



# A Short Note on the Continuous Rokhlin Property and the Universal Coefficient Theorem in $E$ -theory

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*Abstract.* Let  $G$  be a metrizable compact group,  $A$  a separable  $C^*$ -algebra, and  $\alpha: G \rightarrow \text{Aut}(A)$  a strongly continuous action. Provided that  $\alpha$  satisfies the continuous Rokhlin property, we show that the property of satisfying the UCT in  $E$ -theory passes from  $A$  to the crossed product  $C^*$ -algebra  $A \rtimes_{\alpha} G$  and the fixed point algebra  $A^{\alpha}$ . This extends a similar result by Gardella for  $KK$ -theory in the case of unital  $C^*$ -algebras but with a shorter and less technical proof. For circle actions on separable unital  $C^*$ -algebras with the continuous Rokhlin property, we establish a connection between the  $E$ -theory equivalence class of  $A$  and that of its fixed point algebra  $A^{\alpha}$ .

## Introduction

Within the field of  $C^*$ -dynamical systems, the discovery and systematic study of various kinds of Rokhlin-type properties has recently become the driving force behind many new and interesting results. For examples of such instances, see [9, 10, 12, 13]. Although not under that name, the Rokhlin property played important roles already in [2, 6, 7]. Particularly notable for the purpose of this note is the investigation of compact group actions with the Rokhlin property and more specially circle actions with the Rokhlin property; see [3, 4, 8].

Despite the fact that many known results from the realm of finite group actions with the Rokhlin property carry over to the setting of compact groups, there remain some subtle difficulties concerning certain properties, such as the permanence of the UCT. It is known that in many cases, the UCT passes from a  $C^*$ -algebra to its crossed product associated with a Rokhlin action of a finite group; see [13, Theorem 3]. At present, it is unknown whether the UCT passes in all cases, or if such a permanence property holds for actions of compact groups. For example, when confronted with the problem of classifying Rokhlin actions of the circle on Kirchberg algebras by means of  $K$ -theory, this proves to be a rather annoying obstacle. To overcome this, Gardella introduced the continuous Rokhlin property for compact group actions on unital  $C^*$ -algebras in [4]. It turns out that this stronger version is compatible with the UCT for circle actions on nuclear  $C^*$ -algebras. In this short note, we extend the definition of the continuous Rokhlin property to cover the non-unital case. We then prove

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$E$ -theoretic versions of Gardella’s UCT preservation theorem for actions of all metrizable compact groups on separable  $C^*$ -algebras by using a somewhat more conceptual approach, enabling shorter and less technical proofs. We point out that, after the submission of this note, Gardella has updated the preprint [4] to a newer version, and he has independently obtained conclusions similar to the ones in this note. In this newer version, Gardella indicates that his proof for circle actions can be modified in order to show that the continuous Rokhlin property preserves the UCT in  $KK$ -theory for all compact metrizable group actions on separable, unital  $C^*$ -algebras. The UCT in  $KK$ -theory is, in general, indeed a stronger requirement for a  $C^*$ -algebra than UCT in  $E$ -theory; see [14].

We also show that for any strongly continuous  $C^*$ -dynamical system  $(A, \alpha, \mathbb{T})$  with the continuous Rokhlin property on a unital  $C^*$ -algebra,  $A$  is  $E$ -equivalent to  $A^\alpha \oplus SA^\alpha$ . This particularly yields an  $E$ -theoretic version of Gardella’s observation that  $K_*(A) \cong K_*(A^\alpha) \oplus K_{*+1}(A^\alpha)$ , whenever either  $\alpha: \mathbb{T} \rightarrow \text{Aut}(A)$  has the continuous Rokhlin property or  $K_*(A)$  is finitely generated and  $\alpha$  has the ordinary Rokhlin property; see [3, Theorem 5.5].

