

## CORONAL INSTABILITIES

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**ABSTRACT.** The interest in the stability of coronal structures derives from their observed lifetime (much longer than the relevant hydromagnetic timescale) coupled with their active behavior. This fact implies that these structures must be globally stable with respect to fast and destructive instabilities and, at the same time, must allow some local, non-disrupting, dissipative process to take place. In highly magnetized media as solar and stellar coronae a large number of plasma instabilities can occur. The present review will concentrate on those governed by the magnetohydrodynamic (MHD) equations with the inclusion of the effects of finite resistivity and viscosity and the use of an energy equation where radiation, mechanical heating and thermal conduction are considered.

### 1. INTRODUCTION.

The stability of the magnetic structures observed in the solar and stellar coronae has received a great deal of attention in the last decade, since observations have clearly shown that the magnetic field plays a fundamental role. In fact the geometric arrangement of coronal plasma is directly related to the structure of the magnetic field, which, in active regions, confines and collimates the emissive plasma in a myriad of loops and arcades (Vaiana and Rosner, 1978). These structures maintain their identity for a long time (in some examples more than a day) (Foukal, 1976, Krieger, 1977) and on the other hand appear to be the site of high energy active phenomena, like the presence of extended heating, the recurrence of flares, the appearance and disappearance of prominences, the occurrence of coronal mass ejections, etc.

The dynamical behavior of coronal structures is not surprising, the tendency of a magnetized plasma to be unstable being well known (Bateman, 1978). The nature of the instability depends on the features of the equilibrium configuration and on the boundary conditions the perturbations must satisfy. The challenge in the last years has been to understand precisely the link between the nature, and therefore the effects and the timescales, of the instability and the physical conditions of the structure in which the instability evolves. In this paper we will concentrate on phenomena governed by the fluid equations neglecting the influence both of the microinstabilities and of most kinetic effects on the macroinstabilities. Of course this influence is important and can be invoked to explain a number of observed features; however within the MHD approximation we have to deal with an already difficult problem and, at the same time, we are able to describe the properties of the most dangerous and energetic instabilities, such as the ideal magnetic instability, the resistive instability, the Kelvin-Helmholtz instability and the thermal instability.

We want to stress that each type of instability is driven by a different physical mechanism and therefore we can study separately the properties of each instability by considering conditions in which only one mechanism at the time is dominant. In reality this separation is quite artificial and the nature of the growing perturbation is governed by the timescales on which each driving mechanism acts. When the

timescales are comparable, the coupling between different types of instability produces the appearance of new features in the resulting growing perturbation.

We believe it is worth to outline here that the instabilities which will be reviewed in the present paper have been extensively studied for laboratory purposes with the conclusion that they are quite sensitive to the particular regime in which the plasma is found. The operation of transferring results obtained for one particular regime to different ones is, generally speaking, unsafe. This is the reason why for coronal applications many original calculations have been made and only the properties of these "coronal instabilities" will be discussed.

In the next section we will briefly summarize what is known on coronal instabilities and in the last section we will discuss the lines for future work.

## 2. NATURE OF THE INSTABILITIES.

We assume that the collisional magnetohydrodynamic theory is applicable, that the plasma radiates and is mechanically heated according to some known laws and that the only non-ideal effects which are important are finite constant and isotropic resistivity and shear viscosity and finite parallel thermal conduction. The dynamics of such a plasma is then described by the following set of equations:

$$\partial\rho/\partial t + \text{div}(\rho\mathbf{v}) = 0 \quad 1)$$

$$\rho(\partial/\partial t + \mathbf{v}\cdot\text{grad})\mathbf{v} = -\text{grad}p + (\text{curl}\mathbf{B})\times\mathbf{B}/4\pi - \rho\mathbf{g} + \nu\text{div grad}\mathbf{v} \quad 2)$$

$$(\partial/\partial t + \mathbf{v}\cdot\text{grad})(p/\rho^\gamma) = (\gamma-1)\rho^{1-\gamma}(E_H - E_R + \text{div}(\mathbf{b}\kappa\mathbf{b}\text{grad}T)) \quad 3)$$

