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GLOBAL GENERATION OF ADJOINT BUNDLES

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1. Introduction

In 1988, I. Reider proved that for a smooth projective surface X and an ample line bundle L on X, $K_X + 3L$ is globally generated and $K_X + 4L$ is very ample ([12]). In fact his theorem is much stronger than this (see [12] for detail). Recently a lot of results have been obtained about effective base point freeness (cf. [1, 3, 8, 13, 14, 15]). In particular J. P. Demailly proved that $2K_X + 12n^nL$ is very ample for a smooth projective n-fold X and an ample line bundle L on X. [2] will give a good overview for these recent results. The motivation of these works is the following conjecture posed by T. Fujita.

CONJECTURE ([4]). *Let X be a smooth projective n-fold defined over $\mathbf{C}$ and let L be an ample line bundle on X. Then $K_X + (n+1)L$ is generated by global sections and $K_X + (n+2)L$ is very ample.*

We note that Fujita's conjecture is trivial if L is very ample by induction on $\dim X$. In the above situation, it is easy to see that $K_X + (n+1)L$ is nef and $K_X + (n+2)L$ is ample by using the theory of extremal rays (Mori theory cf. ([10, 6]). Moreover by using the base point free theorem ([7, p. 581, Theorem 6.1]), $K_X + (n+1)L$ is semiample, i.e. there exists a positive integer m such that $m(K_X + (n+1)L)$ is generated by global sections. The number $n+1$ is nothing but the maximal length of extremal rays of smooth projective n-folds. In this paper, we shall prove the following theorem.

THEOREM 1. *Let X be a smooth projective variety over $\mathbf{C}$ of dimension n and let L be an ample line bundle on X. Then $K_X + mL$ is generated by global sections on X for every*

$$m \geq n(n+1)/2 + 1.$$

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The following two corollaries are the immediate consequence of Theorem 1.

COROLLARY 1. *Let X be a smooth projective variety of dimension n defined over $\mathbf{C}$ such that the canonical bundle K_X is ample. Then mK_X is generated by global sections for every $m \geq n(n+1)/2+2$.*

COROLLARY 2. *Let X be a smooth projective variety of dimension n defined over $\mathbf{C}$ such that the anticanonical bundle $-K_X$ is ample. Then $-mK_X$ is generated by global sections for every $m \geq n(n+1)/2$.*

Our method is extremely simple. We hope this method is applicable to obtain effective bound for very ampleness of adjoint bundles.

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2. Proof of Theorem 1

In the proof of Theorem 1 we shall use singular hermitian metrics as in [1]. But our proof is mainly algebraic. For example we do not use Monge-Ampère equation.

2.1. Singular hermitian metrics and a vanishing theorem

DEFINITION 1. Let L be a line bundle over a complex manifold M. A singular hermitian metric h on L is given by

$$h = e^{-\varphi} h_0,$$

where h_0 is a C^∞-hermitian metric on L an $\varphi \in L^1_{\mathrm{loc}}(M)$ is an arbitrary function, called the weight of the metric with respect to h_0.

We define a closed current $\mathrm{curv}\, h$ by

$$\mathrm{curv}\, h = \mathrm{curv}\, h_0 + \sqrt{-1}\, \partial\bar{\partial}\varphi,$$

where $\mathrm{curv}\, h_0$ is the curvature form of the hermitian metric h_0 and $\partial\bar{\partial}$ is taken in the sense of current. We call $\mathrm{curv}\, h$ the curvature current of the singular hermitian line bundle (L, h). It is easy to see that $\mathrm{curv}\, h$ is independent of the choice of h_0 and φ.

DEFINITION 2. Let T be a positive (1,1) current on a complex manifold M. T is said to be strictly positive, if for every point $x \in M$, there exists a neighbourhood U of x and a C^∞ Kähler form ω on U such that $T - \omega$ is a positive (1,1)-current on U.