## 1 The Continuous Rokhlin Property

**Definition 1.1** Let  $A$  be a separable  $C^*$ -algebra. We denote the path algebra of  $A$  by

$$A_c = \mathcal{C}_b([1, \infty), A) / \mathcal{C}_0([1, \infty), A).$$

Let  $G$  be a metrizable, locally compact group let and  $\alpha: G \rightarrow \text{Aut}(A)$  be a strongly continuous action. Let  $\mathcal{C}_{b,\alpha}([1, \infty), A)$  be the  $C^*$ -subalgebra of  $\mathcal{C}_b([1, \infty), A)$  consisting of those points on which the induced action of  $\alpha$  is continuous. Observe that  $\mathcal{C}_0([1, \infty), A) \subset \mathcal{C}_{b,\alpha}([1, \infty), A)$ . We call

$$A_{c,\alpha} = \mathcal{C}_{b,\alpha}([1, \infty), A) / \mathcal{C}_0([1, \infty), A)$$

the continuous path algebra of  $A$  with respect to  $\alpha$ . Then  $A$  embeds as constant paths both into  $A_c$  and  $A_{c,\alpha}$ .

**Remark 1.2** In fact, the action  $\alpha$  on  $A$  clearly extends to a (possibly discontinuous) action  $\alpha_c$  on all of  $A_c$  given by  $\alpha_{c,g}([(b_t)_{t \geq 1}]) = [(\alpha_g(b_t))_{t \geq 1}]$  for all  $g \in G$  and  $[(b_t)_{t \geq 1}] \in A_c$ . It follows from [5, Lemma 1.8] that  $A_{c,\alpha}$  coincides with the  $C^*$ -subalgebra of  $A_c$  consisting of the elements  $x$  such that the map  $G \rightarrow A_c, g \mapsto \alpha_{c,g}(x)$  is continuous.

**Definition 1.3** As in the case of sequence algebras (see [11]), we define the central path algebra of  $A$  by

$$F_c(A) = A_c \cap A' / \text{Ann}(A, A_c).$$

One checks that both  $A_c \cap A'$  and  $\text{Ann}(A, A_c)$  are  $\alpha_c$ -invariant  $C^*$ -subalgebras of  $A_c$ , so there is an induced (possibly discontinuous) action  $\tilde{\alpha}_c$  on  $F_c(A)$  via

$$\tilde{\alpha}_{c,g}(x + \text{Ann}(A, A_c)) = \alpha_{c,g}(x) + \text{Ann}(A, A_c)$$

for all  $g \in G$ . The continuous central path algebra of  $A$  with respect to  $\alpha$  is defined as the  $C^*$ -subalgebra  $F_{c,\alpha}(A)$  of  $F_c(A)$  consisting of the elements  $x$  such that the map  $G \rightarrow F_c(A), g \mapsto \tilde{\alpha}_{c,g}(x)$  is continuous.

**Remark 1.4** The  $C^*$ -algebra  $F_c(A)$ , and hence also  $F_{c,\alpha}(A)$ , is unital. Indeed, if  $h \in A$  is a strictly positive contraction, then the class of the path  $b_t = h^{1/t}$  for  $t \geq 1$  defines a unit for  $F_c(A)$ .

**Remark 1.5** Let  $\alpha: G \rightarrow \text{Aut}(A)$  be a strictly continuous action of a locally compact metrizable group on a separable  $C^*$ -algebra. In view of Remark 1.2, we get a well-defined  $*$ -homomorphism

$$A \otimes_{\max} F_c(A) \rightarrow A_c, \quad a \otimes (x + \text{Ann}(A, A_c)) \mapsto ax,$$

which restricts to an equivariant  $*$ -homomorphism

$$(A \otimes F_{c,\alpha}(A), \alpha \otimes \tilde{\alpha}_c) \rightarrow (A_{c,\alpha}, \alpha_c).$$

The image of  $a \otimes \mathbf{1}$  under both these maps is  $a$ , for all  $a \in A$ .

**Notation 1.6** Given a compact group  $G$ , we denote by  $\sigma$  the canonical  $G$ -shift action on  $\mathcal{C}(G)$  given by  $\sigma_g(f)(h) = f(g^{-1}h)$  for all  $g, h \in G$  and  $f \in \mathcal{C}(G)$ .

