

CORRESPONDENCE.

MR. SANG'S SEVEN-FIGURE LOGARITHMS.

In the July Number of the *Journal* we inserted a letter from Mr. Sang communicating an error in his new table of seven-place logarithms. A similar communication made by Mr. Sang to the *Athenæum* about the same time drew forth three letters, which we here reproduce together with Mr. Sang's reply. It will be noticed that Mr. Glaisher points out another error in Mr. Sang's Table, namely in log 38962.

LOG OF 52943.

Vega's ten-place logarithms (Leipzig, 1794), which follow Vlacq's, point out Vlacq's error in this and ninety-nine other logarithms. It is remarkable that though Vega gives the log correctly as 7238085468, Hülsse's abridgment of Vega (Leipzig, 1840) has the error 7238086. But the error has not escaped detection, for Schrön's tables (Braunschweig, 1861) give correctly 7238085. Seeing that, according to Mr. Sang, Callet, Sherwin, Hutton, Babbage, Taylor, Shortrede, John Newton (to which we may add W. Gardiner, 1742), give Vlacq's error in this case, it might be worth while verifying the other ninety-nine cases mentioned by Vega.

ALEXANDER J. ELLIS.

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10 June 1872.

In the current number of the *Athenæum*, Mr. Sang publishes an error in log 52943 in his table which was communicated to him by Dr. A. W. Whitecom, of Milwaukee, U.S. The error occurs in Vlacq's original ten-figure table (where 7238085868 is printed instead of 7238085468), and is reproduced in Callet's (1795), Sherwin's, Hutton's, Babbage's, Taylor's, Shortrede's, and John Newton's tables; and Mr. Sang adds, that "it is remarkable that this error should have so long escaped detection, and all the more credit is due to its discoverer."

The discovery of the error, however, is not new, as the logarithm is printed correctly in Vega's 'Thesaurus,' &c., Lipsiæ, 1794 (folio), the error in Vlacq being pointed out in the preface. In the small seven-place editions of Vega and in Schrön's logarithms (1860) log 52943 is also printed correctly.

In tom. iv. of the *Annales de l'Observatoire Impérial de Paris* (1858), a table of errata in Vlacq's 'Arithmetica Logarithmica,' found by comparison with the great French MS. tables, is given by M. Lefort; it contains 452 errata (including those previously given in Vega's 'Thesaurus'), and the error in log 52943 is pointed out. I may mention that Lefort's errata list is intended to be supplementary to that in Vlacq's 'Arithmetica,' and at the last meeting of the Royal Astronomical Society I read a paper (printed in the *Monthly Notices* for May 1872), on the errors in Vlacq's table, in which I examined how far these lists taken together included all the known

errors; and chiefly by means of a copy of Vlacq in the library of the Royal Observatory at Greenwich, I was enabled to add seventeen errata, not given by Vlacq or Lefort. The total number of errors found in Vlacq thus amounts to 603, which probably includes very nearly all that exist; this cannot be regarded as a great number when it is considered that the table was the result of an original calculation, and that more than 2,100,000 printed figures are liable to error.

The occurrence of the error in $\log 52943$ in Babbage's tables is very remarkable, as it is stated in the preface that they were read three times with the folio edition (1794) of Vega.

As I do not infer from Mr. Sang's letter that Dr. Whitcom claimed to have discovered the error in question himself, I have little doubt it came to light by a comparison between Sang's tables and Vega's errata list. In a seven-figure table the error only produces an additional error of 064 (the error, if 5 be taken as last figure, being + 468, and if 6, - 532), so that no one could detect it by the use of seven-figure logarithms. It may be remarked that although copies of Vlacq's tables are, comparatively speaking, common in England, as most of our chief libraries were in existence in 1628, in America they are very scarce; Vega's logarithms being almost exclusively used for work requiring ten figures.

There is another last-place error in Mr. Sang's tables, viz., in $\log 38962$, the last figure of which should be 2 instead of 3; in this case, also, the logarithm in Vlacq is misprinted, being 5906413420 instead of 5906412420. This error is also pointed out by Vega.

In a paper, 'Notices respecting some errors common to many Tables of Logarithms,' (*Memoirs of the Astronomical Society*, tom. iii., 1829,) Babbage has pointed out six errors, of which this is one, that occur in most tables of logarithms; out of twenty-two the only tables free from them are the editions of Vega, the later editions of Callet and Hutton, and Babbage's own; none of the errors occur in Schrön's tables, but five out of the six are uncorrected in Shortrede's: all are noticed in Lefort's errata list.

It should be stated that of the errors found by Vega, some are corrected in the text, while some are given on a page following the introduction; the error in $\log 52943$ is of the former class, that in $\log 38962$ of the latter.

J. W. L. GLAISHER.

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11 June 1872.

Mr. Sang is, I think, in error, when he states (*Athenæum*, June 8, 1872) that the mistake in Vlacq's tables has hitherto escaped detection. I had recently in my hands the late Mr. Babbage's copies of Hutton's Mathematical Tables (1804), and his own Tables (1829), in which are ink corrections of some logarithms which have been incorrectly given; and when I had seen Mr. Sang's letter, I again consulted these works, to see if the mistake in question had been noted by Mr. Babbage. The error is not indicated in the one or the other; but having access, through the courtesy of Col. Babbage,*

* I have this gentleman's permission to make use of his father's note.

to a copy of the Russian edition of Babbage's logarithms, I there came upon the desired correction, with a reference to the authority for the same: it is simply, "52943 is given as 7238086, it is 7238085, Bremiker's logs., pref.;" then the date of his noting it, 1862.

