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Diffraction of planetary waves by an infinite strip

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In this paper we construct the Wiener-Hopf equation for diffraction of a planetary wave by an infinite strip and obtain its solution in the form of an integral representation. We also discuss the asymptotic character of the diffracted wave using the saddle point method.

1. Introduction

In what seems to be the first attempt to discuss diffraction of a planetary wave, Siew and Hurley [1] have recently studied diffraction of planetary waves by a half plane. This is a two part boundary value problem which is usually easy to handle. We discuss a similar three part boundary value problem, namely the diffraction of a planetary wave by an infinite strip. We follow the analysis of Siew and Hurley [1] and obtain the Wiener-Hopf equation relevant to our problem in the Fourier transformed plane. This equation can be split up very easily into two involved integral equations which for large λ take the form of two algebraic equations. These two are then solved and next the stream function is obtained taking the inverse Fourier transform. Finally the diffracted wave is obtained by considering the large R behaviour of the stream function.

2. Basic equation

We consider the planetary waves produced in a thin layer of liquid of depth h on the surface of a rotating sphere. The well known β plane approximation holds for the planetary waves, provided that the wave number

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n is large. In this case the stream function

(1) $$\psi = -\frac{g\zeta}{f}$$

with the velocity component

(2) $$u = \frac{\partial\psi}{\partial y}\ , \quad v = -\frac{\partial\psi}{\partial x}$$

satisfies the equation

(3) $$\left\{\frac{\partial}{\partial t}\left(\nabla^2 - a^2\right) + \beta\frac{\partial}{\partial x}\right\} = 0\ .$$

Here (x, y) are the rectangular coordinates, (u, v) are the corresponding velocity components, ζ the surface elevation which is assumed to be small, g the acceleration due to the gravity and $f = 2\Omega\sin\chi$ where χ is the latitude and Ω is the angular velocity of the sphere. Also $\nabla^2 = \frac{\partial^2}{x^2} + \frac{\partial^2}{y^2}$, $a^2 = \frac{f^2}{gh}$ and it has been assumed that

$$f = f_0 + \beta y\ .$$

We remark that $\frac{L^2 f^2}{gh}$ is small, where L is the wave length. Then

(4) $$\frac{\partial}{\partial t}\nabla^2\psi + \beta\frac{\partial\psi}{\partial x} = 0\ ,$$

in which case the planetary waves are referred to as being divergenceless. The condition that the plane wave

(5) $$\psi = e^{i(lx+my-\sigma t)}$$

should satisfy (3) yields a dispersion relation

(6) $$(l+\gamma)^2 + m^2 = \gamma^2 - a^2\ ,$$

where $\gamma = \beta/2\sigma$. In this case the wave number locus is a circle of radius $\sqrt{\gamma^2 - a^2}$ and centre $(-\gamma, 0)$ as shown in Figure 1 (on the opposite page).

3. Formulation of the problem

Let the impermeable strip be inclined at an angle α to the y-axis and let us take a new rectangular system (X, Y, Z) where X is the

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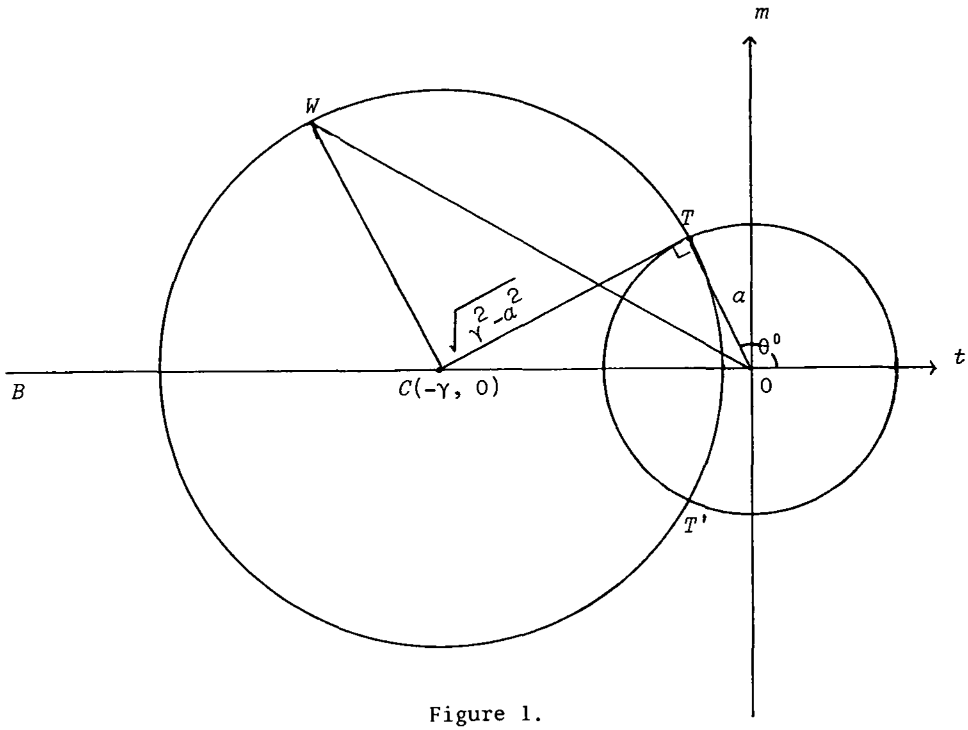