DEFINITION 3. Let L be a line bundle on a complex manifold M and let h be a singular hermitian metric on L. The L^2-sheaf $\mathscr{L}^2(L, h)$ is the sheaf defined by

$$\mathscr{L}^2(L, h)(U) = \{\sigma \in \Gamma(U, L) \mid h(\sigma, \sigma) \in L^1_{\mathrm{loc}}(U)\}.$$

We shall recall the following theorem.

THEOREM 2 ([11] (see also [1, p. 333, Theorem 4.5]). *Let X be a smooth projective variety and let L be a line bundle on X. Let h be a singular hermitian metric on L such that* curv h *is strictlty positive. Then $\mathscr{L}^2(L, h)$ is a coherent sheaf of $\mathscr{O}_X$ module and*

$$H^p(X, \mathscr{O}_X(K_X) \otimes \mathscr{L}^2(L, h)) = 0$$

holds for every $p \geq 1$.

2.2. Construction of singular hermitian metrics

Let X be a smooth projective variety of dimension n defined over $\mathbf{C}$ and let L be an ample line bundle on X. Let $x \in X$ be a point. We shall construct a singular hermitian metric on some multiple of L with sufficiently large singularity at x and (semi) positive curvature in the sense of current.

LEMMA 1. *For sufficiently large $H^0(X, \mathscr{O}_X(m(2n+1)L) \otimes \mathscr{M}_x^{\otimes 2mn})$ is not zero, where $\mathscr{M}_x$ denotes the ideal sheaf of x.*

Proof. Let us consider the exact sequence:

$$0 \to H^0(X, \mathscr{O}_X(m(2n+1)L) \otimes \mathscr{M}_x^{\otimes 2mn}) \to H^0(X, \mathscr{O}_X(m(2n+1)L)) \to \mathscr{O}_X(m(2n+1)L)/\mathscr{M}_x^{\otimes 2mn}.$$

$\mathscr{O}_X/\mathscr{M}_x^{\otimes 2mn}$ is a skyscraper sheaf of rank $\binom{2mn+n-1}{n}$. On the other hand by Serre's vanishing theorem and Riemann-Roch theorem, we see that

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$$\dim H^0(X, \mathcal{O}_X((2n+1)mL)) = \frac{(2n+1)^n L^n}{n!} m^n + O(m^{n-1})$$

holds. Since

$$\binom{2mn+n-1}{n} = \frac{2^n n^n}{n!} m^n + O(m^{n-1})$$

holds, we see that

$$H^0(X, \mathcal{O}(m(2n+1)L) \otimes \mathcal{M}_x^{\otimes 2mn}) \neq 0$$

holds for sufficiently large m. Q.E.D.

Remark 1. *There is no particular reason to use the number* 2 *in Lemma* 1, *i.e. for any fixed positive integer* N, *we can prove that*

$$H^0(X, \mathcal{O}_X(m(Nn+1)L) \otimes \mathcal{M}_x^{\otimes Nmn}) \neq 0$$

for a sufficiently large m. *This fact will be used later.*

For simplicity we set

$$\Lambda_m = | H^0(X, \mathcal{O}_X(m(2n+1)L) \otimes \mathcal{M}_x^{\otimes 2mn}) |.$$

We consider Λ_m as a linear subsystem of $| m(2n+1)L |$. We set

$$B_m = \mathrm{Bs}\, \Lambda_m.$$

Let us take a $\mathbf{C}$ basis $\sigma_0, \ldots, \sigma_N$ of $H^0(X, \mathcal{O}_X(m(2n+1)L) \otimes \mathcal{M}_x^{\otimes 2mn})$. Then $\sigma_0, \ldots, \sigma_N$ generates the ideal sheaf of the scheme B_m over $\mathcal{O}_X$. Let h be a C^∞ hermitian metric of L such that curv h is a Kähler form on X. We define a singular hermitian metric H_x of $\mathcal{O}_X(2m(n+1)L)$ by

$$H_x = \frac{h^{\otimes m(2n+1)}}{\sum_{k=0}^N h^{\otimes m(2n+1)}(\sigma_k, \sigma_k)}.$$

