FORUM

The Cocked Hat

from J. B. Parker

1. INTRODUCTION. The cocked hat problem is a difficult one and though Captain Cotter in a recent article,[1] has gone a long way towards clarifying it, I believe that the fundamentals of the subject are more difficult than this article suggests. A brief treatment of these, together with an indication of how they may be applied in practice, may be worth while.

2. ERRORS. Errors of three types may occur in practice.

2.1 *Systematic errors*. These errors affect all observations, and are constant in sense and magnitude. If known, they may be removed.

2.2 *Random errors*. These are chance errors arising from a number of different sources (e.g. setting up, observations and plotting). Their magnitude and sense varies from one observation to the next. Experience in a wide number of fields, including that of astronomical navigation does not conflict with the assumption that they follow a Gaussian distribution reasonably well. Their most important properties are:

2.2.1 In the absence of systematic error, errors in either sense are equally likely.

2.2.2 Large errors are less likely than small ones.

2.2.3 The error pattern can be characterized by a single quantity σ, called the standard deviation (or equivalently, by d, the 50 per cent error, where $d=2\sigma/3$ approximately).

2.3 *Blunders*. These are the 'stupid mistakes' referred to by Captain Cotter.

3. THE COCKED HAT. Three advantages of taking three position lines rather than two are:

3.1 The existence of a single blunder can be detected (by the size of the cocked hat). This rests on the reasonable assumption that the chance of blundering more than once is negligibly small.

3.2 If only systematic errors are present, a good fix may yet be obtained if the geometry is reasonable (e.g. Fig. 5 of Ref. 1).

3.3 If the errors are wholly random, the resulting fix (the most probable position) is slightly more accurate (i.e. has a smaller zone of error).

It is doubtful whether 3.2 and 3.3 justify the extra labour involved in taking a third position line. 3.2 assumes that random errors are non-existent, which is contrary to fact. In general, systematic errors are less than random errors, otherwise they would have been spotted and allowed for.

A theory of the cocked hat taking into account an unknown systematic error, a roughly known random error, and the (small) possibility of blunder fails on the grounds that there is insufficient information to assess the most likely ship's position. This is reasonable on intuitive grounds. Two pieces of information are required for position, one to give information about the systematic error,

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leaving none over for the random component. With four or more position lines a sound theory could be constructed, but it would be very difficult to base practical recommendations in it, save in degenerate cases.

For this reason, what theory there is[2] assumes that errors are random ones, with a known 50 per cent error *d*, but that blunders may occur. In defence of these assumptions we argue first that, if systematic errors are present, they will be small (and therefore compoundable with random errors), a good navigator having used his experience to assess them; secondly that the navigator has, from past experience, at any rate a rough knowledge of what *d* is.

4. The object of the theory. Theory can answer the following questions:

4.1 How big does the cocked hat have to be before the navigator rejects it as subject to blunder?

4.2 If the cocked hat passes the blunder criterion 4.1, what is the best position to choose for the fix?

4.3 What is the accuracy of the position obtained at 4.2?

Such a theory has been given for the astronomical position line case.[2] It would require trivial modification to deal with bearings.

5. Further considerations. Before illustrating section 4 by means of an example, some of the points raised by Captain Cotter (in the order in which they appear in his paper) are considered.

5.1 Captain Cotter (*loc. cit.*, p. 224) obviously respects the third group of navigators most, and I agree, arguing only that groups 3 and 4 should be combined! But what is the size of the group 3 navigator's circle? Is it constant, depending on past experience, or is it a function of the size of the cocked hat? Theory can help to settle this point.

5.2 'The authors of the *Admiralty Navigation Manual* . . . state that the chance of falling outside the cocked hat is one in four' (*loc. cit.*, p. 227). Great care must be exercised here. What is true, as Captain Cotter proves, is that, if three position lines are taken (no systematic error being present) then on three occasions out of every four the resulting cocked hat will not include the true position. But it is false to argue backwards 'Given a cocked hat, there is only a one in four chance of the true position lying inside'. By inspection of Fig. 1, it is clear that there is a much larger proportion of small cocked hats which exclude the true position T than large cocked hats. *The error zone associated with Fix F is determined not by the size of the cocked hat, but by the quantity d (or σ).* It is certainly true, *on average* (averaging over both large and small cocked hats) that the true position lies inside only once in four times, but this proportion is more favourable for large cocked hats and less favourable for small ones. In the degenerate case, when all lines happen

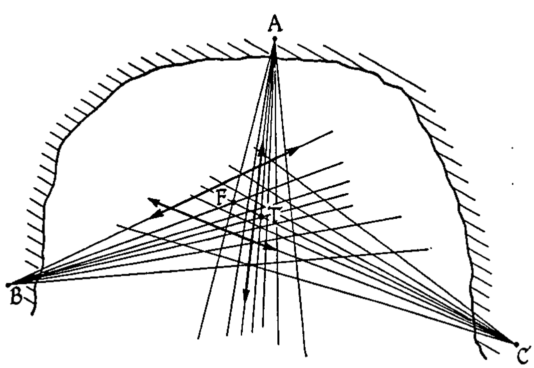

Fig. 1. Cocked hat for equal bearing errors from A, B and C.

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to pass through a point, the chance of the true position lying inside the cocked hat is zero.