Figure 1.

perpendicular to the plate. In this coordinate system, let us suppose that the plate occupies the space

$$X = 0 , \quad 0 < Y < l , \quad -\infty < Z < \infty .$$

Then the equations corresponding to (2) and (3) take the form

(7) $$U = \frac{\partial\psi}{\partial Y} , \quad V = -\frac{\partial\psi}{\partial X}$$

and

$$\frac{\partial}{\partial t}\left[\frac{\partial^2\psi}{\partial X^2} + \frac{\partial^2\psi}{\partial Y^2} - \alpha^2\psi\right] + \beta\cos\alpha\,\frac{\partial\psi}{\partial X} - \beta\sin\alpha\,\frac{\partial\psi}{\partial Y} = 0 ,$$

where U and V are the velocity components in x and y directions respectively. We define the total stream functions as

(9) $$\psi = \{\psi_j+\psi(X, Y)\}e^{-i\sigma t} ,$$

where

(10) $$\psi_j = e^{ik\cos(\theta-\alpha)X+ik\sin(\theta-\alpha)Y}$$

represents the incident wave whose wave number vector has magnitude k and is inclined at an angle θ to the X-axis. Angle θ must satisfy the relation $\theta_0 < \theta \leq 2\pi - \theta_0$ where

$$\theta_0 = \pi - \text{arc sin}\left\{\frac{\sqrt{\gamma^2 - a^2}}{\gamma}\right\}$$

in order that the stream function should correspond to a planetary wave. In terms of X and Y coordinates (8) may be rewritten as

$$\frac{\partial^2\psi}{\partial X^2} + \frac{\partial^2\psi}{\partial Y^2} - \alpha^2\psi + 2i\gamma\cos\alpha\,\frac{\partial\psi}{\partial X} - 2i\gamma\sin\alpha\,\frac{\partial\psi}{\partial Y} = 0 . \tag{11}$$

The boundary condition that the total normal velocity must vanish on the strip implies

$$\frac{\partial\psi}{\partial Y}(0, Y) = -\frac{\partial\psi_i}{\partial Y} = -ik\sin(\theta-\alpha)e^{ik\sin(\theta-\alpha)Y} \quad (0 < Y < l) . \tag{12}$$

In addition the stream function must satisfy the Sommerfeld Radiation Condition at infinity.

4. The Wiener-Hopf equation

We define the Fourier transform of $\psi(X, Y)$ in Y as

$$\bar{\psi}(X, Y) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \psi(X, Y)e^{i\lambda Y}dY . \tag{13}$$

The Fourier transform of (11) reads

$$\frac{\partial^2\bar{\psi}}{\partial X^2} + 2i\gamma\cos\alpha\,\frac{\partial\bar{\psi}}{\partial X} - (\lambda^2+2\gamma\lambda\sin\alpha+a^2)\bar{\psi} = 0 . \tag{14}$$

The solution of (14) satisfying the radiation condition can be formally written as

$$\bar{\psi}(X, \lambda) = A(\lambda)\exp\{-i\gamma\cos\alpha X - \sqrt{(\lambda+\gamma\sin\alpha)^2 - b^2}\,|X|\} , \tag{15}$$

where $b^2 = \gamma^2 - a^2$ and use has been made of the continuity of $\frac{\partial\psi}{\partial Y}$ (and hence of $\bar{\psi}$) at $X = 0$.

