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Foliations with radial Kupka set and pencils of Calabi–Yau hypersurfaces

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Dedicated to Professor César Camacho on the occasion of his 60th birthday

Abstract

We show that holomorphic singular codimension one foliations of the complex projective space with a Kupka singular set of radial type and verifying some global hypotheses have rational first integral. The generic elements of such pencils are Calabi–Yau.

1. Introduction

1.1 Let $\mathcal{F}$ be a codimension one holomorphic singular foliation of the complex projective space $\mathbb{P}^n$. The foliation is defined by a class of sections $\omega \in \mathrm{H}^0(\mathbb{P}^n, \Omega^1(d+2))$, where $d = d(\mathcal{F})$ is its degree, that is, the number of tangencies of leaves of $\mathcal{F}$ with a generic projective line. It is always possible to suppose that $\mathrm{codim}_{\mathbb{C}}(\omega)_0 \geqslant 2$ and we put $\mathrm{sing}(\mathcal{F}) := \{p \in \mathbb{P}^n \mid \omega(p) = 0\}$.

In this paper we consider dimension $n \geqslant 3$ and our main hypothesis is that there is a codimension two connected component of $\mathrm{sing}(\mathcal{F})$ which is a *Kupka set* $K(\mathcal{F})$, defined by

$$K(\mathcal{F}) := \{p \in \mathbb{P}^n \mid \omega(p) = 0, d\omega(p) \neq 0\}.$$

We consider *compact* Kupka sets of foliations of $\mathbb{P}^n$. From [GL90] and references therein, it is known that there is an open covering $\{U_i\}_i$ of $K(\mathcal{F})$ and a collection of submersions $\psi_i : U_i \to \mathbb{C}^2$ with $K(\mathcal{F}) \cap U_i = \psi_i^{-1}(0,0)$, as well as a holomorphic 1-form $\eta_{pq} := px\,dy - qy\,dx$, with isolated zero at $(0,0)$, such that $\mathcal{F}_{|U_i}$ is represented by $\psi_i^*(\eta_{pq})$, where p, q are integers with $1 \leqslant p < q, (p,q) = 1$ or $p = q = 1$. The *transversal type* of $K(\mathcal{F})$ is given by η_{pq} and the *radial type* corresponds to $p = q = 1$; we denote in this case $K(\mathcal{F}) = R(\mathcal{F})$.

It follows from the local product structure that $K(\mathcal{F})$ is smooth and that the other components of $\mathrm{sing}(\mathcal{F})$ do not intersect $K(\mathcal{F})$.

1.2 A fundamental fact about foliations of $\mathbb{P}^n$ with a Kupka set is that $\mathcal{F}$ has the global rational first integral if and only if $K(\mathcal{F})$ is a complete intersection (scheme theoretically) [CL94].

Furthermore, it is conjectured that $K(\mathcal{F})$ is always a complete intersection. There are already several partial positive answers to this conjecture. In fact, in [Cal99, Theorem 3.5] it is proven that a codimension one foliation of $\mathbb{P}^n$, $n \geqslant 3$, whose Kupka set is *not* of radial type, has a rational first integral. Also, for any transversal type, it is proven in [CAS94, Corollary 4.5] that the Kupka set of a foliation is a complete intersection, under the extra hypotheses $d(\mathcal{F}) \leqslant 2n$ and $n \geqslant 6$. At last, in [Bal95] and [Bal99] we find the hypothesis $n \geqslant 6$.

In order to state our result we recall some definitions.

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1.3 The *canonical bundle* of a k-dimensional algebraic variety S is defined by $K_S := \bigwedge^k TS^*$, where TS^* is its cotangent bundle. The triviality of K_S is a main feature of *Calabi–Yau varieties*. Smooth hypersurfaces of $\mathbb{P}^n$ with degree $n+1$ are the first examples of Calabi–Yau varieties (see [GHJ03] for the general theory of Calabi–Yau threefolds).

