

Anelastic Tidal Dissipation in Multi-Layer Planets

Françoise Remus^{1,2,3}, Stéphane Mathis³, Jean-Paul Zahn¹,
and Valéry Lainey²

¹LUTH, Observatoire de Paris – CNRS – Université Paris Diderot,
5 place Jules Janssen, F-92195 Meudon Cedex, France
email: francoise.remus@obspm.fr, jean-paul.zahn@obspm.fr

²IMCCE, Observatoire de Paris – UMR 8028 du CNRS – Université Pierre et Marie Curie
77 avenue Denfert-Rochereau, F-75014 Paris, France
email: lainey@imcce.fr

³Laboratoire AIM Paris-Saclay, CEA/DSM – CNRS – Université Paris Diderot, IRFU/SAP
Centre de Saclay, F-91191 Gif-sur-Yvette, France
email: stephane.mathis@cea.fr

Abstract. Earth-like planets have anelastic mantles, whereas giant planets may have anelastic cores. As for the fluid parts, the tidal dissipation of these regions, gravitationally perturbed by a companion, highly depends on its internal friction and thus its internal structure. Therefore, modeling this kind of interaction presents a high interest to constrain planetary interiors, whose properties are still quite uncertain. Here, we examine the anelastic tidal dissipation in deep planetary interiors, in presence of a fluid envelope, and taking into account its dependence on the rheology.

Taking plausible values for the anelastic parameters, and discussing the frequency-dependence of the anelastic dissipation, we show how this mechanism may compete with the dissipation in fluid layers, when applied to Jupiter- and Saturn-like planets. We also discuss the case of the icy giants Uranus and Neptune. Finally, we show how the results may be implemented to describe the dynamical evolution of planetary systems.

Keywords. planets and satellites: general, (stars:) planetary systems

1. Introduction

Since 1995, a large number of extrasolar planets have been discovered, which display a wide range of physical parameters (Santos *et al.* 2007). The question quite naturally arose of their habitability. Determining factors are the presence of liquid water or a protective magnetic field, which are closely linked to the values of the rotational and orbital parameters of planetary systems. These elements strongly depend on the action of tides, since, once a planetary system is formed, its dynamical evolution is governed by gravitational interactions between its components, be it a star-planet or planet-satellite interaction. By converting kinetic energy into heat, the tides perturb their orbital and rotational properties. The rate at which the system evolves depends on the physical properties of tidal dissipation. Therefore, to understand the past history and predict the fate of a binary system, one has to identify the dissipative processes that achieve this conversion of energy. Studies have been carried out on tidal effects in fluid bodies such as stars and envelopes of giant planets (see, e.g., Ogilvie & Lin 2004; Remus *et al.* 2012b). However, the anelastic planetary regions also contribute to tidal dissipation, be it the mantles of Earth-like planets, or the cores of giant planets. The purpose of our study is

to determine the tidal dissipation in the anelastic central regions of giant planets, taking into account the presence of a fluid envelope.

2. Tidal dissipation of the core of a two-layer planet

2.1. Two-layer model

We will consider a two-body system where the main component A, rotating at the angular velocity Ω , has an anelastic icy/rocky core of complex shear modulus $\tilde{\mu}$, surrounded by a fluid envelope, such as an ocean, stretching out from core’s surface (of mean radius R_c) up to planet’s surface (of mean radius R_p). Both core and envelope are assumed homogeneous, with respective densities ρ_c and ρ_o . This model is represented on Fig. 1.

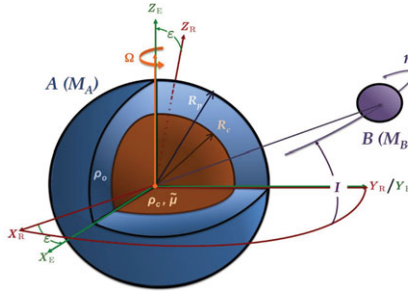


Figure 1. The system is composed by a two-layer main component A, with an homogeneous and incompressible solid core and an homogeneous viscous-free fluid envelope, and a point-mass perturber B orbiting around A. The spin axis of A, perpendicular to its equatorial plane (X_E, Y_E), is assumed to have an obliquity angle ϵ with respect to the total angular momentum of the system (in the direction of Z_R). This latter defines an inertial reference plane (X_R, Y_R), perpendicular to it. B is supposed to move on an elliptical orbit (of eccentricity e), inclined with respect to the inertial plane (by the inclination angle I).

