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BOOK REVIEWS—COMPTES RENDUS CRITIQUES

Géométrie Intégrale, by M. I. Stoka. 64 pages. Mémorial des Sciences Mathématiques, Gauthier-Villars, Paris, 1968. 29 F.

Integral geometry has its origin in the theory of geometrical probabilities known as the problems of M. W. Crofton, and it was developed by E. Cartan and W. Blaschke by introducing in it the group theoretical measures together with the notion of kinetic density in convex bodies. Ch. I of this book is devoted to the definition of measurable groups and presents the necessary and sufficient condition for two parametric groups that belong to a Lie group to have the same integral invariants. Kinetic density is defined by means of differential forms. Ch. II is the application of this material to points and lines on a plane and shows several classical theorems, e.g., the measure of a set of lines which intersect a convex curve is equal to its length. These elementary topics are enhanced in the remaining Chapters to Euclidean and Riemannian spaces and reveal the integral formulas obtained by L. A. Santálo for a two-dimensional Riemannian space of negative constant curvature.

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Differentiable Manifolds, by S. T. Hu. x+177 pages. Holt, New York, 1969. U.S. $12.65.

The object of this book is to provide a text of algebraic topology for a one-semester undergraduate course and it can be used before and after the author's Homology Theory published by Holden–Day, 1966, to provide a one-year course. The introduction is devoted to the definitions of the fundamental concepts of differentiable manifolds. After endowing a paracompact Hausdorff space with differentiable structure, tangent and cotangent frames are defined in order to construct the vector bundles over manifolds. In Ch. II the notions of differential forms and of exterior differentiation are introduced and by means of these the De Rham cohomology group is defined. It is shown that this group is invariant under the induced homomorphisms. Ch. III is a short sketch of Riemannian geometry beginning from inner product, metric and connection. Geodesics and normal coordinate neighbourhoods are used to define the convex neighbourhoods of Riemannian manifolds and this is done by the exponential map of each tangent space to the other. The final chapter is devoted to the proof as well as the formulation of De Rham theorem which is and must be the center of interest to the readers.

Singular homology and cohomology are defined in the first two sections to the extent that they are needed for the purpose. Following the scheme due to Eckman the detailed proof of De Rham theorem is presented and other approaches to and slightly stronger statements of the theorem are formulated as exercises to readers.

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Universal Algebra, by G. Grätzer. xvi+368 pages. Van Nostrand, Princeton, N.J., Toronto, London, Melbourne, 1968. Cdn. $15.

This book is the first complete account of the present state of universal algebra and it will probably remain for a long time the standard textbook of it, both for those who want to get acquainted with the subject as for those who need it as a reference book for their own research. The amount of material contained in it is tremendous and the author deserves highest admiration for all the time and energy he has devoted to writing this book. We give a summary of its content:

Ch. 0. Basic Concepts. Set theory, preliminary definition of algebra, equivalence relations, mappings, partially ordered sets, ideals in semilattices.

Ch. 1. Subalgebras and Homomorphisms. Definition of universal algebra, subalgebra, homomorphism and congruence relation, polynomial symbol (term), the subalgebra lattice and the congruence lattice of an algebra, the homomorphism theorem and the isomorphism theorems.

Ch. 2. Partial Algebras. The notions of chapter 1 are extended to partial algebras and applied to prove the author's and E. T. Schmidt's characterization theorem for congruence lattices.

Ch. 3. Constructions of Algebras. Direct products, subdirect products, direct and inverse limits, reduced products, prime products.

Ch. 4. Free Algebras. Existence, G. Birkhoff's characterization of equational classes, consistency, equational completeness, identities in finite algebras, free algebras generated by partial algebras, free products, word problem.

Ch. 5. Independence. Independence and bases of free algebras, invariants in finite algebras, generalizations.

Ch. 6. Elements of Model Theory. Construction of first order logic, satisfiability, elementary equivalence and elementary extensions, prime products, axiomatic classes.

Ch. 7. Elementary Properties of Algebraic Constructions. Sentences preserved under formation of subalgebras, extensions, chain unions, direct products and subdirect products.

Ch. 8. Free Σ-Structures. The author's theory of free structures over a first order axiom system.