1.4 Let M be an n-dimensional projective smooth variety with a codimension one holomorphic singular foliation $\mathcal{F}$ with $\operatorname{codim}_{\mathbb{C}}\operatorname{sing}(\mathcal{F}) \geqslant 2$. Let $T_{\mathcal{F}}$ be the *tangent sheaf* of $\mathcal{F}$ and consider the line bundle $\bigwedge^{n-1} T^*_{\mathcal{F}}$ which is the dual of the top exterior power. We remark that local sections of $\bigwedge^{n-1} T^*_{\mathcal{F}}$ correspond to top degree holomorphic forms along the leaves of $\mathcal{F}$.

1.5 We recall that a line bundle $\mathcal{L}$ of M is called nef if $c_1(\mathcal{L}) \cdot C \geqslant 0$ for all curves C, where $c_1(\mathcal{L})$ means the first Chern class of $\mathcal{L}$.

We remark that the nefness condition is weaker than ampleness, which has already been used in the theory of holomorphic foliations (e.g. [GL90, Cal99]). For information on the geometrical meaning of Nefness of cotangent bundles of foliations we refer to [McQ00], [Bru04] and [BM01].

At last, we call *sectional* Baum–Bott *indices* the usual indices of singularities of foliations by curves [Bru04] induced in generic plane sections of a codimension one foliation.

Theorem. *Let $\mathcal{F}$ be a codimension one singular foliation of $\mathbb{P}^n$, for $n \geqslant 3$. Suppose that $d(\mathcal{F}) = 2n$ and that $\mathcal{F}$ has a compact connected Kupka set of radial transversal type $R(\mathcal{F})$.*

Denote by $S_{n-2}(\mathcal{F})$ its codimension two singular set and suppose that $S_{n-2}(\mathcal{F}) \setminus R(\mathcal{F})$ has non-positive sectional Baum–Bott indices.

Consider $\sigma : M \to \mathbb{P}^n$ the blowing up along $R(\mathcal{F})$ and $\widehat{\mathcal{F}}$ the transformed foliation of $\mathcal{F}$ by σ, with $\operatorname{codim}_{\mathbb{C}}\operatorname{sing}(\widehat{\mathcal{F}}) \geqslant 2$ *on M. Suppose that $\bigwedge^{n-1} T^*_{\widehat{\mathcal{F}}}$ is Nef.*

Then $S_{n-2}(\mathcal{F}) = R(\mathcal{F})$ and $\deg R(\mathcal{F}) = (n+1)^2$. Moreover, $\mathcal{F}$ is a pencil of hypersurfaces of degree $n+1$ which are smooth along the base locus $R(\mathcal{F})$.

Remark 1. When $\mathcal{F}$ is a generic pencil of hypersurfaces of degree $m \geqslant n+1$ (for instance, a Lefschetz pencil) then $\bigwedge^{n-1} T^*_{\widehat{\mathcal{F}}}$ is Nef.

Remark 2. The Theorem is motivated by [MS02, Theorem 2] applied to a degree 4 foliation $\mathcal{F}$ of $\mathbb{P}^2$. The hypotheses of that paper imply that the transformed foliation $\widehat{\mathcal{F}}$ under blow up of k radial points has Morse singularities and that $T^*_{\widehat{\mathcal{F}}}$ is Nef. The conclusion is that $\mathcal{F}$ is a pencil of cubics, smooth at the $k = 9$ base points.

On the other hand, there is an example [Lin02] of a one parameter family of foliations $\mathcal{F}_\lambda$ of degree 4 in the plane such that the singularities are either 12 radial points or singularities with local holomorphic first integral (and non-positive Baum–Bott indices). But only for a countable set of parameters are such foliations pencils of elliptic curves. In this example, $T^*_{\widehat{\mathcal{F}_\lambda}}$ are not Nef.

2. Preliminaries

2.1 An adjunction formula

A codimension one foliation $\mathcal{F}$ of an n-dimensional smooth projective variety M can be represented by a non-trivial integrable section of $\Omega^1_M \otimes N_{\mathcal{F}}$ where $N_{\mathcal{F}}$ is the normal bundle of $\mathcal{F}$.