We note that the branch points of $\sqrt{(\lambda+\gamma\sin\alpha)^2 - b^2} = \sqrt{(\lambda-\lambda_1)(\lambda-\lambda_2)}$ are $\lambda_1 = -\gamma\sin\alpha + b$, $\lambda_2 = -\gamma\sin\alpha - b$, and we shall choose the branch

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which is real as $\lambda \to \infty$. To calculate $A(\lambda)$, we define two functions $f(y)$ and $g(y)$ as

$$(16) \qquad \frac{\partial\psi}{\partial Y}(0, Y) = \begin{cases} f(Y) & \text{for } Y < 0 ,\ Y > l , \\ -ik\sin(\theta-\alpha)e^{ik\sin(\theta-\alpha)Y} & \text{for } 0 < Y < l , \end{cases}$$

and

$$(17) \qquad \frac{\partial\psi}{\partial Y}(0^+, Y) - \frac{\partial\psi}{\partial X}(0^-, Y) = \begin{cases} 0 & , Y < 0 ,\ Y > l , \\ g(Y) , & 0 < Y < l . \end{cases}$$

Transforming (16) and using (15) we get

$$(18) \quad i\bar{f}_-(\lambda) + \frac{ik\sin(\theta-\alpha)}{\sqrt{2\pi}[\lambda+k\sin(\theta-\alpha)]}\left\{1-e^{i[\lambda+k\sin(\theta-\alpha)l]}\right\} + ie^{i\lambda l}\bar{f}_+(\lambda) = \lambda A(\lambda) .$$

Similarly, transforming (17) and using (15) we obtain

$$(19) \qquad -2\sqrt{(\lambda+\gamma\sin\alpha)^2-b^2}\, A(\lambda) = \bar{g}(\lambda) ,$$

where subscript '+' denotes a function regular for

$$\mathrm{Im}\lambda > \varepsilon'\left\{\frac{\gamma_0\cos^2\theta}{\sqrt{\gamma_0^2\cos^2\theta-a^2}} - \cos\theta\right\} \quad \text{where} \quad \gamma = \frac{\beta}{2\sigma_0^2}\left(\sigma_0-i\varepsilon\right) = \gamma_0 - i\varepsilon'$$

and '-' denotes a function regular for $\mathrm{Im}\lambda < \mathrm{Im}\lambda_2$. (18) and (19) give

$$i\bar{f}_-(\lambda) + ie^{i\lambda l}\bar{f}_+(\lambda) + \frac{ik\sin(\theta-\alpha)}{\sqrt{2\pi}[\lambda+k\sin(\theta-\alpha)]}\left\{1-e^{i[\lambda+k\sin(\theta-\alpha)l]}\right\} =$$

$$= -\frac{\lambda\bar{g}(\lambda)}{2\sqrt{(\lambda+\gamma\sin\alpha)^2-b^2}} .$$

Above we have the required Wiener-Hopf equation to be solved for $\bar{f}_+(\lambda)$ and $\bar{f}_-(\lambda)$, the quantities needed to solve the problem, as is manifest from (15) and (18).

5. Method of solution

Let us start with the Wiener-Hopf equation

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$$i\mathcal{F}_-(\lambda) + \frac{ik\sin(\theta-\alpha)}{\sqrt{2\pi}[\lambda+k\sin(\theta-\alpha)]}\left[1-e^{i[\lambda+k\sin(\theta-\alpha)l]}\right] + ie^{i\lambda l}\mathcal{F}_+(\lambda) = -\frac{\lambda g(\lambda)}{2\sqrt{(\lambda-\lambda_1)(\lambda-\lambda_2)}} \tag{20}$$

where

$$\sqrt{(\lambda-\lambda_1)(\lambda-\lambda_2)} = \sqrt{(\lambda+\sin\alpha)^2-b^2} .$$