2.2. Tidal dissipation of the core

The tidal perturbation exerted by B on the anelastic core of A results not only in its deformation (first treated by Dermott 1979 for a two-layer planet), but also in the dissipation of the tidal energy into heat, leading to a lag angle δ between the line of centers and the tidal bulge.

Acting as an overload on the core, the tidally deformed fluid shell, modifies both the tidal deformation and dissipation of the core. The quality factor Q_c , inversely proportional to core’s tidal dissipation, takes then a different form than in the fully-solid case:

$$Q_c^2 = 1 + \frac{9 \bar{\mu}_2^2}{4\alpha^2 A^2 D^2} \left[1 + \frac{(B + \bar{\mu}_1) \left(\frac{2\alpha C}{3} + \bar{\mu}_1 \right)}{\bar{\mu}_2^2} \right]^2 \tag{2.1}$$

with

$$\bar{\mu}_1 + i \bar{\mu}_2 = \frac{19 \tilde{\mu}}{2 \rho_c g_c R_c} . \tag{2.2}$$

This expression has been derived by Remus *et al.* (2012a); it depends on:

- the planet’s internal structure through the quantities α, A, B, C and D which are functions of the ratios of radii R_c/R_p and densities ρ_o/ρ_c ,
- the core’s rheological parameters through the effective shear modulus $\bar{\mu} = \bar{\mu}_1 + i \bar{\mu}_2$,

- the tidal forcing frequency σ through the effective shear modulus $\bar{\mu} \equiv \bar{\mu}(\sigma)$.

One may note that no assumption has been made on the rheology of the core, except that it is linear under the small tidal perturbations (i.e. the core's material obeys Hooke's law). Hence, it is valid for any linear rheological model.

Before evaluating the tidal dissipation for specific cases, let us introduce the effective tidal dissipation factor Q_{eff} defined by

$$Q_{\text{eff}} = \left(\frac{R_p}{R_c}\right)^5 \times \frac{k_2(R_p)}{k_2(R_c)} \times Q_c, \quad (2.3)$$

where $k_2(R_p)$ (resp. $k_2(R_c)$) is the potential Love number at the surface of the planet (resp. core). In the following applications, $k_2(R_c)$ is calculated from Eq. (79) of Remus *et al.* (2012a), and the value of $k_2(R_p)$ is taken from Gavrilov & Zharkov (1977) ($\{0.379, 0.341, 0.104, 0.127\}$, for Jupiter, Saturn, Uranus and Neptune in this order).

3. Application to giant planets

3.1. The case of the gas giants Jupiter and Saturn

Using astrometric data covering more than a century, Lainey *et al.* (2009, 2012) succeeded in determining from observations the effective tidal dissipation in Jupiter ($Q_{\text{Jupiter}} = (3.56 \pm 0.56) \times 10^4$) and Saturn ($Q_{\text{Saturn}} = (1.682 \pm 0.540) \times 10^3$) respectively. As mentioned in the corresponding reference, such a high dissipation in Saturn is about 10 times the usual value estimated from theoretical arguments, but it may account for the huge thermal emission of Enceladus and would be compatible with a new model of satellite formation in which the Saturnian satellites formed at the outer edge of the main rings (Charnoz *et al.* 2011). These values of tidal dissipation are higher than predicted by up-to-date models of tides in fluid planets. For instance Ogilvie & Lin (2004) studied the tidal dissipation in a rotating giant planet resulting from the excitation by the tidal potential of inertial waves in the convective upper part of the planet; they found that the quality factor depends strongly on the tidal frequency, displaying a high number of resonances, and that it averages around $Q_{\text{eff}} \approx 5 \times 10^5$. However the core of the planet was just invoked to provide a reflecting boundary. Similar results were obtained by Wu (2005), considering a coreless Jupiter.

In the two-layer model that we present here, the core plays an active role in the dissipation, through its viscoelasticity. Since the composition of giant planets is poorly constrained (Guillot 2005), we explore the effective tidal dissipation of Jupiter's and Saturn's core for a large range of viscoelastic parameters, adopting the Maxwell rheological model. As shown in Fig. 2, our model predicts quality factors that are compatible with those observed by Lainey *et al.* (2009, 2012), for plausible values of the rheological parameters (η, G) - we refer to Remus *et al.* (2012a), and references therein, for a discussion on the rheology of giant planets' cores. The other parameters (see the legend) are compatible with the internal structure models of Guillot (1999) for Jupiter and Hubbard *et al.* (2009) for Saturn. Moreover, in the frequency interval of Saturnian satellites, our model shows the same smooth dependence on tidal frequency than observed (see Fig. 3).