For instance, in $\mathbb{P}^n$ we have $N_{\mathcal{F}} = \mathcal{O}_{\mathbb{P}^n}(d(\mathcal{F}) + 2)$, where $d(\mathcal{F})$ is its degree. From the exact sequence of sheaves

$$0 \to T_{\mathcal{F}} \to T_M \to \mathcal{J}_{\operatorname{sing}(\mathcal{F})} \cdot N_{\mathcal{F}} \to 0,$$

by dualising and taking the top exterior powers, we obtain the following isomorphism of line bundles of M:

$$\bigwedge^{n} TM^* = \bigwedge^{n-1} T^*_{\mathcal{F}} \otimes N^*_{\mathcal{F}}.$$

Hence we have

$$\bigwedge^{n-1} T^*_{\mathcal{F}} = K_M \otimes N_{\mathcal{F}}.$$

2.2 Blowing up of a radial Kupka set

Let $\sigma_{R(\mathcal{F})} : M \to \mathbb{P}^n$ be the blow up along the radial Kupka set of $\mathcal{F}$ (as remarked, $R(\mathcal{F})$ is a smooth variety).

Let $E = \sigma^{-1}_{R(\mathcal{F})}(R(\mathcal{F}))$ be the exceptional divisor, and denote by $\widehat{\mathcal{F}}$ the transformed foliation of $\mathcal{F}$ by $\sigma_{R(\mathcal{F})}$ (with singularities of codim $\geqslant 2$). We assert the following line bundle isomorphism:

$$N_{\widehat{\mathcal{F}}} = \sigma^*_{R(\mathcal{F})}(N_{\mathcal{F}}) \otimes \mathcal{O}_M(-2E).$$

In fact, on open sets $U = (x, y, z_1, \ldots, z_{n-2})$, with local submersion $\psi : U \to \mathbb{C}^2 = (x, y)$, $\mathcal{F}_{|\,U}$ can be induced by $\psi^*(\eta)$, where $\eta = (x + \text{h.o.t})\, dy - (y + \text{h.o.t})\, dx$. In local coordinates we have

$$\sigma_{R(\mathcal{F})}(x, t, z_1, \ldots, z_{n-2}) = (x, xt, z_1, \ldots, z_{n-2}) = (x, y, z_1, \ldots, z_{n-2}),$$

thus $\sigma^*_{R(\mathcal{F})}(\psi^*(\eta)) = x^2 \cdot \hat{\omega}$, where locally $E = \{x = 0\}$ and $\hat{\omega}$ induces $\widehat{\mathcal{F}}$. Therefore

$$\sigma^*_{R(\mathcal{F})}(N^*_{\mathcal{F}}) = N^*_{\widehat{\mathcal{F}}} \otimes \mathcal{O}_M(-2E)$$

and dualising we prove the assertion. On the other hand, as is well known,

$$K_M = \sigma^*_{R(\mathcal{F})}(K_{\mathbb{P}^n}) \otimes \mathcal{O}(E).$$

Combining these line bundle isomorphisms with our adjunction formula we obtain

$$\bigwedge^{n-1} T^*_{\widehat{\mathcal{F}}} = \sigma^*_{R(\mathcal{F})}\left(\bigwedge^{n-1} T^*_{\mathcal{F}}\right) \otimes \mathcal{O}_M(-E).$$

3. Proof of the Theorem

Lemma 3.1. *Let $\mathcal{F}$ be a codimension one foliation of $\mathbb{P}^n$ satisfying the hypotheses of the Theorem. Then* $\deg R(\mathcal{F}) = (n+1)^2$.

Proof. If $n > 3$ any extra component of $S_{n-2}(\mathcal{F})$ would intersect $R(\mathcal{F})$, contradicting the local product structure of $\mathcal{F}$ along $R(\mathcal{F})$. Then it suffices to consider two cases:

(1) $S_{n-2}(\mathcal{F}) = R(\mathcal{F})$ and

(2) $n = 3$ and $S_1(\mathcal{F}) \neq R(\mathcal{F})$.