**Definition 1.7** Let  $A$  be a separable  $C^*$ -algebra and let  $G$  be a compact group. Let  $\alpha: G \rightarrow \text{Aut}(A)$  be a strongly continuous action. We say that  $\alpha$  has the continuous Rokhlin property, if there exists a unital and equivariant  $*$ -homomorphism

$$\phi: (\mathcal{C}(G), \sigma) \rightarrow (F_{c,\alpha}(A), \tilde{\alpha}_c).$$

**Remark 1.8** Let  $(A, \alpha, G)$  be a strongly continuous  $C^*$ -dynamical system as above. In the special case that  $G = \mathbb{T}$ , the action  $\alpha$  has the continuous Rokhlin property if and only if there is a unitary  $u \in F_{c,\alpha}(A)$  with  $\tilde{\alpha}_{c,\xi}(u) = \xi \cdot u$  for all  $\xi \in \mathbb{T}$ . This follows from the observation that  $u = \phi(\text{id}_{\mathcal{C}(\mathbb{T})})$  yields such a unitary, and conversely every such unitary generates an equivariant copy of the dynamical system  $(\mathcal{C}(\mathbb{T}), \sigma)$ . In particular, this explains the analogy of the above definition to [4, Definition 3.1].

## 2 Preservation of the UCT in $E$ -theory

In this section, we show that the UCT in  $E$ -theory is preserved under forming crossed products or fixed point algebras associated to compact group actions with the continuous Rokhlin property.

**Definition 2.1** (compare to [5, Section 7]) A separable  $C^*$ -algebra  $A$  satisfies the UCT in  $E$ -theory, if for every separable  $C^*$ -algebra  $B$ , the natural map  $E_*(A, B) \rightarrow \text{Hom}(K_*(A), K_*(B))$  gives rise to a short exact sequence

$$0 \longrightarrow \text{Ext}_{\mathbb{Z}}^1(K_*(A), K_*(B)) \longrightarrow E_*(A, B) \longrightarrow \text{Hom}(K_*(A), K_*(B)) \longrightarrow 0.$$

As is the case for  $KK$ -theory, this is (a posteriori) the same as being  $E$ -equivalent to an abelian  $C^*$ -algebra. For nuclear  $C^*$ -algebras,  $E$ -theory coincides with  $KK$ -theory.

In particular, a nuclear C\*-algebra satisfies the UCT in KK-theory if and only if it satisfies the UCT in E-theory.

**Definition 2.2** Let  $A$  and  $B$  be separable C\*-algebras. We say that  $A$  E-dominates  $B$ , if there are  $x \in E(B, A)$  and  $y \in E(A, B)$  with  $x \otimes y = 1$  in  $E(B, B)$ . If  $\phi: B \rightarrow A$  is a \*-homomorphism such that  $E(\phi) \otimes y = 1$  in  $E(B, B)$  for some  $y \in E(A, B)$ , then we say that  $\phi$  induces E-dominance of  $A$  over  $B$ .

The following result is proved in [1, Corollary 23.10.8] for KK-theory, but an analogous proof yields the same for E-theory.

**Proposition 2.3** Let  $A$  and  $B$  be separable C\*-algebras. Assume that  $A$  E-dominates  $B$ . If  $A$  satisfies the UCT in E-theory, then so does  $B$ .

**Notation 2.4** Let  $(A, \alpha, G)$  and  $(B, \beta, G)$  be two strongly continuous C\*-dynamical systems. For an equivariant \*-homomorphism  $\psi: (A, \alpha) \rightarrow (B, \beta)$ , we denote by  $\tilde{\psi}: A \rtimes_{\alpha} G \rightarrow B \rtimes_{\beta} G$  the induced \*-homomorphism between the crossed products.

**Theorem 2.5** Let  $A$  be a separable C\*-algebra and let  $G$  be a metrizable compact group. Let  $\alpha: G \rightarrow \text{Aut}(A)$  be a strongly continuous action. Assume that  $\alpha$  has the continuous Rokhlin property. Then  $A$  E-dominates  $A \rtimes_{\alpha} G$ . In particular, the UCT passes from  $A$  to its crossed product with respect to  $\alpha$ .