3.2. The case of the ice giants Uranus and Neptune

As in gas giants, the standard three-layer models for the interior structure of ice giants predict the presence of an anelastic rocky core (e.g., Hubbard *et al.* 1991; Podolak *et al.* 1995; Guillot 1999). Recent three-dimensional simulations of Neptune's and Uranus' dynamos predict that the intermediate "icy" layer, located between the rocky core and the

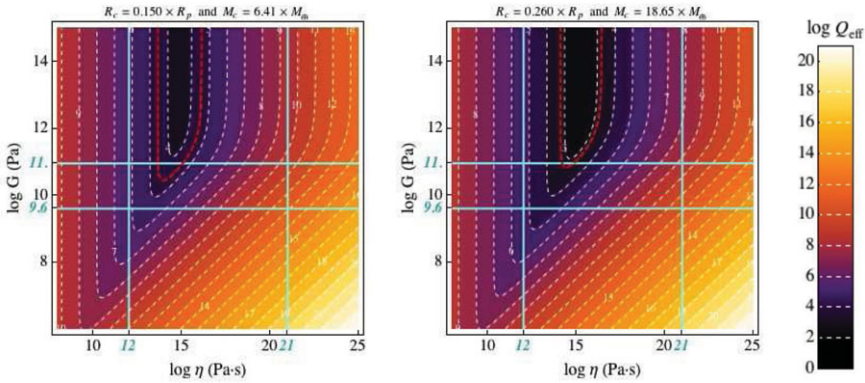


Figure 2. Dissipation quality factor Q_{eff} as a function of the viscoelastic parameters G and η , of a two-layer gas giant, using the Maxwell model. **Left:** for a Jupiter-like planet at the tidal frequency of Io. **Right:** for a Saturn-like planet at the tidal frequency of Enceladus. The red dashed lines indicate the value of $Q_{\text{eff}} = \{3.56 \times 10^4, 1.682 \times 10^3\}$ (for Jupiter and Saturn, respectively) determined by Lainey *et al.* (2009, 2012). The blue rectangle corresponds to the reference values taken by the viscoelastic parameters G and η for an unknown mixture of ice and silicates. We assume the values of $R_p = \{10.97, 9.14\} R_{\oplus}$, $M_p = \{317.8, 95.16\} M_{\oplus}$, $R_c = \{0.15, 0.26\} R_p$, $M_c = \{6.41, 18.65\} M_{\oplus}$, and $k_2(R_p) = \{0.379, 0.341\}$, for Jupiter and Saturn in this order.

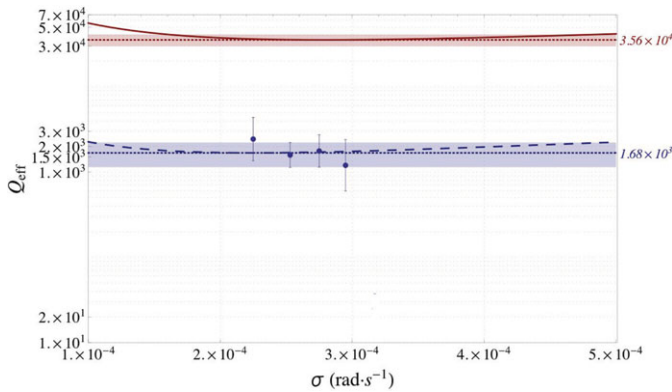


Figure 3. Dependence of the effective dissipation factor Q_{eff} on the tidal frequency σ for Jupiter-like (red solid line) and Saturn-like (blue dashed line) giant planets. The dotted lines (and their corresponding zone of uncertainty) indicate the mean value of $Q_{\text{eff}} = (3.56 \pm 0.56) \times 10^4$ (Jupiter) and $Q_{\text{eff}} = (1.682 \pm 0.540) \times 10^3$ (Saturn) determined by Lainey *et al.* (2009, 2012). The blue points correspond to the value of Q_{eff} of Saturn perturbed by the tide-raising satellites Enceladus, Thetys, Dione and Rhea (Lainey *et al.* 2012). We assume the same values of R_p , M_p , R_c , M_c , and $k_2(R_p)$ than in Fig. 2, and the value of $G = \{2.72, 10.51\} \times 10^{10}$ (Pa), and $\eta = \{8.65, 25.0\} \times 10^{13}$ (Pa · s⁻¹) for the viscoelastic parameters of Jupiter’s and Saturn’s core respectively.

