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Labeyrie's (1970, 1976) speckle interferometry permits an astronomical object to be resolved to a limit approaching $\lambda / \mathrm{D}$ (where $\lambda$ is the mid-band wavelength of the light accepted by a telescope having a pupil aperture of diameter D) in the presence of severe atmospheric seeing. As Dainty (1973) has pointed out, this resolution limit is virtually independent of the accuracy to which the telescope is figured - the seeing can actually improve a telescope's resolution! Even though a true image of the object cannot usually be constructed by Labeyrie's data reduction procedures, nevertheless the autocorrelation of (the distribution of brightness over) the object can always be reconstructed - provided the extent (width, apparent angular diameter) of the object does not exceed that of the isoplanatic patch (cf. Bates and Gough, 1975). The autocorrelation is useful because its extent is necessarily twice that of the object.

True images of certain objects can be formed straightforwardly by speckle interferometry - this is related to holography (cf. Bates and Gough, 1975). By superimposing the individual speckles apparent in a narrow-band, short exposure of an isolated object whose angular diameter is somewhat greater than $\lambda / \mathrm{D}$, Lynds et $a l$. (1976) have suggested that an actual image of the object is obtained. Unfortunately, when used unaided, such imaging techniques can only be applied to very limited classes of objects.

All but a very few individual stars are unresolvable in the largest telescopes, even when seeing is perfect, implying that diffraction-limited images of the great majority of star clusters are collections of Airy discs. The essential information in any such image can be represented by an array of two-dimensional delta functions, whose relative positions and intensities correspond to the relative positions and brightnesses of the Airy discs.

We have realised that by combining Labeyrie's original technique with the approach taken by Lynds et al., and employing data reduction procedures developed by X-ray crystallographers (Baldwin and Warner,

1976, have already made interesting astronomical use of these), we should be able to reproduce accurate representations of those star clusters that could be imaged, if the seeing conditions could be perfect, with existing telescopes in the conventional way. We have carried out simulations in our optical laboratory, using a previously developed technique (Gough and Bates, 1974), and have already reported results of preliminary experiments in which only optical processing was used (Bates et al., 1978).

To take full advantage of speckle interferometry we feel it is essential to use digital techniques. Accordingly, we have placed a CCD-camera (Cady et $a l ., 1978$ ) at the focus of our simulated telescope, and have interfaced it to the EAI 590 Hybrid System in our Computer Laboratory (note that CCD stands for Charge Coupled Device). The theoretical foundations of our new method of imaging, the experimental apparatus and the computational procedures are described in detail elsewhere (Bates and Milner, 1978; Milner, 1978). Here, we explain the physical basis of our approach and delineate the essential steps of the data reduction procedures.

## 1. SPECKLE MASK PROCESSING

We call a narrow-band, short exposure of an astronomical object a speckle image. The short exposure of $\gamma$ Orionis shown by Lynds et $a l$. (1976) is typical of the many speckle images that have been recently reported in the literature. For our purposes, the essential characteristic of a speckle image is that many of its constituent speckles axe distinct (i.e. they can be recognised as individual entities).

Suppose that the cluster shown in Figure 1 is being viewed (each circle in the figure represents an individual star, positioned at the centre of the circle and having a relative intensity denoted by the number immediately above the circle). Our CCD-camera has a dynamic range of $2^{8}$, which is why the maximum intensity of each of our images is normalised to 255. We use the letter $C$ to denote the pattern of delta functions corresponding to the circles shown in Figure 1.

The graphics facility in our computer laboratory incorporates a storage oscilloscope, so that the visual display has a low dynamic range. Consequently, when the display threshold is set to reveal most of a speckle image, neighbouring speckles are almost completely merged into each other. However, by applying a gradually decreasing threshold to the display, the brightest points in each of the brightest speckles are conveniently revealed.

When the cluster shown in Figure 1 is simulated optically and is viewed through simulated seeing, the brightest points in each of the brightest speckles in a typical speckle image appear as shown in Figure 2. The relative intensities of each of these brightest points
(they are two-dimensional delta functions, in effect) are the same as the relative intensities of the corresponding speckles.

We call Figure 2 a speckle mask because, when we perform this processing purely optically (cf. Bates et al., 1978), we place a transparent sheet over the speckle image, affix a black dot to the sheet over the centre of each of the brightest speckles (the diameter of the dot is proportional to the brightness of the corresponding speckle) and then photograph the sheet - the result is a mask suitable for performing an optical cross-correlation operation.

By cross-correlating the speckle image with the speckle mask, we obtain the correlation image, in which the brightest speckles are superimposed upon each other. The idea put forward by lynds et al. (1976) is that, since the brightest speckles can be interpreted as distorted images of the object (when it is isolated and somewhat larger than $\lambda / \mathrm{D}$ ), an improved image is formed by superimposing these speckles. Our extension of this idea is the suggestion that the brightest star in the cluster can generally be associated with the brightest point in each of the brightest speckles. We therefore expect the correlation image to be a distorted image of the cluster.

By gradually reducing the threshold applied to the display of the correlation image, we uncover individual stars in the cluster. The fainter stars are often hidden in the noisy background which permeates most correlation images. However, it is usually only necessary to recognise a few of the stars in the cluster, in order to be able to recover the remainder by the data reduction procedure described in the next section.

Figure 3 shows the recognisable stars abstracted from the correlation image which was formed by correlating the speckle mask shown in Figure 2 with its corresponding speckle image.

We give the name basic star pattern to the image consisting of delta functions having the same relative positions and intensities as the isolated stars that are recognisable in a correlation image. Figure 4 shows the basic star pattern derived from the correlation image shown in Figure 3.