Case (1). Let H_2 be a generic 2-plane in $\mathbb{P}^n$, intersecting $R(\mathcal{F})$ transversally. The tangencies of H_2 with $\mathcal{F}$, at non-singular points, give rise to Morse type singularities of $\mathcal{G} = i^*(\mathcal{F})$, where $i : H_2 \to \mathbb{P}^n$ is the inclusion. Then the singularities of $\mathcal{G}$ are the tangencies of $\mathcal{F}$ with H_2, denoted $\{q_1, \ldots, q_r\}$ and the points of $R(\mathcal{F}) \cap H_2 = \{p_1, \ldots, p_m\}$, where $m = \deg R(\mathcal{F})$, the p_j being radial singularities of $\mathcal{G}$. By the Baum–Bott formula:

$$\sum_i BB(\mathcal{G}, q_i) + \sum_j BB(\mathcal{G}, p_j) = (d(\mathcal{G}) + 2)^2 = (d(\mathcal{F}) + 2)^2.$$

On the other hand, the Morse singularities have zero Baum–Bott indices and for a radial singularity the index is 4. Therefore we obtain

$$4 \deg R(\mathcal{F}) = (d(\mathcal{F})+2)^2 = 4(n+1)^2.$$

Case (2). Let $S_1(\mathcal{F}) = R(\mathcal{F}) \cup X$, where $X \neq \emptyset$ has non-positive sectional Baum–Bott indices. Keeping the notation of Case (1), let $R(\mathcal{F}) \cap H_2 = \{p_1, \dots, p_m\}$, $X \cap H_2 = \{r_1, \dots, r_s\}$ and $q_1, \dots, q_r$ the Morse points of $\mathcal{G} = i^*(\mathcal{F})$. Again, the Baum–Bott formula gives

$$(d(\mathcal{F})+2)^2 = \sum_i BB(\mathcal{G}, q_i) + \sum_j BB(\mathcal{G}, p_j) + \sum_k BB(\mathcal{G}, r_k) \leqslant 4 \deg R(\mathcal{F}).$$

That is, $\deg R(\mathcal{F}) \geqslant 16$.

Let $\sigma_{R(\mathcal{F})} : M \to \mathbb{P}^3$ be the blowing up of the Kupka set and $\mathcal{H} = \sigma^*_{R(\mathcal{F})}(H_2)$ the total transform of H_2. Since $\bigwedge^2 T^*_{\widehat{\mathcal{F}}}$ is Nef, by a theorem of Kleiman [K66] we have

$$0 \leqslant c_1\Big(\bigwedge^2 T^*_{\widehat{\mathcal{F}}}\Big)^2 \cdot \mathcal{H}.$$

If $E = \sigma^{-1}_{R(\mathcal{F})}(R(\mathcal{F}))$, we have (§ 2.2):

$$\bigwedge^2 T^*_{\widehat{\mathcal{F}}} = \sigma^*_{R(\mathcal{F})}\Big(\bigwedge^2 T^*_{\mathcal{F}}\Big) \otimes \mathcal{O}_M(-E).$$

Hence

$$\begin{aligned} 0 &\leqslant \left[c_1\Big(\sigma^*_{R(\mathcal{F})}\Big(\bigwedge^2 T^*_{\mathcal{F}}\Big)\Big)^2 - 2 \cdot c_1\Big(\sigma^*_{R(\mathcal{F})}\Big(\bigwedge^{n-1} T^*_{\mathcal{F}}\Big)\Big) \cdot E + E^2\right] \cdot \mathcal{H} \\ &= c_1\Big(\bigwedge^2 T^*_{\mathcal{F}}\Big)^2 \cdot H_2 + E^2 \cdot \mathcal{H} = c_1\Big(\bigwedge^2 T^*_{\mathcal{F}}\Big)^2 \cdot H_2 - \deg R(\mathcal{F}), \end{aligned}$$