convective atmosphere, is a stably stratified conductive fluid (Stanley & Bloxham 2004, 2006). From there, Redmer *et al.* (2011) studied the electric conductivity of warm dense water taking into account the phase diagram of water, and concluded that part of this shell is in the superionic state, i.e. a two-component system of both a conducting proton fluid and a crystalline oxygen solid, extending to about 0.42-0.56 of the planet radius. Thus, it seems reasonable to assume for our two-layer model that the solid central region extends from the rocky core surface up to somewhere in the superionic shell.

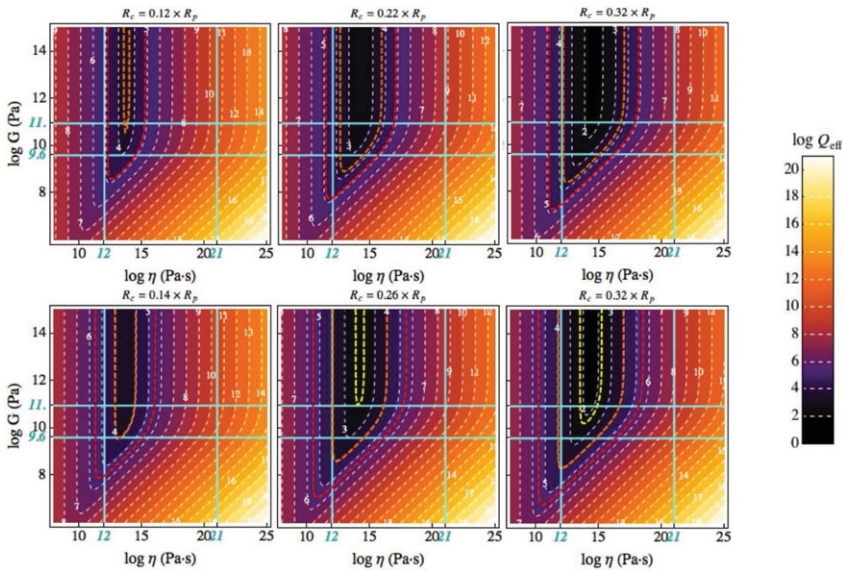


Figure 4. Dissipation quality factor Q_{eff} as a function of the viscoelastic parameters G and η , of a two-layer ice giant, using the Maxwell model. **Top:** for a Uranus-like planet at the tidal frequency of Miranda, with three different core sizes $R_c = \{0.12, 0.22, 0.32\} R_p$. **Bottom:** for a Neptune-like planet at the tidal frequency of Triton, with three different core sizes $R_c = \{0.14, 0.26, 0.32\} R_p$. The orange and red dashed lines indicate, respectively, the lowest and highest values of Q_{eff} from formation scenarios: $Q_{\text{eff}} = \{5, 72\} \times 10^3$ for Uranus (Gavrilov & Zharkov 1977; Goldreich & Soter 1966) and $Q_{\text{eff}} = \{0.9, 33\} \times 10^4$ for Neptune (Zhang & Hamilton 2008; Banfield & Murray 1992). The yellow dashed line indicates the value of $Q_{\text{eff}} = 1.7 \times 10^2$ from a study of Neptune's internal heat (Trafton 1974). The blue rectangle corresponds to the reference values taken by the viscoelastic parameters G and η for an unknown mixture of ice and silicates. We assume the values of $R_p = \{3.98, 3.87\} R_{\oplus}$, $M_p = \{14.24, 16.73\} M_{\oplus}$ and $k_2(R_p) = \{0.104, 0.127\}$.