Multiplying (20) throughout by $\sqrt{\lambda-\lambda_1}\, e^{i\lambda l}$,

$$i\mathcal{F}_+(\lambda)\sqrt{\lambda-\lambda_1} + \frac{\lambda e^{i\lambda l}g(\lambda)}{2\sqrt{\lambda-\lambda_2}} + ie^{i\lambda l}\mathcal{F}_-(\lambda)\sqrt{\lambda-\lambda_1} = \frac{A'}{\sqrt{2\pi}}\sqrt{\lambda-\lambda_1}\,\frac{e^{ik\sin(\theta-\alpha)l}-e^{-i\lambda l}}{\lambda+k\sin(\theta-\alpha)} \tag{21}$$

where

$$A' = ik\sin(\theta-\alpha) . \tag{22}$$

Equation (21) may be rewritten as

$$i\mathcal{F}_+(\lambda)\sqrt{\lambda-\lambda_1} - \frac{A'}{\sqrt{2\pi}}\frac{e^{ik\sin(\theta-\alpha)l}\sqrt{\lambda-\lambda_1}}{\lambda+k\sin(\theta-\alpha)} + U_+(\lambda) + V_+(\lambda) = -\frac{\lambda e^{-i\lambda l}g(\lambda)}{2\sqrt{\lambda-\lambda_2}} - U_-(\lambda) - V_-(\lambda) \tag{23}$$

where

$$\begin{aligned} U_+(\lambda) + U_-(\lambda) &= ie^{-i\lambda l}\mathcal{F}_-(\lambda)\sqrt{\lambda-\lambda_1} , \\ V_+(\lambda) + V_-(\lambda) &= \frac{A'}{\sqrt{2\pi}}\frac{e^{-i\lambda l}\sqrt{\lambda-\lambda_1}}{\lambda+k\sin(\theta-\alpha)} . \end{aligned} \tag{24}$$

Similarly, rearrangement of (20) after multiplication by $\sqrt{\lambda-\lambda_2}$ gives

$$i\mathcal{F}_-(\lambda)\sqrt{\lambda-\lambda_2} + \frac{A'}{\sqrt{2\pi}}\left[\frac{\sqrt{\lambda-\lambda_2}-\sqrt{-\lambda_2-k\sin(\theta-\alpha)}}{\lambda+k\sin(\theta-\alpha)}\right] + R_-(\lambda) - S_-(\lambda) = -\frac{\lambda g(\lambda)}{2\sqrt{\lambda-\lambda_1}} - \frac{A'}{\sqrt{2\pi}}\frac{\sqrt{-\lambda_2-k\sin(\theta-\alpha)}}{\lambda+K\sin(\theta-\alpha)} + S_+(\lambda) - R_+(\lambda) \tag{25}$$

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where

$$R_+(\lambda) + R_-(\lambda) = ie^{i\lambda l}\mathcal{F}_+(\lambda)\sqrt{\lambda-\lambda_2} ,$$

(26)

$$S_+(\lambda) + S_-(\lambda) = \frac{A'}{\sqrt{2\pi}} \sqrt{\lambda-\lambda_2} \frac{e^{i[\lambda+k\sin(\theta-\alpha)l]}}{\lambda+k\sin(\theta-\alpha)} .$$

All the functions appearing on the left hand side of (23) and the right hand side of (25) are '+' functions while those on the other sides are '-' functions. The behaviour of the functions $\frac{\partial\psi}{\partial X}$ and $\frac{\partial\psi}{\partial Y}$ and application of the extended form of Liouvilles' Theorem shows that both sides of equations (23) and (25) are constants. Thus

$$(27) \qquad i\mathcal{F}_+(\lambda)\sqrt{\lambda-\lambda_1} - \frac{A'}{\sqrt{2\pi}} \sqrt{\lambda-\lambda_1} \frac{e^{ik\sin(\theta-\alpha)l}}{\lambda+k\sin(\theta-\alpha)} + U_+(\lambda) + V_+(\lambda) = C_1$$

and

$$(28) \quad i\mathcal{F}_-(\lambda)\sqrt{\lambda-\lambda_2} + \frac{A'}{\sqrt{2\pi}} \left[\frac{\sqrt{\lambda-\lambda_2}-\sqrt{-\lambda_2-k\sin(\theta-\alpha)}}{\lambda+k\sin(\theta-\alpha)}\right] + R_-(\lambda) - S_-(\lambda) = C_2 .$$