Let us define a closed current T by

$$T = \mathrm{curv}\, H_x.$$

Let $\Phi_x : X - \cdots \rightarrow \mathbf{P}^N$ be the rational map defined by

$$\Phi_x(p) = [\sigma_0(p) : \ldots : \sigma_N(p)].$$

Then T is expressed by

$$T = \Phi^* \omega_{FS},$$

where ω_{FS} denotes the Fubini-Study Kähler form of $\mathbf{P}^N$. Because Φ is only a rational map, we need to explain a little bit more the precise meaning of $\Phi^* \omega_{FS}$. Let $G \subset X \times \mathbf{P}^N$ be the graph of the rational map Φ. Let π_i $(i = 1,2)$ denote the restriction of the first and second projections on G respectively. Then $\Phi^* \omega_{FS}$ is defined by

$$\Phi^* \omega_{FS} = (\pi_1)_* \pi_2^* \omega_{FS}.$$

This implies that T is a closed positive current on X.

We shall analyze T.

2.3. Basic invariant

Let H_x be the singular hermitian metric of $L^{\otimes m(2n+1)}$ constructed in 2.2. Let us define a function on φ on X by

$$\varphi = -\frac{1}{2m} \log\Big(\frac{H_x}{h^{\otimes m(2n+1)}}\Big).$$

For $t \in [0, 1]$, we define an ideal sheaf $\mathscr{I}(t)$ by

$$\mathscr{I}(t) := \mathscr{L}^2(\mathscr{O}_X, e^{-t\varphi}).$$

If $s \leq t$, then

$$\mathscr{I}(t) \subset \mathscr{I}(s)$$

holds. By increasing t from 0 to 1, we obtain a strictly decreasing sequence of ideals:

$$\mathscr{O}_{X,x} \supset \mathscr{I}_{1,x} \supset \mathscr{I}_{2,x} \supset \cdots \supset \mathscr{I}_{k,x}$$

We set

$$\alpha = \sup\{t \in [0, 1] \mid \mathscr{I}(t)_x = \mathscr{O}_{X,x}\}$$

and

$$V = \text{the union of irreducible components of } V\mathscr{I}_1 \text{ containing } x,$$

where $V\mathscr{I}_1$ denotes the zero variety of $\mathscr{I}_1$. We note that V is nonempty because $1/(\sum_{i=1}^n |z_i|^2)^n$ is not locally integrable near the origin in $\mathbf{C}^n$. Then V is a reduced (but may not be irreducible) subvariety of X.

2.4. Case: codim $V = n$

In this case $V = \{x\}$. Let us define a singular hermitian metric h_x of $\mathcal{O}_X((n+1)L)$ by

$$h_x = H_x^{\frac{\alpha+\varepsilon}{2m}} h^{(n+1-(n+\frac{1}{2})(\alpha+\varepsilon))},$$

where ε is a sufficiently small positive number. Then since

$$\text{curv}\, h_x = \frac{\alpha+\varepsilon}{2m} T + \left(n + 1 - \left(n + \frac{1}{2}\right)(\alpha+\varepsilon)\right)\text{curv}\, h,$$

h_x has strictly positive curvature. By Theorem 2, we have

$$H^p(X, \mathcal{O}_X(K_X \otimes \mathcal{L}^2(L^{\otimes(n+1)}, h_x))) = 0$$

holds for every $p \geq 1$. We note that x is an isolated point in the zero variety of $\mathcal{I}_1$. Hence

$$H^0(X, \mathcal{O}_X(K_X + (n+1)L)) \to \mathcal{O}_X(K_X + (n+1)L)/\mathcal{M}_x$$

is surjective. Hence $K_X + (n+1)L$ is generated by global sections at x.

2.5. Case: codim $V < n$

Let X_1 be a minimal dimensional irreducible component of V and let n_1 be the dimension of X_1. For the first we assume that X_1 is nonsingular at x. The following lemma is an easy consequence of Serre's vanishing theorem.

LEMMA 2. *The restriction morphism*

$$\phi : H^0(X, \mathcal{O}_X(\nu L)) \to H^0(X_1, \mathcal{O}_{X_1}(\nu L \mid X_1))$$

is surjective for every sufficiently large ν.

