

**Compactness-type problems in
topological vector spaces.
A treatment mainly from the viewpoint
of non-standard analysis**

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This thesis derives from an attempt to apply the techniques of non-standard analysis to problems in the theory of topological vector spaces; in particular, to compactness-type problems in these spaces. However, because these methods are not always advantageous the thesis divides into two parts: Chapters 1 to 4 and Chapters 5 and 6. The initial chapters are written essentially from the viewpoint of non-standard analysis whilst the later work employs only standard techniques.

Chapter 1 is entirely expository. In Chapter 2 the basic non-standard concepts which we find useful in functional analysis are introduced and a number of preliminary theorems are established including a non-standard version of Helly's Theorem. Non-standard proofs are then given for the main compactness results. We continue in Chapter 3 to consider a class of generalizations of weak compactness for subsets of locally convex spaces. If we replace weak compactness by instances of these generalizations we can drop the completeness assumption from the statement of many theorems (in particular, we prove a generalized version of Eberlein's classical theorem). We also consider generalizations of semi-reflexivity and reflexivity and characterize these properties in terms of these new notions as well as in terms of known concepts. Chapter 4 depends very much on the ideas of the preceding chapter. It deals with continuous linear maps

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between topological vector spaces and in it several theorems of Grothendieck and Ringrose are re-proved and generalized.

The main purpose of Chapter 5 is to show that if X is a smooth Banach space with a certain property, then its conjugate space X' is isomorphic to a rotund space. This result clarifies an observation of Day. Finally in Chapter 6 some problems related to almost reflexivity are considered.

References

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