where we have used the projection formula [Ful98, Proposition 2.5]. By § 2.1

$$\bigwedge^2 T^*_{\mathcal{F}} = \mathcal{O}_{\mathbb{P}^3}(d(\mathcal{F})-2) = \mathcal{O}_{\mathbb{P}^3}(4)$$

and therefore $\deg R(\mathcal{F}) \leqslant 16$. □

Lemma 3.2. *Under the hypotheses of the Theorem, if S is a hypersurface containing $R(\mathcal{F})$ then $\deg S \geqslant n+1$; moreover, if $\deg S = n+1$ then S is smooth along $R(\mathcal{F})$.*

Proof. Suppose $R(\mathcal{F}) \subset S$ and consider a plane section $C_S := S \cap \Pi$, with Π intersecting $R(\mathcal{F})$ transversally. Lemma 3.1 gives $\deg R(\mathcal{F}) = (n+1)^2$ and C_S contains the points $p_1, \dots, p_{(n+1)^2}$ of $R(\mathcal{F}) \cap \Pi$.

Denote by $\nu_i = \nu(C_S, p_i) \geqslant 1$ the algebraic multiplicity of C_S at points of $R(\mathcal{F}) \cap \Pi$. Keeping the notation of Lemma 3.1, let $\widehat{\mathcal{F}}$ be the transformed foliation by $\sigma_{R(\mathcal{F})}$. Denote by $\widehat{C}_S$ the strict transform of C_s by $\sigma_{R(\mathcal{F})\,|\widehat{\Pi}} : \widehat{\Pi} \to \Pi$, and let $E \cap \widehat{\Pi} = \bigcup_{i=1} E_i$ denote the union of exceptional curves of $\widehat{\Pi}$. We have

$$\begin{aligned} c_1\Big(\bigwedge^{n-1} T^*_{\widehat{\mathcal{F}}}\Big) \cdot \widehat{C}_S &= c_1\Big(\sigma^*_{R(\mathcal{F})}\Big(\bigwedge^{n-1} T^*_{\mathcal{F}}\Big) \otimes \mathcal{O}_M(-E)\Big) \cdot \Big(\sigma_{R(\mathcal{F})}{}^*_{|\widehat{\Pi}}(C_S) - \sum_{i=1}^{(n+1)^2} \nu_i \cdot E_i\Big) \\ &= (n+1)\deg C_S - \sum_{i=1}^{(n+1)^2} \nu_i. \end{aligned}$$

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The Nefness of $\bigwedge^{n-1} T^*_{\widehat{\mathcal{F}}}$ implies that $\deg C_S \geqslant n+1$, with equality only if $\nu_i = 1$ for all i. $\square$

Note that by the same reasoning we can prove that there is no projective line $l \subset \Pi$ passing by more than $(n+1)$ points of $R(\mathcal{F}) \cap \Pi$, there is no conic of Π passing by more than $2(n+1)$ points of $R(\mathcal{F}) \cap \Pi$, etc.

We recall now some definitions that will be used in the proof of the next proposition (for more details see [OSS78, ch. II]). Let V be a 2-bundle on $\mathbb{P}^n$ with *even* first Chern class $c_1(V)$. Put

$$V_{\text{norm}} := V\left(-\frac{c_1(V)}{2}\right);$$

in this case $c_1(V_{\text{norm}}) = 0$. By definition V is *stable* if $H^0(\mathbb{P}^n, V_{\text{norm}}) = 0$; and V is *semistable* if

$$H^0(\mathbb{P}^n, V_{\text{norm}}) \neq 0 \quad \text{and} \quad H^0(\mathbb{P}^n, V_{\text{norm}}(-1)) = 0.$$

On the other hand, the *discriminant* of V is the integer number

$$\Delta(V) = c_1(V)^2 - 4c_2(V).$$

It is invariant with respect to tensoring with $\mathcal{O}(k)$; in particular $\Delta(V) = \Delta(V_{\text{norm}})$.