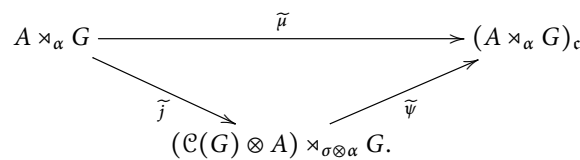
**Proof** Since  $\alpha$  has the continuous Rokhlin property, we can find an equivariant and unital \*-homomorphism

$$\phi: (\mathcal{C}(G), \sigma) \rightarrow (F_{c,\alpha}(A), \tilde{\alpha}_c).$$

In view of Remarks 1.5 and 1.2,  $\phi$  induces an equivariant and unital \*-homomorphism

$$\psi: (\mathcal{C}(G) \otimes A, \sigma \otimes \alpha) \rightarrow (A_{c,\alpha}, \alpha_c) \quad \text{via} \quad f \otimes a \mapsto \phi(f) \cdot a.$$

Consider the equivariant embedding  $j: (A, \alpha) \rightarrow (\mathcal{C}(G) \otimes A, \sigma \otimes \alpha)$  given by  $a \mapsto \mathbf{1} \otimes a$ . Then the composition  $\psi \circ j$  coincides with the canonical embedding  $\mu: A \rightarrow A_{c,\alpha}$ . Since both of these \*-homomorphisms are equivariant, we can form the induced maps between the crossed products, and compose them with the canonical \*-homomorphism from  $A_{c,\alpha} \rtimes_{\alpha_c} G$  to  $(A \rtimes_{\alpha} G)_c$  (which we will suppress from notation) to obtain the following commutative diagram:



Now the C\*-algebra  $(\mathcal{C}(G) \otimes A) \rtimes_{\sigma \otimes \alpha} G$  is isomorphic to  $A \otimes \mathcal{K}(L^2(G))$  because it is a well-known fact that  $\mathcal{C}(G) \rtimes_{\sigma} G \cong \mathcal{K}(L^2(G))$  and that  $\sigma \otimes \alpha$  is conjugate to  $\sigma \otimes \text{id}_A$ .

The above commutative diagram yields two E-theory elements  $x = E(\tilde{j}) \in E(A \rtimes_{\alpha} G, A)$  and  $y = E(\tilde{\psi}) \in E(A, A \rtimes_{\alpha} G)$  with  $x \otimes y = E(\tilde{\mu}) = 1$  in  $E(A \rtimes_{\alpha} G, A \rtimes_{\alpha} G)$ . In particular, we have verified Proposition 2.3 for  $A$  and  $A \rtimes_{\alpha} G$ . ■

**Theorem 2.6** *Let  $A$  be a separable  $C^*$ -algebra and  $G$  a metrizable compact group. Let  $\alpha: G \rightarrow \text{Aut}(A)$  be a strongly continuous action. Assume that  $\alpha$  has the continuous Rokhlin property. Then the canonical embedding  $\iota: A^\alpha \rightarrow A$  induces  $E$ -dominance of  $A$  over  $A^\alpha$ . Moreover, the UCT passes from  $A$  to its fixed point algebra with respect to  $\alpha$ .*

**Proof** As we have observed in the proof of Theorem 2.5, there exists an equivariant and unital  $*$ -homomorphism

$$\psi: (\mathcal{C}(G) \otimes A, \sigma \otimes \alpha) \rightarrow (A_{c,\alpha}, \alpha_c).$$

Consider the group action  $\sigma \otimes \alpha$  on  $\mathcal{C}(G) \otimes A \cong \mathcal{C}(G, A)$ , which is given (under this identification) by

$$(\sigma \otimes \alpha)_g(f)(h) = \alpha_g(f(g^{-1}h)) \quad \text{for all } g, h \in G \text{ and } f \in \mathcal{C}(G, A).$$