LEMMA 3. *Let $x \in X_1$ be a regular point of X_1, then*

$$H^0(X_1, \mathcal{O}_{X_1}(2n+1)m_1 L \mid X_1) \otimes \mathcal{M}_x^{\otimes 2m_1 n}) \neq 0$$

holds for some $m_1 \gg 1$.

To prove Lemma 3, we need the following lemma.

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LEMMA 4. *Let M be a smooth projective n-fold and let F be a nef and big line bundle on M. Then for every $q \geq 1$.*

$$\dim H^q(M, \mathcal{O}_M(\nu F)) \leq O(\nu^{n-1})$$

holds as ν tends to infinity.

Proof of Lemma 4. By Kodaira's lemma ([8, Appendix]) there exists an effective divisor E and a positive integer ν_0 such that both $\nu_0 F - E$ and $\nu_0 F - E - K_X$ is ample. Then by Kodaira's vanishing theorem, we have an isomorphism:

$$H^q(M, \mathcal{O}_M(\nu F)) \simeq H^q(E, \mathcal{O}_M(\nu F|_E))$$

for every $q \geq 1$ and $\nu > \nu_0$. Since

$$\dim H^q(E, \mathcal{O}_E(\nu F|_E)) = O(\nu^{n-1}),$$

this completes the proof of Lemma 4. Q.E.D.

Proof of Lemma 3. Let $\mu : \hat{X}_1 \to X_1$ be a resolution of singularity. By Lemma 4, we have

$$H^0(\tilde{X}_1, \mathcal{O}_{\tilde{X}1}(m_1(2n+1)\mu^*(L \mid X_1) \otimes \mathcal{M}_y^{\otimes 2m_1 n})) \neq 0$$

holds for every $m_1 \gg 1$ and $y \in \hat{X}_1$. If X_1 is normal then this completes the proof of Lemma 3. Suppose that X_1 is nonnormal. Let D_1 be the codimension 1 singular locus of X_1 and let $\bar{D}_1$ denote $\mu^{-1}(D_1)$. Then we have for every fixed positive integer a,

$$H^0(\tilde{X}_1, \mathcal{O}_{\tilde{X}1}(\mu^*(m_1(2n+1)L) - a\tilde{D}_1) \otimes \mathcal{M}_y^{\otimes 2m_1 n})) \neq 0$$

for every $m_1 \gg 1$ and $y \in \tilde{X}_1$. If we take a sufficiently large this completes the proof of Lemma 3. Q.E.D.

Since L is ample, by Serre's vanishing theorem, if we take m_1 sufficiently large

$$H^1(X, \mathcal{O}_X(m_1(2n+1)L) \otimes \mathcal{O}_X(-X_1) \otimes \mathcal{M}_y) = 0$$

for every $y \in X$, where $\mathcal{O}_X(-X_1)$ denotes the ideal sheaf of X_1. This implies that for every $y \in X - X_1$

$$H^0(X, \mathcal{O}_X(m_1(2n+1)L)) \to H^0(X_1, \mathcal{O}_{X_1}(m(2n+1)L \mid X_1)) \oplus \mathcal{O}_X / \mathcal{M}_y$$

is surjective if we take m_1 sufficiently large. Hence taking m_1 sufficiently large, if

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necessary, by Noetherian induction we may assume that the linear subsystem

$$|\phi^* H^0(X_1, \mathcal{O}_{X_1}(m_1(2n+1)L \mid X_1 \otimes \mathcal{M}_x^{\otimes 2m_1 n}))|$$

of $|m_1(2n+1)L|$ does not have base points on $X - X_1$. Let $\tau_0, \ldots, \tau_M$ be a basis of $\phi^* H^0(X_1, \mathcal{O}_{X_1}(m_1(2n+1)L) \otimes \mathcal{M}_x^{\otimes 2m_1 n})$. We define a singular hermitian metric $H_{1.x}$ by

$$H_{1,x} = \frac{h^{\otimes m_1(2n+1)}}{\sum_{j=0}^{M} h^{\otimes m_1(2n+1)}(\tau_j, \tau_j)}.$$