In Bremiker's edition (Berolini, 1852), which is only a "six-place" table, the log is given 723809, which is correct to that number of places; but in his 'Logarithmic Tables of Numbers and Trigonometrical Functions, by Baron von Vega, translated from the fortieth of Dr. Bremiker's thoroughly revised and enlarged edition, by W. L. F. Fischer, M.A., Prof. in the Univ. of St. Andrews (Berlin, 1857),' it is correctly given, 7238085; and this is the value also given in J. Dupuis' 'Tables de Logarithmes à sept décimales, d'après Callet, Vega, Bremiker, &c., deuxième tirage, Paris, 1863.' This puts back Mr. Sang's date about fifteen years. In this gentleman's seven-place logarithms (1871), we have 7238086, without the mark given by Mr. Babbage to indicate that the log was, as he thought, nearer 6 than 5 in the last place, showing that Mr. Sang had accepted Vlacq's results; and this we might have expected, as he informs us that his own independent calculations are confined to numbers ranging from 10000 to 200000. That his own work was carried on independently of Vlacq's tables, will appear from the following simple test: employing Vlacq's erroneous log to find $\log 105886$; we should have it equal $\log 52943 + \log 2$, *i.e.* equal $4.7238085868 + .3010299913$, *i.e.* equal 5.0248835781 , whereas it ought to be 5.0248835381 . Now, had Mr. Sang merely copied Vlacq here, he would have given to seven-places 5.0248386 instead of 5.0248385 , which he correctly gives.

R. TUCKER.

The following is Mr. Sang's reply.

Allow me to offer my thanks to your Correspondents for the interest they have taken in this matter, and particularly to Mr. Glaisher, for the indication of the error in $\log 38962$, which had been caused by an omission to correct that logarithm in my copy of Vlacq, although noticed in my list. Mr. Glaisher inclines to the idea that Dr. Whitcom had found the mistake in some published list of errata. I rather think that he had found it in the course of his actuarial work. The logarithm in question is the middle one of the three 8003, 8086, 8167; the difference of the extremes being even, while the middle one is not half their sum. This can occur only once in seven-place logarithmic tables from 10000 to 100000, the solitary instance being in logs 12735, 6, 7, which are 9989; 9331; 0671; or, to 12 places, 89 49300; 30 50394; 71 48810. It does not again occur up to 200000. Now a careful computer, in the habit of looking to the difference both before and after, having to deal with some number between the above limits, could hardly fail to observe the anomaly.

As to the previous detection of the error, I could only consult the works on my own shelves, which include a copy of John Newton's eight-place table, kindly lent me by Mr. Peter Gray, and also one of Vega (1783), containing both of the errors.

I shall be grateful to any one having access to printed or MS. lists

of errors who may take the trouble to compare the same with my book, and shall gladly send him a copy for the first notice of each error. The second part of my work is quite independent of and has not yet been compared with any other table.

EDWARD SANG.

In a letter to Mr. Peter Gray, dated 7 Oct. 1872, Mr. Glaisher says, "I have no doubt Professor Whitcom did not claim to have found out the error in log 52943 himself, but obtained it (as I did that in log 38962) by comparison with Vega." He further mentions that Bruhns gives log 52943 correct, and that Hülse's edition of Vega, 1840, has log 52943 *wrong*, altho' the previous editions have it right: also, that Dr. Bremiker in the preface to his 41st edition traces Babbage's error in log 52943 to Gardiner.

[In connection with this subject we may mention that our next Number will contain an article by Mr. Glaisher on "Errors in Tables of Logarithms of Numbers," being an abstract of a paper lately read by him before the Royal Astronomical Society.—Ed. *J. I. A.*]

AMERICAN TEN-YEAR NONFORFEITURE POLICIES.

To the Editor of the Assurance Magazine.

SIR,—I send for publication, as likely to interest your readers, a statement of certain serious objections to the anomalous contract known in America as the Ten-Year Nonforfeiture Life Policy. These objections, always held and consistently acted upon by a few, have at last come to be widely recognized. The alteration of opinion has been manifested practically within a year or two past by changes in the plans of business of three, at least, of the larger companies.

A life policy calling for the payment of a limited number of annual premiums, is, of course, no novelty. Every writer on life contingencies furnishes, explicitly or implicitly, formulæ proper to be used in such cases. Secured contracts of this sort, forming an intermediate class between paid-up policies and those at annual premiums, are, in themselves, unobjectionable; presenting, apparently, no disadvantages which are not shared by one or other of those more usual forms of policy. By "secured contracts," I mean those which provide for forfeiture in case of lapse, or for such other penalty as will indemnify the society for whatever loss it may sustain by the default. Of this class are the ten-premium policies (now) written by some of the American companies, which promise the issue, in case of lapse, of a paid-up policy for "an equitable sum," to be determined by the company. This equitable sum, it must be presumed, is to be ascertained by making the present value of all liabilities on the new policy equal to that of those on the old, less proper damages for the breaking of the contract. Policies of this class, under which the society is, except in case of death, certain to receive all the premiums contemplated, or else a proper amount of profit-money out of the reserve value, may, for convenience, be called *secured ten-premium policies*.

A second variety of ten-premium policy was brought out some years ago by a prominent American company. At age 30, for