Using the asymptotic limits $\lambda \to \infty$ in the above equations, one concludes that the constant C_1 is equal to zero. Thus (27) becomes

$$(29) \qquad i\mathcal{F}_+(\lambda)\sqrt{\lambda-\lambda_1} - \frac{A'}{\sqrt{2\pi}} \sqrt{\lambda-\lambda_1} \frac{e^{ik\sin(\theta-\alpha)l}}{\lambda+k\sin(\theta-\alpha)} + U_+(\lambda) + V_+(\lambda) = 0 .$$

Calculating the asymptotic behaviours of $U_+(\lambda)$, and so forth, and putting them in (29) and (28) gives

$$(30) \quad i\mathcal{F}_+(\lambda)\sqrt{\lambda-\lambda_1} - \frac{A'}{\sqrt{2\pi}} \sqrt{\lambda-\lambda_1} \frac{e^{ik\sin(\theta-\alpha)l}}{\lambda+k\sin(\theta-\alpha)} - iT(\lambda)\mathcal{F}_-(\lambda_1) + \frac{A'}{\sqrt{2\pi}} R_1(\lambda) = 0$$

and

$$(31) \quad i\mathcal{F}_-(\lambda)\sqrt{\lambda-\lambda_2} + \frac{A'}{\sqrt{2\pi}} \frac{\sqrt{\lambda-\lambda_2}-\sqrt{-\lambda_2-k\sin(\theta-\alpha)}}{\lambda+k\sin(\theta-\alpha)} - iT'(-\lambda)\mathcal{F}_+(\lambda_2) + \frac{A'}{\sqrt{2\pi}} e^{ik\sin(\theta-\alpha)l} R_2(-\lambda) = C .$$

(30) and (31) are two algebraic equations which result in

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$$(32)\quad i\bar{f}_+(\lambda) = \frac{A'}{\sqrt{2\pi}}\frac{e^{ik\sin(\theta-\alpha)l}}{\lambda+k\sin(\theta-\alpha)} - \frac{R_1(\lambda)}{\sqrt{\lambda-\lambda_1}} +$$
$$+ \frac{T(\lambda)T\left(-\lambda_1\right)}{G\sqrt{\lambda-\lambda_1}}\left[\frac{\sqrt{\lambda_2-\lambda_1}e^{ik\sin(\theta-\alpha)l}}{\lambda_2+k\sin(\theta-\alpha)} - R_1\left(\lambda_2\right)\right] +$$
$$+ \frac{T(\lambda)\sqrt{\lambda_2-\lambda_1}}{G\sqrt{\lambda-\lambda_1}}\left[-\frac{\sqrt{\lambda_1-\lambda_2}-\sqrt{-\lambda_2-k\sin(\theta-\alpha)}}{\lambda_1+k\sin(\theta-\alpha)} - e^{ik\sin(\theta-\alpha)l}R_2\left(-\lambda_1\right)+C\right]$$

and

$$(33)\quad i\bar{f}_-(\lambda) = -\frac{1}{\lambda+k\sin(\theta-\alpha)}\left[1-\frac{\sqrt{-\lambda_2-k\sin(\theta-\alpha)}}{\lambda_1+k\sin(\theta-\alpha)}\right] - \frac{e^{ik\sin(\theta-\alpha)l}R_2(-\lambda)}{\sqrt{\lambda-\lambda_2}} +$$
$$+ \frac{T'(-\lambda)T\left(\lambda_2\right)}{G\sqrt{\lambda-\lambda_2}}\left[-\frac{\sqrt{\lambda_1-\lambda_2}-\sqrt{-\lambda_2-k\sin(\theta-\alpha)}}{\lambda_1+k\sin(\theta-\alpha)} - e^{ik\sin(\theta-\alpha)l}R_2\left(-\lambda_1\right)\right] +$$
$$+ \frac{T'(-\lambda)}{G}\frac{\sqrt{\left(\lambda_1-\lambda_2\right)\left(\lambda_2-\lambda_1\right)}}{\sqrt{\lambda-\lambda_2}}\left[\frac{e^{ik\sin(\theta-\alpha)l}}{\lambda_2+k\sin(\theta-\alpha)} - \frac{R_1\left(\lambda_2\right)}{\sqrt{\lambda_2-\lambda_1}}\right] +$$
$$+ \frac{C}{\sqrt{\lambda-\lambda_2}}\left[1+\frac{T'(-\lambda)T\left(\lambda_2\right)}{G}\right].$$