$$\partial\mathbf{B}/\partial t = \text{curl}(\mathbf{v}\times\mathbf{B}) - (c^2/4\pi)\eta\text{div grad}\mathbf{B} \quad 4)$$

where  $\rho$  is the plasma density,  $p$  the pressure,  $\mathbf{v}$  and  $\mathbf{B}$  the velocity and magnetic field respectively,  $T$  the temperature,  $\gamma$  the specific heat ratio.  $E_R$  represents the losses per unit mass due to radiation,  $E_H$  the heating and  $\mathbf{b}\kappa\mathbf{b}$  the parallel to the magnetic field thermal conduction coefficient. Finally  $\eta$  is the resistivity and  $\nu$  the shear viscosity.

It is easy to identify in Eqs 1-4) the terms which can be responsible for the the onset of an instability.

The magnetic term in Eq 2) can drive an ideal magnetic instability due to the curvature of the magnetic lines. Generally speaking, all the cylindrical or toroidal pinches have the tendency to disrupt on the very short Alfvén time scale. In the laboratory, stabilization is achieved by a combination of strong axial applied fields and close-fitting concentric conductors. Such conditions are unrealistic in modelling coronal plasmas, and it has been necessary to perform completely new calculations to study under which conditions realistic equilibrium configurations, when the influence of the photospheric boundary is taken into account, can be stable.

The resistive term in Eq. 4) is generally very small with respect to the ideal convective term and therefore negligible everywhere except in the regions close to the zeroes of  $\text{curl}(\mathbf{v}\times\mathbf{B})$ . When these zeroes are present in the structure and the ideal instabilities either are stabilized or evolve on time scales longer than the Alfvén time, the resistive term leads to reconnection of the magnetic lines and consequent conversion of the magnetic energy into other forms of energy.

The  $\mathbf{v}\cdot\text{grad}\mathbf{v}$  term can originate a Kelvin-Helmholtz instability whose source lies in the energy stored in the kinetic energy of relative motion of the different layers when a sheared velocity field is present. The magnetic field line tension can in some cases play the role of the surface tension as stabilizing effect. On the other hand, in the presence of a magnetic field the possible interplay of magnetic and K.-H. instabilities can considerably change the nature of the growing perturbation with respect to the static or non-magnetized case.

The gravitational term can be responsible for the onset of Rayleigh-Taylor instability and Kruskal-Schwarzschild instability. We will not be concerned with these kinds of processes in this paper, since, at least in the solar case, the gravitational scale length is bigger than the typical height of the coronal structures.

The balance between gains ( $E_H$ ) and losses ( $E_R$ ) can also be destabilizing if its temperature and density dependence are appropriate, namely when  $E_H - E_R$  is a monotonically increasing function of the temperature and therefore, for example, to a local decrease of the temperature corresponds a local decrease of  $E_H - E_R$ . A thermal instability develops when the thermal conduction, which tends to stabilize by smoothing the perturbation temperature gradient, is not effective in balancing the destabilizing term.

Let us now review some of the most interesting results obtained recently in the study of the instabilities mentioned above.

2a. Magnetic instabilities. As far as the ideal magnetic instabilities are concerned one has to choose a realistic equilibrium configuration and proper photospheric boundary conditions to mimic the effect of the much denser photosphere on the perturbations arising in corona. The coronal magnetic field is embedded in the high-inertia photospheric plasma, which effectively anchors the magnetic lines, producing the so-called line tying. The relatively long photospheric Alfvén time is taken to mean that the medium below the boundary, which models the transition region between photosphere and corona, cannot move on the relevant coronal time scale. This circumstance, in turn, ties the magnetic field in the photosphere against perpendicular motions and maintains the continuity of the component of the magnetic field perpendicular to the boundary at the ends (Van Hoven *et al.* 1981, Velli *et al.* 1988a). As far as the equilibrium is concerned both magnetic arcades and loops are usually described in cylindrical geometry with the  $z$ -axis parallel to the photospheric surface (assumed planar), when modelling an arcade, and perpendicular to it in the case of a loop. Various authors have shown that ideal stability can be achieved for these cylindrical equilibria when line-tying is taken into account.