Now it is straightforward to check that a function  $f \in \mathcal{C}(G, A)$  is fixed under  $\sigma \otimes \alpha$  if and only if it is of the form  $f(h) = \alpha_h(a)$  for some  $a \in A$ . In particular, we have an isomorphism

$$\kappa: A \rightarrow (\mathcal{C}(G) \otimes A)^{\sigma \otimes \alpha} \quad \text{via} \quad \kappa(a)(h) = \alpha_h(a).$$

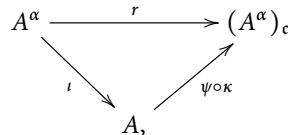
Recall the equivariant embedding  $j: A \rightarrow \mathcal{C}(G) \otimes A$  given by  $a \mapsto \mathbf{1} \otimes a$ . From the definition of  $\kappa$ , it is obvious that one has  $\kappa \circ \iota = j \circ \iota$ .

Observe that by compactness of  $G$ , the fixed point algebra of the path algebra  $(A_{c,\alpha})^{\alpha_c}$  coincides with the path algebra of the fixed point algebra  $(A^\alpha)_c$ . Indeed, if  $\nu$  is the normalized Haar measure on  $G$ , then we have

$$[(b_t)_{t \geq 1}] = \int_G \alpha_{c,g}([(b_t)_{t \geq 1}]) d\nu(g) = \left[ \left( \int_G \alpha_g(b_t) d\nu(g) \right)_{t \geq 1} \right] \in (A^\alpha)_c$$

for all  $[(b_t)_{t \geq 1}] \in (A_{c,\alpha})^{\alpha_c}$ .

Let  $r: A^\alpha \hookrightarrow (A^\alpha)_c$  denote the canonical inclusion. Combining all these observations, we obtain a commutative diagram



which yields two  $E$ -theory elements  $x = E(\iota) \in E(A^\alpha, A)$  and  $y = E(\psi \circ \kappa) \in E(A, A^\alpha)$  with  $x \otimes y = E(r) = 1$  in  $E(A^\alpha, A^\alpha)$ . In particular, we have verified that  $\iota$  induces  $E$ -dominance of  $A$  over  $A^\alpha$ . ■

### 3 Circle Actions and the Continuous Rokhlin Property

In this section, we examine the special case of circle actions more closely. This is demonstrated in the next theorem:

**Theorem 3.1** *Let  $A$  be a separable unital  $C^*$ -algebra and let  $\alpha: \mathbb{T} \rightarrow \text{Aut}(A)$  be a strongly continuous action. Assume that  $\alpha$  has the continuous Rokhlin property. Then  $A$  is  $E$ -equivalent to  $A^\alpha \oplus SA^\alpha$ .*

**Proof** Denote by  $\iota: A^\alpha \rightarrow A$  the canonical embedding. It follows from [3, Theorem 3.11] that there is an automorphism  $\theta \in \text{Aut}(A^\alpha)$  and an equivariant isomorphism

$$\phi: (A^\alpha \rtimes_\theta \mathbb{Z}, \tilde{\theta}) \rightarrow (A, \alpha) \quad \text{with} \quad \phi|_{A^\alpha} = \iota.$$

Identifying  $A$  with  $A^\alpha \rtimes_\theta \mathbb{Z}$  by the above isomorphism, we apply the Pimsner–Voiculescu exact sequence in  $KK$ -theory [1, Theorem 19.6.1]. For every separable  $C^*$ -algebra  $B$ , there is a six-term exact sequence of the form

$$\begin{array}{ccccc} KK(B, A^\alpha) & \xrightarrow{1-KK(\theta)} & KK(B, A^\alpha) & \xrightarrow{KK(\iota)} & KK(B, A) \\ \uparrow & & & & \downarrow \\ KK(B, SA) & \xleftarrow{KK(S\iota)} & KK(B, SA^\alpha) & \xleftarrow{1-KK(S\theta)} & KK(B, SA^\alpha). \end{array}$$