The results for $f_\pm(\lambda)$ in the above two equations, (32) and (33), are now substituted in (18) to obtain the quantity $A(\lambda)$. On account of (15), on taking inverse Fourier transforms, we obtain

$$(34)\quad \psi(X, Y) = \int\left[\frac{\sqrt{-\lambda_2-k\sin(\theta-\alpha)}}{\lambda\sqrt{\lambda-\lambda_2}[\lambda+k\sin(\theta-\alpha)]} - \frac{e^{ik\sin(\theta-\alpha)l}R_2(-\lambda)}{\lambda\sqrt{\lambda-\lambda_2}} + \frac{T'(-\lambda)T\left(\lambda_2\right)Q_1}{G\lambda\sqrt{\lambda-\lambda_2}}\right.$$
$$+ \frac{T'(-\lambda)}{G}\frac{\sqrt{\left(\lambda_1-\lambda_2\right)\left(\lambda_2-\lambda_1\right)}}{\lambda\sqrt{\lambda-\lambda_2}}Q_2 - \frac{e^{i\lambda l}R_1(\lambda)}{\lambda\sqrt{\lambda-\lambda_1}} +$$
$$+ \frac{e^{i\lambda l}T(\lambda)T'\left(-\lambda_1\right)\sqrt{\lambda_2-\lambda_1}Q_2}{G\lambda\sqrt{\lambda-\lambda_1}} + \frac{T(\lambda)\sqrt{\lambda_2-\lambda_1}Q_1}{G\lambda\sqrt{\lambda-\lambda_1}} +$$

$$+\frac{1}{A'}\left\{\frac{C_2}{\lambda\sqrt{\lambda-\lambda_1}\sqrt{\lambda-\lambda_2}G}\left[\sqrt{\lambda-\lambda_1}\left(G+T'(-\lambda)T(\lambda_2)\right)\right]+e^{i\lambda l}T(\lambda)\sqrt{\lambda_2-\lambda_1}\sqrt{\lambda-\lambda_2}\right\}\Bigg] \times$$

$$\times \exp\left\{-i\gamma\cos\alpha x-\sqrt{(\lambda+\gamma\sin\alpha)^2-b^2}|x|-i\lambda Y\right\}d\lambda ,$$

where

$$Q_1 = \frac{\sqrt{\lambda_1-\lambda_2}-\sqrt{-\lambda_2-k\sin(\theta-\alpha)}}{\lambda_2+k\sin(\theta-\alpha)} - \frac{R_2(-\lambda_1)e^{ik\sin(\theta-\alpha)l}}{\lambda\sqrt{\lambda-\lambda_2}} , \tag{35}$$

$$Q_2 = \frac{e^{ik\sin(\theta-\alpha)l}}{\lambda_2+k\sin(\theta-\alpha)} - \frac{R_1(\lambda_2)}{\sqrt{\lambda_2-\lambda_1}} , \tag{36}$$

and

$$G = \sqrt{(\lambda_1-\lambda_2)(\lambda_2-\lambda_1)} - T(\lambda_2)T'(-\lambda_1) . \tag{37}$$

Equation (34) can also be written as

$$\psi(X, Y) = \int N(\lambda)\exp\left\{-i\gamma\cos\alpha X-\sqrt{(\lambda+\gamma\sin\alpha)^2-b^2}|X|-i\lambda Y\right\}d\lambda \tag{38}$$

where

$$N(\lambda) = A'^{*}\left[\frac{\sqrt{-\lambda_2-k\sin(\theta-\alpha)}}{\lambda\sqrt{\lambda-\lambda_2}[\lambda+k\sin(\theta-\alpha)]} - \frac{e^{ik\sin(\theta-\alpha)l}R_2(-\lambda)}{\lambda\sqrt{\lambda-\lambda_2}}\right. \tag{39}$$

$$+\frac{T'(-\lambda)T(\lambda_2)Q_1}{G\lambda\sqrt{\lambda-\lambda_2}}+\frac{T'(-\lambda)\sqrt{(\lambda_1-\lambda_2)(\lambda_2-\lambda_1)}}{\lambda\sqrt{\lambda-\lambda_2}}Q_2-\frac{e^{i\lambda l}R_1(\lambda)}{\lambda\sqrt{\lambda-\lambda_1}}$$