Then as before, curv $H_{1,x}$ is a closed current. Let ε be a sufficiently small positive number. Let $\varphi_{1,t}$ be the function on X defined by

$$\varphi_{1,t} = \log(H_x^{\frac{\alpha-\varepsilon}{2m}}(H_{1,x}^{\frac{t}{2m_1}})h^{-(n+\frac{1}{2})(\alpha+t-\varepsilon)})$$

and let α_1 be the positive number defined by

$$\alpha_1 = \sup(t \in \mathbf{R} \mid e^{-\varphi_{1,t}} \in L^1_{\mathrm{loc}}(X, x)\}.$$

We set

$$\mathcal{I}^{(1)}(t) = \mathcal{L}^2(\mathcal{O}_X, e^{-\varphi_{1,t}}).$$

Then by increasing t we obtain a strictly decreasing sequence of ideals

$$\mathcal{O}_{X,x} \supset \mathcal{I}_{1,x}^{(1)} \supset \cdots.$$

We set

$$\mathcal{I}_1^{(1)} = \lim_{t \downarrow 0} \mathcal{I}^{(1)}(\alpha_1 + t).$$

Then the stalk $(\mathcal{I}_1^{(1)})_x$ of $\mathcal{I}_1^{(1)}$ at x is $\mathcal{I}_{1,x}^{(1)}$ by the Noetherian property of coherent analytic sheaves. Let X_2' be the subscheme $V\mathcal{I}_1^{(1)}$ of X. Then by the construction X_2' is a subscheme of X_1. Let X_2 be a minimal dimensional irreducible component of X_2' containing x.

LEMMA 5. *We have the inequality*:

$$\alpha_1 \leq n_1/n + O(\varepsilon).$$

To prove this lemma we need the following elementary lemma:

LEMMA 6. *Let b be a positive number. Then*

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$$\int_0^1 \frac{r_2^{2n_1-1}}{(r_1^2 + r_2^{4m_1n})^b}\, dr_2 = r_1^{\frac{n_1}{m_1n}-2b} \int_0^{r_1^{-\frac{1}{2m_1n}}} \frac{r_3^{2n_1-1}}{(1 + r_3^{4m_1n})^b}\, dr_3$$

holds, where

$$r_3 = r_2 / r_1^{1/2m_1n}.$$

Suppose that x is a regular point of X_1. Let $(z_1, \dots, z_n)$ be a local coordinate on a neighbourhood U of x in X such that

$$U \cap X_1 = \{p \in U \mid z_{n_1+1}(p) = \cdots = z_n(p) = 0\}.$$

We set $r_1 = (\sum_{i=n_1+1}^{n} |z_i|^2)^{1/2}$ and $r_2 = (\sum_{i=1}^{n_1} |z_i|^2)^{1/2}$. Then there exists a positive constant C such that

$$\sum_{j=0}^{M} |\tau_j|^2 \leq C(r_1^2 + r_2^{4m_1n})$$

holds on a neighbourhood of x, where

$$|\tau_j|^2 = h(\tau_j, \tau_j).$$

We note that there exists a positive integer l such that

$$\sum_{i=0}^{N} |\sigma_i|^2)^{-1} = O(1/r_1^l)$$

on a neighbourhood of generic point of $X_1 \cap U$. Then by Lemma 6, we have the inequality $\alpha_1 \leq n_1/n + O(\varepsilon)$.

For the next, suppose that x is a singular point of X_1.

Let $\pi : \tilde{X} \to X$ be an embedded resolution of X_1 and let X_1^* be the strict transform of X_1.

LEMMA 7. *Let x_1 be a point on $\pi^{-1}(x)$. Then there exist global sections*

$$\tau_0, \dots \tau_M \in H^0(X, \mathcal{O}_X(m_1(2n+1)L))$$

such that

$$\pi^*(\tau_j)|_{X_1^*} \in H^0(X_1^*, \mathcal{O}_{X_1^*}(\pi^*(m_1(2n+1)L)) \otimes \mathcal{M}_{x_1}^{\otimes 2m_1n})$$

holds for every j and $\{\tau_0, \dots, \tau_M\}$ is a basis for such sections.