In the case of an arcade stability is found when the field is force-free and all the magnetic lines reach the photosphere, that is if the axis of the cylinder is in the photosphere (Cargill *et al.* 1986). When the field is not force free, radial pressure gradients can drive an interchange-type instability which has been found using a ballooning transformation with a large wavenumber perpendicular to the magnetic field (Hood 1986).

A loop has been described as a cylindrical structure where currents are flowing, embedded in a much more extended potential field (Chiuderi *et al.* 1980). Radial pressure gradients can be present (Chiuderi and Einaudi 1981) as suggested by some of the Skylab EUV observations (Foukal 1978). The potential blanket and the pressure gradients improve the stability, which can be achieved only when line-tying is considered (Einaudi and Van Hoven 1981, 1983). Raadu (1972) was the first to suggest the importance of the line-tying, while Hood and Priest (1979, 1980) studied the constant shear field of Gold and Hoyle (1960). All these works have shown, using an energy principle, that a line-tied coronal loop goes unstable once a critical twist (or ratio of average poloidal to longitudinal flux) is exceeded. The critical twist depends on the length of the loop and on the kind of magnetic equilibrium adopted. Recently Velli *et al.* (1988b) have realized the importance of studying the growth rate and the spatial profile of the growing perturbations in order to determine the capacity of the small plasma resistivity to enhance instability producing reconnection of magnetic lines. As discussed above, resistive field reconnection involves a localized break-down of the frozen-in-field constraint of infinite conductivity MHD. In the infinite length or periodic cases this break-down occurs in narrow layers around the surfaces where  $mB_\theta/r + kB_z = 0$  (Coppi *et al.* 1966), where a  $\theta$  and  $z$  dependence  $\sim \exp(i(m\theta + kz))$  of the perturbed quantities has been assumed. Line-tying however excludes a simple harmonic dependence in the axial direction. It has been suggested (Mok and Van Hoven 1982) that in this case only  $\theta$ -independent ( $m=0$ ) perturbations are resistively unstable in configurations where the field component  $B_z$  vanishes at some point. Mok and Van Hoven (1982) and Velli and Hood (1988) have found that the instability is localized in vicinity of such points and that line-tying does not influence its properties with respect to the infinite or periodic case. However also  $\theta$ -dependent perturbations can be resistively unstable, as Velli *et al.* (1988b) have shown for the resistive kink ( $m=1$ ) by studying the behaviour of the operator  $B \cdot \text{grad}$  on the ideal eigenfunction at marginal stability. They have found that the derivative along the magnetic field vanishes on the same magnetic surface as in the infinite case but only at the center of the cylinder and when an inversion of the axial component of the field is present.

The properties of the  $m=1$  resistive instability have been extensively studied for laboratory purposes (Coppi *et al.* 1976, Ara *et al.* 1978, Finn and Manheimer 1982). We want to point out two results which

have important coronal implications. The first is that close to the ideal marginal stability the interplay of ideal and resistive effects markedly modifies the plasma behaviour (Batistoni *et al.* 1985) producing an instability which grows on a faster time scale than the well-known tearing mode. The second one concerns the effects of finite pressure which, even at very small values of the plasma  $\beta$ , can either transform the tearing mode into a more robust and faster growing interchange-type instability or can stabilize it, depending on the geometry of the equilibrium configuration (Valdettaro *et al.* 1988).

