

topology. The text is written with admirable clarity. The logical thread of the argument is carried by a sequence of definitions and theorems, but this is liberally interspersed with "asides" of a less formal nature giving additional explanations and motivations. Each chapter is preceded by a historical and bibliographical note. Nonetheless this text may be difficult for the average student if he satisfies only the prerequisites stated by the author, namely knowledge of the topology of the real line and properties of real valued functions. For throughout the text (including the exercises) there are very few concrete examples, and it would appear desirable for the student to have some backlog of experience with sets in Euclidean spaces to enable him to appreciate the more abstract ideas treated here.

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Disquisitiones Arithmeticae, by Carl Friedrich Gauss, 1801; English translation, by Arthur A. Clarke, S. J. Yale University Press, New Haven and London, 1966. xx + 472 pages. \$12.50.

At the beginning of 1795 a young man not yet eighteen happened upon a result he recognized as beautiful: an odd prime  $p$  is a factor of  $x^2 + 1$  for some integer  $x$  if and only if the prime is of the form  $4n + 1$ . He surmised a connection with properties even more profound, and strove to discover the underlying principles and to find a proof. Succeeding, he was so enthralled he could not let these questions be. The young man was of course Gauss; and the book or saga, he wrote will still be read with delight in the year 3000. I choose three items from Gauss' notes listed at the end of the book. Gauss discovered the quadratic reciprocity law, by experiment, in March 1795, and completed its first proof on April 8, 1796. He proved that "a circle is geometrically divisible into 17 parts" on March 30, 1796; and so soon after must have solved the 2000-year old problem of Euclidean constructibility of regular polygons (included in Sect. VII of the *Disquisitiones*). It was this discovery, Bell tells us, which decided the young man to choose mathematics rather than philology.

Gauss is very special to mathematicians, and this first English translation is an event, even after 165 years. One's native tongue always comes easier, and so now many of us will have a real opportunity to become much better acquainted with a very great man. Myself, even though I had previously plowed through much of the original Latin, the French translation (1807), and the German one (1885), and had read various accounts of parts of Gauss' work (especially in H. J. S. Smith's Report on the Theory of Numbers, 1860-65), I found in reading this translation items I had forgotten or did not know, - and it was just delightful to browse through Gauss' reasoning and his approach to various questions, here and there.

There are, unfortunately, errors in the translation, and these disturbed me perhaps more than ordinarily because I felt a translation

of Gauss should be perfect. I hope the errors will not keep people from reading the book, -but I hope no one will think the errors are due to Gauss. My impression of the French and German translations is that they followed Gauss' words and style fairly closely, correcting a few minor errors and giving preference when a choice had to be made to the terms which later usage established. Gauss wrote with dignity and care, and for a young man pioneering, amazingly well. He liked to express his thinking in sentences of several lines - which seem to me to read smoothly and to show the relative importance of ideas and their inter-coherence by the manner in which he placed them. For some reason the translator has broken up many of Gauss' sentences into two. For example the 32 sentences in Gauss' dedication and preface have become 56. In several cases that I have studied the coherence and emphasis of Gauss' thought have been lost.

I give one example. Gauss has in Art. 11, "Generaliter perspicuum est., aequationem  $X = 0$ , quando  $X$  functio incognitae  $x$ , huius formae  $x^n + Ax^{n-1} + Bx^{n-2} + \text{etc.} + N$ ,  $A, B, C$ , etc. integri, atque  $n$  integer positivus, (ad quam formam omnes aequationes algebraicas reduci posse constat) radicem rationalem nullam habere, si congruentiae  $X \equiv 0$  secundum ullum modulum satisfieri nequeat. Sed hoc criterium, quod hic sponte se nobis obtulit, in Sect. VIII fusius pertractabitur." I translate almost literally: "In general it is clear that the equation  $X = 0$ , where  $X$  is a function of the unknown  $x$  of the form ... with  $A, B, C$ , etc., integers and  $n$  a positive integer, (to which form all algebraic equations can be reduced), has no rational root if the congruence  $X \equiv 0$  cannot be satisfied for any one modulus. But this criterion which is given us here by itself will be repeatedly applied in Sect. VIII." Clarke's translation: "Suppose  $X$  is a function in unknown  $x$  of the form ... where  $A, B, C$ , etc., are integers,  $n$  a positive integer (it is clear that all algebraic equations can be reduced to this form). In general it is clear that in the equation  $X = 0$  there exists no rational root unless the congruence  $X \equiv 0$  can be satisfied for some modulus. But this omission will be discussed more fully in Section VIII." Thus Gauss has been made to say something silly; and even if "some" is made "every", we are not hearing Gauss.

As I read the early parts of the book I consulted the Latin wherever the English or reasoning seemed askew, and thus drew up the following list of "corrections" - most of them minor. Perhaps the publishers will arrange for a list of corrections which could be sent to readers of the book upon inquiry. I abbreviate Art. h, line k, as (h;k), and change p to Q as  $p \rightarrow q$ . (1;5) append of one another; (5;5) many  $\rightarrow$  several; (7;6) and (22;8) make a new sentence beginning For; (16;6) many various  $\rightarrow$  more than one; (17;7) insert prime; (22;7) equal  $\rightarrow$  unchanged; (32;24) when none of the auxiliary congruences is solvable  $\rightarrow$  when any of the auxiliary congruences is not solvable; (36;15) more  $\rightarrow$  several; (38;12)  $p^m \rightarrow p^{m-1}$  (twice); (39;20) We may suppose; (42;9) form  $\rightarrow$  terms; (42;10) many  $\rightarrow$  more; (42;24, 34 et seq.) needs recasting with exponent (Gauss' *dimensio*) instead of power; (46;6) as long as it is

finite  $\rightarrow$  as soon as ended. Also needing revision are (8;6, 7), (39;24-27), (64;3-6). The book has 366 articles.

I noticed in Art. 9 the phrases "in the indeterminate  $x$ " and "in several indeterminates" (which Clarke translated "with undetermined  $x$ " and "of many undetermined variables"). I had long had the impression that the word indeterminate, to designate the letter by means of which a polynomial is expressed, originated late in the nineteenth century. It now seems that this important term is due to Gauss.

While everyone will recognize Gauss' abbreviation Q.E.D., perhaps some would appreciate a translation of Q.E.F. (quod erat faciendum) and Q.E.A. (quod est absurdum).

It may seem a small thing, but alterations of familiar technical terms can be annoying: it would have sounded better to me if the translator had turned Gauss' *primus ad  $m$*  into the familiar prime to  $m$  rather than prime relative to  $m$ ; if he had preferred to the modulus  $m$  or modulo  $m$  (phrases everyone associates with Gauss) instead of relative to the modulus  $m$ ; if he had translated *incongruus* as incongruent rather than noncongruent; and if, in Art. 157 et seq., he had one form containing another instead of implying it, even though Gauss did use *implicans*.

Father Clarke has done a great service to all of us in making Gauss' book available. I hope all mathematicians will partake of the delight to be found in this great book.

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