$$+\frac{e^{i\lambda l}T(\lambda)T'(-\lambda_1)\sqrt{\lambda_2-\lambda_1}Q_2}{G\lambda\sqrt{\lambda-\lambda_1}}+\frac{T(\lambda)\sqrt{\lambda_2-\lambda_1}Q_1}{G\lambda\sqrt{\lambda-\lambda_1}}$$

$$\left.+\frac{1}{A'}\left[\frac{C_2}{\lambda\sqrt{\lambda-\lambda_1}\sqrt{\lambda-\lambda_2}G}\left\{\sqrt{\lambda-\lambda_1}\left(G+T'(-\lambda)T(\lambda_2)\right)+e^{i\lambda l}T(\lambda)\sqrt{\lambda_2-\lambda_1}\sqrt{\lambda-\lambda_2}\right\}\right]\right]^{*} .$$

6. Asymptotic behaviour of the solution

We now use the saddle point method to determine the behaviour of $\psi(X, Y)$ in (34) at a large distance from the origin. Putting $X = R\cos r$,

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$Y = R\sin t$, it can be seen easily that the single saddle point is at

$$\lambda_s = \bar{\gamma}\sin\alpha + b\sin\tau .$$

Thus

$$(40) \quad \psi \sim \frac{C}{i\cos\tau G(b\sin\tau-\gamma\sin\alpha)}\Bigg[(1+\sin\tau)\{G+T'(\gamma\sin\alpha-b\sin\tau)+T(\lambda_1)\} + +i\sqrt{2}be^{il(b\sin\tau-\gamma\sin\alpha)}T(\gamma\sin\alpha-b\sin\tau)(1+\sin\tau)\Bigg] +$$

$$+ \frac{A'\sqrt{-\lambda_2-k\sin(\theta-\alpha)}}{\sqrt{b+b\sin\tau}(b\sin\tau-\gamma\sin\alpha)[b\sin\tau-\gamma\sin\alpha)+k\sin(\theta-\alpha)]}$$

$$+ \frac{1}{(b\sin\tau-\gamma\sin\alpha)\sqrt{b+b\sin\tau}}\Bigg[-A'e^{ik\sin(\theta-\alpha)l}R_2(\gamma\sin\alpha-b\sin\tau) +$$

$$+ \frac{T'(\gamma\sin\alpha-b\sin\tau)T(\gamma\sin\alpha-b\sin\tau)Q_1}{G} + \frac{iT'(\gamma\sin\alpha-b\sin\tau)2b}{G}Q_2\Bigg] +$$

$$+ \frac{e^{i(b\sin\tau-\gamma\sin\alpha)}}{\sqrt{b\sin\tau-b}(b\sin\tau-\gamma\sin\alpha)}\Bigg[R_1(-\gamma\sin\alpha+b\sin\tau) + \frac{T(b\sin\alpha-\gamma\sin\tau)T'(-\lambda_1)Q_3}{G} +$$

$$+ \frac{iT(b\sin\tau-\gamma\sin\alpha)\sqrt{2b}\sqrt{\sin\tau-1}Q_4}{G}\Bigg] \times \exp\Bigg[-iR\{\gamma\cos(\alpha+\tau)+b\} + \frac{i\pi}{4}\Bigg]|\cos\tau|\frac{1}{\sqrt{R}}$$

$$+ \psi_R + \psi_P ,$$

where ψ_R and ψ_P are the contributions from the poles at $-k\sin(\theta-\alpha)$ and at 0 . (40) gives the required diffracted wave.