2b. Kelvin-Helmholtz instability. As already remarked, this instability arises when the different layers of a stratified heterogeneous fluid are in relative horizontal motion. Its properties have been widely studied in the hydrodynamic framework (Chandrasekhar 1961), whereas the magnetic effects have been scarcely considered, only in the ideal limit and adopting elementary magnetic configurations. There have been some efforts to study analytically the properties of resistive instabilities in presence of a velocity field aligned with the magnetic field (Hoffman 1975, Dobrowolny *et al.* 1983, Paris and Sy 1983). Recently Einaudi and Rubini (1986, 1988), using a numerical approach, have found that the nature of the instability in presence of velocity and magnetic sheared fields is determined by the ratio  $R$  between the magnetic and the velocity shear scales and the ratio  $V$  between the amplitude of the velocity field and the Alfvén velocity. Depending on the values of  $R$  and  $V$  an interplay between magnetic and fluid effects arises and dissipative processes as resistivity and viscosity can play different roles. In particular, resistivity can drive reconnection of magnetic lines even when the fluid effects are dominant on a time scale shorter than in the static case. Viscosity can enhance the growth rate of the perturbation, in contrast to its stabilizing effect on all other instabilities, when the sheared velocity field is subject to the so-called "cat's-eye" instability (Drazin and Reid 1981). In this case both reconnection of magnetic and stream lines occurs, leading to the formation of magnetic islands and small scale vortices. This instability has been used by Carbone *et al.* (1987) to explain some observed coronal features associated to solar surges.

2c. Thermal instability. The possible occurrence of thermal instabilities in coronal structures has been investigated by many authors since the pioneering paper by Field (1965), who found that isobaric perturbations grow provided the length of the isothermal structure  $L \geq 3 \times 10^{-11} T^{13/4} / P$  with  $P$  the pressure and  $T$  the temperature. The driving mechanism of such instability is the form of the solar radiation function at high temperatures. The non-linear evolution leads to the formation of a thin prominence-like condensation (Oran *et al.* 1982). The results obtained in the isothermal one-dimensional case cannot be applied to the corona because the presence of the transition region has a dramatic influence on the stability for three reasons. First of all  $L$  is not a free parameter as in the isothermal case but is related to the pressure and the apex temperature. Secondly, the typical radiative time scales in the transition region are much shorter than those in the corona. Finally, the static and stationary solution of Eq. 3) reproduces the observed temperature profile provided the heating dominates the radiation in the corona while it is negligible in the transition region (Chiuderi *et al.* 1981). As a result, the radiation drives an instability which mainly influences the transition region leading to the formation of low-lying cold loops, rather than prominence-like structures (Klimchuk *et al.* 1987 and references therein). The quasi-isothermal coronal part can be destabilized only if the heating has an appropriate pressure and temperature dependence and in this case a linear growing perturbation similar to the isothermal one has been found by Demoulin and Einaudi (1988).

The only magnetic effect considered in all these one-dimensional studies is the channelling of the heat flux along the magnetic lines. In a three-dimensional magnetic structure the thermal conduction can vanish in the same points as  $\text{curl}(\mathbf{v} \times \mathbf{B})$ , namely where the perturbation propagation vector is perpendicular to the field. When this is the case, the growing perturbation is localized in the regions around these points (Chiuderi and Van Hoven 1979, Van Hoven *et al.* 1987), and a local decrease of the temperature can lead to an increase of the value of the resistivity and to a consequent acceleration of the reconnection process (Steinolfson and Van Hoven 1984).

### 3. CONCLUSIONS.

The above discussion on the nature of the various instabilities which can arise in a coronal plasma has clearly shown that the understanding of the evolution of the coronal structures is far from being satisfactory. The knowledge of which physical mechanisms can influence the growth of an instability,

and therefore can determine the dynamical behaviour of the coronal plasmas, has improved consistently in the last few years. The actual non-linear development of the relevant instability, however, is in many cases unclear because only highly idealized computations have been performed. In particular the choice of the initial equilibrium configurations, in which curvature, flows and localized currents effects should be included, has been determined by computational convenience rather than by realistic considerations. Moreover the boundary conditions adopted in very few cases have properly modelled the photospheric line-tying or the presence of nearby regions. Finally the thermal properties of coronal plasmas should be considered more in detail.

The problem is that the observations are unable at the moment to provide direct details on the small scale structure of coronal features. Only a joint observational and theoretical effort can lead to a better insight on the dynamics of coronal structures through observations triggered by theoretical results on one hand and theoretical work establishing connections between unknown and observable quantities.

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