has meant to the community and us personally during his career. We hope that he may accept this volume as an expression of our appreciation of this.

December 2018

Søren Asmussen Fima Klebaner Olle Nerman Vladmir Vatutin

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