## 2. CORRELATION PROCESSING

Figure 5 shows the autocorrelation of the cluster shown in Figure 1. The autocorrelation can be obtained in practice by processing a large number of speckle images according to Labeyrie's (1970) original prescription. Labeyrie's form of speckle interferometry has a sound and simple, yet widely applicable, theoretical basis. So one can be confident that the autocorrelation is as accurate as the observational conditions permit. However, the true image cannot be recovered unequivocally from the autocorrelation unless extra information is

## ${ }^{58}$

## 134 ${ }_{1}^{140}$ ${ }^{255}$ ${ }^{38}$

Figure l. Star cluster C.


Figure 3. Recognisable stars abstracted from correlation image.


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Figure 5. Autocorrelation A.


Figure 2. Speckle mask.
${ }^{74}$


Figure 4. Basic star pattern $\overline{\mathrm{B}}$.
${ }^{255} \quad 158$
$\stackrel{148}{0}$
$\stackrel{74}{\circ}$

Figure 6. Inversion $\overline{\mathrm{B}}_{\mathrm{i}}$ of basic star pattern.
available (cf. Bates and Gough, 1975).

It is convenient to denote the autocorrelation and the basic star pattern by the symbols $A$ and $\bar{B}$ respectively. It is also convenient to use a subscript $i$ to denote the inversion of a pattern (e.g. for any pattern $P$, its inversion $P_{i}$ is defined to be its reflection about its origin - its origin is always taken to be the position of the brightest star in P). Figure 4 shows $B$. To make clear what we mean by inversion, $\bar{B}_{i}$ is shown in Figure 6.

The interpretation of the basic star pattern rests on much less firm theoretical foundations than does Labeyrie's original form of speckle interferometry. Consequently, $\bar{B}$ cannot be expected to be as accurate as A. In fact, the relative positions and intensities of the stars in $\vec{B}$ are likely to be significantly in error. However, as we argue below, $\bar{B}$ can often be expected to contain sufficient information to permit resolution of the ambiguities inherent in $A$.

The stars making up the pattern $C$ of the cluster can be separated into two patterns, $B$ and $R$ say :
$C=B+R$
Pattern $B$ contains those stars that have been recognised in $\bar{B}$ (but $B$ is free of the errors present in $\bar{B}$ ). Pattern $R$ contains the remainder of the stars in the cluster. Thus, A can be written as
$A=B * B+R * R+B * R+R * B$
where the asterisk denotes correlation. If there are $n$ stars in the pattern $R$ then $R * B$ consists of $n$ replicas of $B_{i}$. Each of these replicas is similar to $\bar{B}_{i}$. So, provided that $\bar{B}_{i}$ is not too distorted a version of $\mathrm{B}_{\mathbf{i}}$, each of these replicas (which we call matching patterns) can be identified by inspection of $A$.

Figure 7 shows $A$ (same as Figure 5) with $\bar{B}_{i}$ (same as Figure 6) superimposed upon it (in the form of black dots) in such a way as best to show up a matching pattern in A. Careful scrutiny of Figure 7 confirms that there is no other matching pattern. So we deduce that $\mathrm{n}=1$. The single star in R is positioned (with respect to the origin of $C$ ) at the same point as the black dot marked 21 is positioned (in Figure 7) with respect to the origin of $A$.

Since the autocorrelation can be taken as accurate, the matching pattern identified in Figure 7 is an accurate version of $B_{i}$ (see Figure 8). The basic star pattern has thus been corrected - it only had to be accurate enough to allow the matching pattern to be identified and to distinguish it from the inverted pattern - the latter consists of those circles marked by the horizontal arrows shown in Figure 7. The whole cluster $C$, as shown in Figure 1, is thus reconstructed.

Suppose that only the three brightest stars in the cluster had been identified in $\bar{B}$, which would then have been almost symmetrical about a line passing through the brightest star. It would have been impracticable to distinguish $\mathrm{B}_{\mathrm{i}}$ from B . There would have been additional ambiguity if only two stars had been identified in the basic star pattern. However, it should now be clear that a basic star pattern need contain only two stars if the brighter stars in the cluster are arranged appreciably asymmetrically, as in general they are likely to be. It should also be clear that any possibility of ambiguity becomes apparent as the data reduction proceeds. If it is found that more stars are needed in the basic pattern, then many speckle images may be subjected to speckle mask processing so that many correlation images can be averaged (this improves the signal-to-noise ratio, thereby increasing the probability of identifying more stars in the basic pattern).

## 3. RADIO ASTRONOMICAL PROCESSING

At high radio astronomical frequencies, the visibility phase cannot be measured as accurately (using a synthesis telescope) as can the fringe visibility. By Fourier transforming the square of the latter, the autocorrelation of the observed radio sources is constructed. If these sources contain a cluster $C$ of radio stars of small angular diameter, then the autocorrelation $A$ of this cluster can be abstracted by inspection from the autocorrelation of the whole radio source field.

Provided the measured visibility phase is not too inaccurate, Fourier transforming the observed complex visibility provides an image of the cluster which may be sufficiently well defined that distorted versions of the more intense stars in the cluster can be recognised. This permits a basic star pattern $\bar{B}$ to be formed. An accurate version of $C$ can then be constructed in exactly the manner described in the previous section.
4. CONCLUSIONS

The data reduction introduced here is a two-dimensional analogue of a procedure which we developed earlier (Bates and Napier, 1972). The present procedure represents an enormous computational simplification, although it is less general (cf. Bates, 1978). It has recently occurred to us that the principle of isomorphic replacement, as employed by X-ray crystallographers (cf. Lipson and Cochran, 1966), could be invoked to form a true image of a continuous distribution of brightness (e.g. a nebula) set as a background to a cluster containing at least one variable star (e.g. either a closely-orbiting pair that would be resolvable under perfect seeing conditions, or a spectroscopic binary).


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