The proof is the same as Lemma 3. Let x_1 be a point on the strict transform X_1^* such that $\pi(x_1) = x$. Let $(z_1^1, \dots, z_n^1)$ be a local coordinate on a neighbourhood

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$\tilde{U}$ of x_1 such that

$$\tilde{U} \cap X_1^* = \{p \in \tilde{U} \mid z^1_{n_1+1}(p) = \cdots z^1_n(p) = 0\}.$$

We define $\tilde{\tau}_1$, $\tilde{\tau}_2$ similarly as above. Then there exists a constant C such that

$$\pi^*(\sum_{j=0}^{M} |\tau_j|^2) \leq C(\tilde{r}_1^2 + \tilde{r}_2^{Am_1n})$$

holds. Then again by Lemma 6 and the uppersemicontinuity of the multiplicity, we have the inequality $\alpha_1 \leq n_1/n + O(\varepsilon)$.

If $X_2 = \{x\}$, then as before we have that

$$H^0(X, \mathcal{O}_X(K_X + mL)) \to O_X(K_X + mL)/\mathcal{M}_x$$

is surjective for every

$$m > (\alpha + \alpha_1)\left(n + \frac{1}{2}\right).$$

If X_2 is not $\{x\}$ we can continue the same process and obtain the strictly decreasing sequence of subvarieties

$$X \supset X_1 \supset X_2 \supset \cdots.$$

We see that there exists $k \leq n$ such that $X_k = \{x\}$. By Lemma 6, we have that

$$\sum_{i=0}^{k-1} \alpha_i \leq \frac{n(n+1)}{2n} + \varepsilon$$

holds, where $\alpha_0 = \alpha$ and ε is a positive number which we can take arbitrarily small. This implies that

$$H^0(X, \mathcal{O}_X(K_X + mL)) \to O_X(K_X + mL)/\mathcal{M}_x$$

is surjective for every

$$m > \frac{n(n+1)}{2n}\left(n + \frac{1}{2}\right).$$

But we improve this estimate as

$$m > \frac{n(n+1)}{2}$$

by replacing $H^0(X, \mathcal{O}_X(m(2n+1)L) \otimes \mathcal{M}_x^{\otimes 2mn})$ by $H^0(X, \mathcal{O}_X(m(Nn+1)L) \otimes \mathcal{M}_x^{\otimes Nmn})$ for a sufficiently large integer N in Lemma 1 (with trivial change of constants in the argument after Lemma 1).

This completes the proof of Theorem 1.

3. A generalization

The proof of Theorem 1 says a little bit more. We shall show the local version of Theorem 1.

DEFINITION 4. Let X be a smooth projective variety and let x be a point on X. Let L be a nef and big line bundle on X. We set for $1 \leq d \leq \dim X$,

$$\mu_d(L, x) = \inf\{(L^d V)^{1/d} \mid V \text{ is a } d\text{-dimensional subvariety of } X \text{ such that } x \in V\}.$$

Now we can state the local version of Theorem 1.

THEOREM 3. *Let X be a smooth projective variety defined over* $\mathbf{C}$ *of dimension n and let L be a nef and big line bundle on X. Let x be an arbitrary point on X. Then $K_X + mL$ is generated by global sections at x for every*

$$m > \sum_{d=1}^{n} \frac{d}{\mu_d(L, x)}.$$

The proof is actually contained in the proof of Theorem 1. Hence we omit it.

Remark 2. Let X be a smooth projective n-fold and let L be an ample line bundle on X. Then the proof of Theorem 1 implies that $K_X + mL$ gives a birational morphism for every $m > n(n+1)$. In fact $K_X + mL$ separates **general** two distinct point on X for every $m > n(n+1)$ by a trivial modification of the proof of Theorem 1.

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