Proposition 3.3. *Under the hypotheses of the Theorem, $R(\mathcal{F})$ is a complete intersection of hypersurfaces of degree $n+1$.*

Proof. First, we show that $R(\mathcal{F})$ is contained in some hypersurface S with $\deg S = n+1$.

In [CAS94] it is proven that Kupka sets are subcanonically embedded. From Serre's construction the normal bundle of $R(\mathcal{F})$ extends as a rank two vector bundle V of $\mathbb{P}^n$, having a holomorphic section s with an exact sequence:

$$0 \to \mathcal{O}_{\mathbb{P}^n} \xrightarrow{\cdot s} V \to \mathcal{J}_{R(\mathcal{F})}(d(\mathcal{F})+2) \to 0, \tag{1}$$

where $\mathcal{J}_{R(\mathcal{F})}$ is the ideal sheaf associated to $R(\mathcal{F})$ and total Chern class

$$c(V) = 1 + (d(\mathcal{F})+2)\cdot \mathbf{h} + \deg R(\mathcal{F}) \cdot \mathbf{h}^2 \ \in \mathbb{Z}[\mathbf{h}]/\mathbf{h}^{n+1} \simeq H^*(\mathbb{P}^n, \mathbb{Z}).$$

Since $d(\mathcal{F}) = 2n$ and $\deg R(\mathcal{F}) = (n+1)^2$, we obtain

$$\Delta(V) = c_1(V)^2 - 4c_2(V) = 0.$$

According to [Bar77], V is non-stable, that is, $H^0(V_{\text{norm}}) \neq 0$.

Tensoring (1) by $\mathcal{O}_{\mathbb{P}^n}(-n-1)$ and taking the long exact sequence of cohomology we get

$$H^0(\mathcal{O}(-n-1)) = 0 \to H^0(V_{\text{norm}}) \simeq H^0(\mathcal{J}_{R(\mathcal{F})}(n+1)) \to 0 = H^1(\mathcal{O}(-n-1)),$$

from which follows the existence of S.

Secondly, let us prove that $R(\mathcal{F})$ is a complete intersection.

Together with Lemma 3.2 we have concluded that $n+1$ is the *minimum* degree of hypersurfaces containing $R(\mathcal{F})$. By [OSS78, Lemma 1.3.4] this is equivalent to the semistability of V.

Let τ be a non-trivial holomorphic section of V_{norm} and $(\tau)_0$ its scheme of zeroes. We assert that $(\tau)_0$ does *not* have codimension one. In fact, if $(\tau)_0$ is a hypersurface of degree k, then from τ we obtain a holomorphic section of $V_{\text{norm}}(-k)$, contradicting the semistability of V.

We consider now the exact sequence

$$0 \to \mathcal{O} \xrightarrow{\cdot \tau} V_{\text{norm}} \to \mathcal{J}_Z \to 0,$$

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where $Z = (\tau)_0$ (either Z is empty or has codimension two). If we suppose that Z is not empty, then $\deg Z = c_2(V_{\text{norm}})$. But $c_1(V_{\text{norm}}) = 0$ and $\Delta(V_{\text{norm}}) = \Delta(V) = 0$ imply $c_2(V_{\text{norm}}) = 0$. Then we conclude that $Z = \emptyset$. Hence V_{norm} is defined by an extension of line bundles on $\mathbb{P}^n$. Then it splits as a sum of line bundles ([OSS78, ch. I, § 2]); the same is true for V. As known, V splits if and only if $R(\mathcal{F})$ is a complete intersection. □

After Proposition 3.3, we conclude that $\mathcal{F}$ coincides with a pencil of degree $n+1$ hypersurfaces using [CL94, Theorem A].

For $n = 3$, applying the Bertini theorem we conclude that the curves of singularities $S_1(\mathcal{F})\backslash R(\mathcal{F})$ intersect the base locus $R(\mathcal{F})$, violating the local product structure of $R(\mathcal{F})$. At this point we see that Case (2) in the proof of Lemma 3.1 in fact does not exist. This completes the proof of the Theorem.

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