Moreover, one can see from the construction of this sequence in [1, Section 19] that the vertical map on the right is given by right-multiplication with an element  $z' \in KK(A, SA^\alpha)$ , which is independent of  $B$ . The same is true for the left vertical map. Now  $E$ -theory naturally factors through  $KK$ -theory, in a way that is compatible with addition and forming Kasparov products; see [1, Corollary 25.5.8]. Let  $z$  denote the  $E$ -theory element induced by  $z'$ . Then we also have a Pimsner–Voiculescu exact sequence in  $E$ -theory:

$$\begin{array}{ccccc} E(B, A^\alpha) & \xrightarrow{1-E(\theta)} & E(B, A^\alpha) & \xrightarrow{E(\iota)} & E(B, A) \\ \uparrow & & & & \downarrow z \\ E(B, SA) & \xleftarrow{E(S\iota)} & E(B, SA^\alpha) & \xleftarrow{1-E(S\theta)} & E(B, SA^\alpha). \end{array}$$

By [4, Proposition 3.11], it follows that  $\theta$  is asymptotically representable. In particular, this implies that  $E(\theta) = 1$  in  $E(A^\alpha, A^\alpha)$ . Thus we can extract a short exact sequence

$$0 \longrightarrow E(B, A^\alpha) \xrightarrow{E(\iota)} E(B, A) \xrightarrow{z} E(B, SA^\alpha) \longrightarrow 0.$$

By Theorem 2.6, there is some  $y \in E(A, A^\alpha)$  with  $E(\iota) \otimes y = 1$  in  $E(A^\alpha, A^\alpha)$ . It follows that the map

$$E(B, A) \rightarrow E(B, A^\alpha \oplus SA^\alpha), \quad x \mapsto x \otimes (y \oplus z)$$

is an isomorphism of abelian groups for all  $B$ . Using surjectivity of this map in the case of  $B = A^\alpha \oplus SA^\alpha$ , we find  $g \in E(A^\alpha \oplus SA^\alpha, A)$  with  $g \otimes (y \oplus z) = 1$  in  $E(A^\alpha \oplus SA^\alpha, A^\alpha \oplus SA^\alpha)$ . On the other hand, we have

$$\begin{aligned} ((y \oplus z) \otimes g) \otimes (y \oplus z) &= (y \oplus z) \otimes (g \otimes (y \oplus z)) \\ &= (y \oplus z) \otimes 1 = y \oplus z \\ &= 1 \otimes (y \oplus z). \end{aligned}$$

Hence, injectivity of the homomorphism

$$E(A, A) \rightarrow E(A, A^\alpha \oplus SA^\alpha), \quad x \mapsto x \otimes (y \oplus z)$$

implies that  $(y \oplus z) \otimes g = 1$  in  $E(A, A)$ . This shows that  $y \oplus z \in E(A, A^\alpha \oplus SA^\alpha)$  is an  $E$ -equivalence. ■

**Remark 3.2** Applying the ordinary (i.e.,  $K$ -theoretic) Pimsner–Voiculescu exact sequence as in the proof of Theorem 3.1, one recovers the isomorphisms  $K_i(A) \cong K_i(A^\alpha) \oplus K_{i+1}(A^\alpha)$  for  $i = 0, 1$ . Moreover, in the case  $i = 0$ , one can see that this isomorphism carries  $[\mathbf{1}_A]_0 \in K_0(A)$  to the pair  $([\mathbf{1}_{A^\alpha}]_0, 0) \in K_0(A^\alpha) \oplus K_1(A^\alpha)$ .

**Remark 3.3** We further remark that unitality of  $A$  is probably not necessary to obtain the result in Theorem 3.1. The only obstacle is to define a suitable notion of asymptotic representability for automorphisms on non-unital  $C^*$ -algebras, and extending the results [3, Theorem 3.11] and [4, Proposition 3.11] to the non-unital case. Considering that this has been done in the finite group case (see [12]), this should also be possible for circle actions in a similar fashion. The proposed definition [4, Definition 3.8] could potentially be the right candidate, but since it involves invariant unitaries coming from the multiplier algebra, it could be too strict a requirement to yield such a duality for non-unital  $C^*$ -algebras.

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