We note that $-k\sin(\theta-\alpha)$ lies between λ_1 and λ_2 if $-k\sin(\theta-\alpha) < \lambda_s$, an inequality satisfied in the regions I and II of Figure 2. In the shadow region II we observe that

$$\psi_R = -\exp\{-i2\gamma\cos\alpha X-ik\cos(\theta-\alpha)X+ik\sin(\theta-\alpha)Y\}$$

as expected. In region I, the calculation of the residue at $\lambda = -k\sin(\theta-\alpha)$ gives

$$\psi_R = -\exp\{-i2\gamma\cos\alpha X-ik\cos(\theta-\alpha)X+ik\sin(\theta-\alpha)Y\}$$

which represents a reflected wave. Also calculation of the residue at 0 gives

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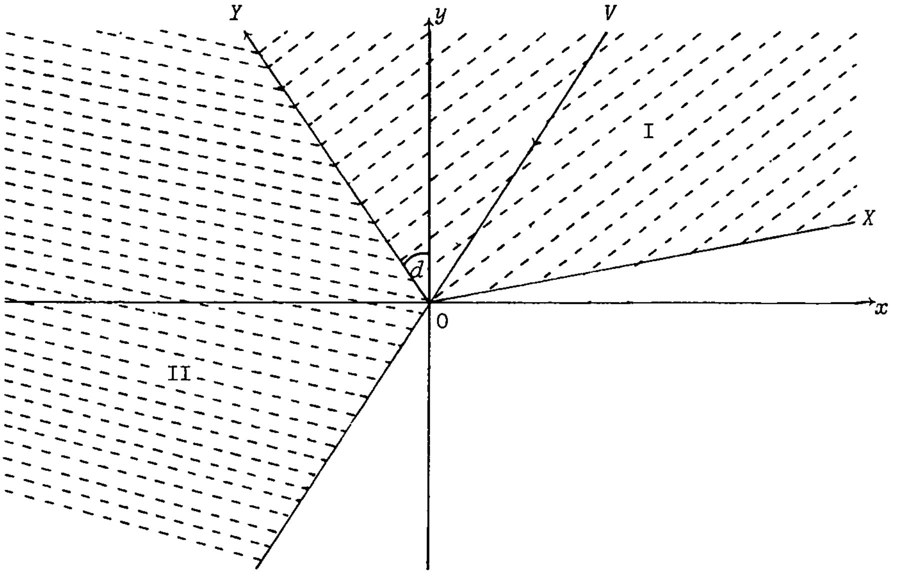

Figure 2.

$$\psi_p = D\exp\{-i\gamma\cos\alpha X - i\gamma\sqrt{\cos^2\alpha-\cos^2\theta}\,|X|\}\ ,$$

where

$$D = N(0)$$

Now we can also calculate the quantity C that we need to complete the solution. We utilize the fact that the total energy flux in the direction normal to the plate is zero. This gives

$$\psi_p = 0\ , \quad D = N(0) = 0\ ,$$

or

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$$C = -\frac{A'}{\sqrt{2\pi}}\left|\frac{\sqrt{-\lambda_2}-k\sin(\theta-\alpha)}{\sqrt{-\lambda_2}k\sin(\theta-\alpha)} - \frac{A'}{\sqrt{2\pi}}\frac{e^{ik\sin(\theta-\alpha)l}R_2(0)}{\sqrt{-\lambda_2}} - \frac{A'}{\sqrt{2\pi}}\frac{R_1(0)}{\sqrt{-\lambda_1}}\right.$$
$$+\frac{Q_1\left\{T'(0)T(\lambda_2)\sqrt{-\lambda_1}+T(0)\sqrt{\lambda_2-\lambda_1}\sqrt{-\lambda_2}\right\}}{\sqrt{-\lambda_1}\sqrt{-\lambda_2}G}$$
$$\left.+\frac{Q_3\left\{T'(0)\sqrt{\lambda_1-\lambda_2}\sqrt{-\lambda_1}+T(0)T'(-\lambda_1)\sqrt{-\lambda_2}\right\}}{G\sqrt{-\lambda_1}\sqrt{-\lambda_2}}\right| \frac{\sqrt{-\lambda_1}\sqrt{-\lambda_2}G}{\sqrt{-\lambda_1}\left|G+T'(0)T(\lambda_2)+\sqrt{-\lambda_2}\sqrt{-\lambda_1}T(0)\right|}$$

We wish to make a remark regarding the result in [1]. We can immediately obtain the conclusion in this paper by approaching the limit $l \to \infty$, in which case all the R's and Q's vanish.

Reference

[1] P.F. Siew and D.G. Hurley, "Diffraction of planetary waves by a semi-infinite plate", *Bull. Austral. Math. Soc.* 6 (1972), 145-156.

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