5.3 The idea of a region of uncertainty (p. 229) is a good one but the choice of a six-pointed star to show it has a number of drawbacks:

5.3.1 What is the 'estimated error' to be used in the construction? A 50 per cent error? A 95 per cent error?

5.3.2 It is not exact. In Fig. 2 the point X is more probable than the point Y, assuming all errors are random ones.

5.3.3 It is relatively awkward to construct.

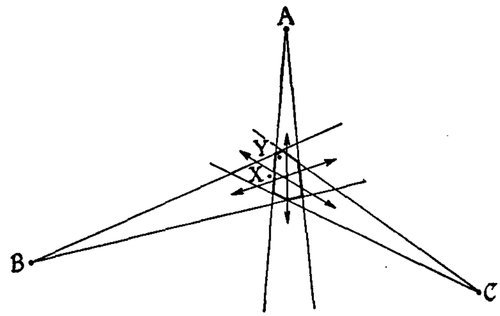

Fig. 2. Six-pointed star.

The theoretical error zones are closed curves surrounding the most probable position, each curve corresponding to a given probability that the true position lies inside it. In the case where the bands of error are parallel (i.e. A, B and C are a long way off) these curves are ellipses but their construction is awkward. The better the cocked hat geometry the nearer are these ellipses to circles, and it can be shown[2] that a circle, whose centre is the incentre of the triangle (a good, easily estimated approximation to the most probable position) and whose radius is $2\frac{1}{2}d$, will contain the true position on roughly 95 per cent of occasions. This is a good deal easier to draw than a hexagonal star, and incidentally provides the answer to the question posed in para. 5.1 above.

6. Example. Consider a case where three bearings (50 per cent error = 2 degrees, estimated) are taken on objects A, B and C distant 10, 12 and 15 miles (Fig. 3). The three bearings form a cocked hat. The linear 50 per cent errors are $2/60 \times 10 = \frac{1}{3}, \frac{2}{5}$ and $\frac{1}{2}$ mile respectively. For simplicity we assume that these are all $\frac{1}{2}$ mile, the least favourable case. Since the linear 50 per cent errors are roughly equal, the incentre is taken as the fix (Table 1 of Ref. 2, p. 248) provided the cocked hat passes the blunder criterion. This criterion (Table 1 of Ref. 2) states that the inradius (the perpendicular distance of the incentre from any of the three sides of the cocked hat) must be less than the 50 per cent linear error ($\frac{1}{2}$ mile) if the cocked hat is to be passed blunder free. If the

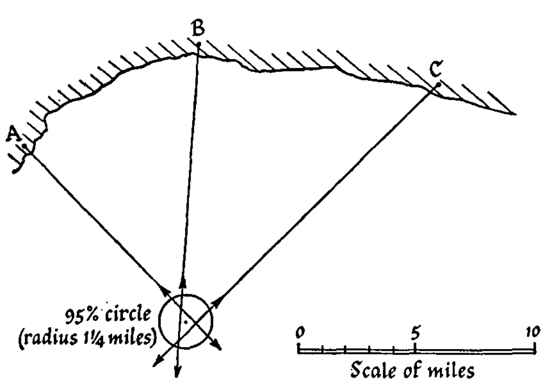

Fig. 3. Evaluation of cocked hat. Bearing errors: 2° (50 per cent error) Inradius = $\frac{1}{4}$ mile (approx). ∴ cocked hat deemed good.

inradius is between $\frac{1}{2}$ and 1 mile the cocked hat is suspect and if greater than 1 mile it should be rejected (in which case the navigator needs further data before he can find his position). Assuming that the cocked hat passes the test, we may assert with 95 per cent confidence that the true position lies in a circle, centred at the incentre, whose radius is $1\frac{1}{4}$ miles. This is irrespective of the actual size of the cocked hat.

7. Summary. Three pieces of information do not throw a great deal of light on position when both systematic and random errors, and possibly blunders as well, may occur. Nevertheless the *size* of the cocked hat can be used as a measure of consistency, and the incentre (a visual estimate requiring no construction, is sufficient) may be taken as the most probable position. A 95 per cent circle may be constructed, and plots started from the most dangerous point on the perimeter (Ref. 1, p. 224) if required.

The *Admiralty Navigation Manual* theorem quoted in para. 5.2, while theoretically correct, is practically of little consequence since its converse for any particular realization of three position lines cannot be used. It is therefore misleading and could well be assigned to the archives!

REFERENCES

1 Cotter, C. H. (1961). The cocked hat. This *Journal*, 14, 223.

2 Parker, J. B. (1952). The treatment of simultaneous position data in the air. This *Journal*, 5, 235.

The Polaroid Procedure for Photographing Radar Screens

from Captain A. Wepster

(*Holland–America Line*)

In the July *Journal* (14, 362) Klerk and Steensma describe a new and ingenious method of radar plotting. The following comment, sent in at the invitation of the editor of the *Journal*, follows closely my remarks on a similar article in the Dutch review *De Zee* (December 1960).

Careful study and consideration of this article and the original photographs that go with it have confirmed my opinion that the reflex plotter is still the most desirable and appropriate instrumental aid to carry out a radar plot. The authors mention five definite advantages in favour of this plotting method. The only objection they mention is eye strain and consequent fatigue. The plotting difficulties mentioned by them for the Strait of Dover will be dealt with later. The photographic plotting method does not solve these difficulties either.

The most obvious disadvantages of the proposed plotting method can be summed up as follows:

(1) *Scale reduction*

for a 12-in. radar display 1:4·3
for a 16-in. radar display 1:5·7

This scale reduction can under certain circumstances have a detrimental effect

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