We explore in Fig. 4 the tidal dissipation of Uranus' and Neptune's core for a large range of values of the viscoelastic parameters, considering the Maxwell rheological model, for different core sizes. The core mass is obtained by integration of the density profiles of Helled *et al.* (2011) up to a given core size. We compare our results to the most pessimistic and optimistic evolution scenarios, in terms of tidal dissipation. In 1966, Goldreich & Soter derived lower bounds to the effective dissipation factor Q_{eff} of the major planets, by analyzing the orbital tidal evolution of their nearest satellites. Calculating more realistic Love numbers $k_2(R_p)$ of these planets, Gavrilov & Zharkov (1977) obtained, for Uranus, a lower limit reduced by one order of magnitude. The case of Neptune's dissipation has been examined by Banfield & Murray (1992) from the study of the dynamical history of its inner satellites. With higher estimations of satellites densities, Zhang & Hamilton (2008) determined a lower bound of the tidal dissipation in Neptune reduced by less than half an order of magnitude.

Since much uncertainties remain on the formation of the Uranian and Neptunian systems, we explore in Fig. 5 the value of Q_{eff} that would be required in Uranus and Neptune to make evolve the semi-major axis of some of their satellites from an unknown initial value over any timescale up to the age of the Solar System (age_{SS}). We use the integrated formula Eq. (31) of Efroimsky & Lainey (2007). This give us lower bounds that are compatible with our results obtained from physical considerations (Fig. 4).

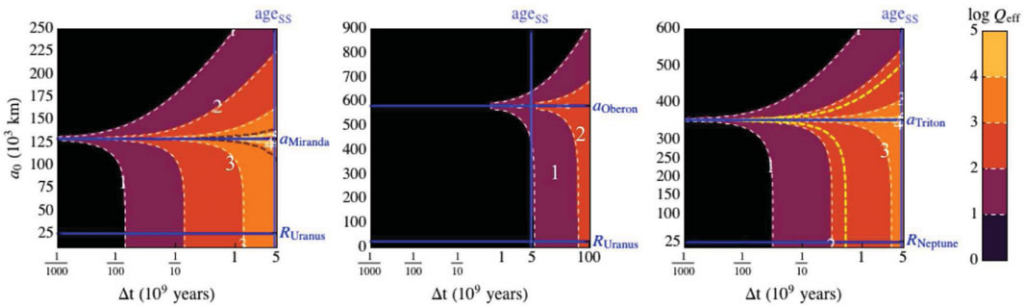


Figure 5. Amplitude of the effective dissipation factor Q_{eff} required by a planet for the evolution of the semi-major of one of its satellites in function of the initial position (a_0) and the timescale of evolution (Δt , in logarithmic scale). **From left to right:** Q_{eff} of Uranus/Neptune for the evolution of the semi-major axis of Miranda/Oberon/Triton's semi-major axis. The brown (resp. yellow) dashed line in the left (resp. right) panel indicates the lower bound predicted by Gavrilov & Zharkov (1977) (resp. Trafton 1974). We assume the same values of R_p and M_p than in Fig. 2. The satellites' masses are $m_{\text{sat}} = \{0.66, 30.1, 214\} \times 10^{20}$ (kg), and their semi-major axis are $a = \{129.39, 583.52, 354.76\} \times 10^3$ (km), for Miranda, Oberon and Triton in this order.

4. Dynamical evolution

Due to dissipation, the tidal torque has non-zero average over the orbit, and it induces an exchange of angular momentum between each component and the orbital motion. This exchange governs the evolution of the semi-major axis, the eccentricity, the inclination of the orbital plane, the obliquity and the angular velocity of each component (see, e.g., Mathis & Le Poncin-Lafitte 2009; Remus *et al.* 2012a). Depending on the initial conditions and on the planet/star mass ratio, the system evolves either to a stable state of minimum energy (with aligned spins, circular orbits and rotations of each body synchronized with the orbital motion) or the planet tends to spiral into the parent star.

5. Conclusion

Our evaluations reveal a much higher dissipation in the solid cores of planets than that found by Ogilvie & Lin (2004) for the fluid envelope of a planet possessing a small solid core. These results seem to be in good agreement with observed properties of Jupiter's and Saturn's system (Lainey *et al.* 2009, 2012). To explain the tidal dissipation observed in the gas giant planets of our Solar System, all processes have to be taken into account. In the case of the ice giants Uranus and Neptune, too much uncertainties remain on their internal structure to give an order of magnitude, other than a minimum value, of tidal dissipation in the solid regions, which constitutes a first step in the study of such planets.

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