ON THE DYNAMICS OF OPEN CLUSTERS

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It has already been pointed out in the literature that due to several causes, open star clusters dissipate with time. For instance, Rosseland showed that when external stars move through a cluster, they cause a perturbation of the motion of the stars in the cluster and could transfer enough momentum to individual stars to cause their escape from the cluster's gravitational field. In this way the cluster will lose stars gradually, i.e., it will dissipate. According to Rosseland the time needed for the star cluster to dissipate following the outlined mechanism is 10^{10} years. However, as pointed out by the author of this article in the supplement to the Russian edition of Rosseland's book, there is another factor that makes the life of the open cluster even shorter: the stars in the cluster have close encounters with each other, as a result of which they exchange kinetic energy and gradually tend towards the most probable distribution, i.e., a Maxwell-Boltzmann distribution. And this, as we shall see shortly, also causes the dissipation of the cluster.

The relaxation time, i.e. the time in which the encounters of the stars in the cluster will lead to statistical equilibrium, is given approximately by the formula:

$$\tau = \frac{3\sqrt{2}}{32\pi n} \quad \frac{v^3}{G^2 m^2 \ln\left(\frac{\rho}{\rho_0}\right)} , \tag{1}$$

where n is the number of stars per unit volume, m the stellar mass, G the gravitational constant, v the average stellar velocity in the cluster, ρ the radius of the cluster, and ρ_0 the distance at which the potential energy of two stars is equal to the average kinetic energy of stars in the cluster, i.e.

$$\rho_0 = \frac{2Gm}{v^2} . \tag{2}$$

Formula (1) was derived for the case of stars with equal masses.

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The average velocity v enters in formula (1) both explicitly and through ρ_0. To find v we shall assume that the cluster is stationary at any given moment of time. We can do that as the time necessary to change the distribution function of the stars in the cluster, considered as a system in phase space, is large compared to the time necessary for a star to cross the cluster. In the case of a stationary system consisting of particles attracted to each other according to Newton's law, using the virial theorem we can write

$$U = 2T, \tag{3}$$

where U is the absolute value of the potential energy of the system, and T is its kinetic energy.

The exact formula for U is:

$$U = \frac{1}{2} \sum_{i \neq k} \frac{Gm^2}{r_{ik}} , \tag{4}$$

where we shall assume again that all stellar masses are equal, and r_{ik} denotes the distance between the i^{th} and the k^{th} star. We shall replace all of the r_{ik}'s with their mean harmonic value which is apparently close to the radius of the cluster ρ. Therefore, approximately:

$$U = \frac{1}{2} \frac{GN(N-1)m^2}{\rho} ,$$

where N is the total number of stars in the cluster. For $N >> 1$,

$$U = \frac{1}{2} \frac{GN^2m^2}{\rho} .$$

On the other hand

$$2T = Nmv^2 .$$

Therefore the virial theorem assumes the form:

$$v^2 = \frac{GNm}{2\rho} . \tag{4}$$

Comparing (4) with (2), we find that

$$\ln\left(\frac{\rho}{\rho_0}\right) = \ln\left(\frac{N}{4}\right); \tag{5}$$

substituting (4) and (5) in (1) and taking into account that

$$n = \frac{N}{\frac{4}{3} \pi\rho^3} ,$$

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we find

$$\tau = \frac{2}{16 \ln \frac{N}{4}} \sqrt{\frac{N\rho^3}{Gm}} \quad . \tag{6}$$

Assuming that for a typical cluster $N = 400$, $\rho = 2$ parsecs, $m = 2\times10^{33}$g, we find the relaxation time to be $\tau \simeq 4\times10^7$ years.

The result of the evolution of the distribution towards a Maxwell-Boltzmann distribution is the appearance of stars with kinetic energy larger than the escape energy for the cluster. Such stars will leave the cluster. The whole question is, what is the percentage of such stars in a cluster with the Boltzmann distribution ? If this percentage is small, then the dissipation of the cluster as a result of this process will be very slow. It is apparent that the ratio of the number of stars which can escape in a relaxation time τ to the total number of stars in the cluster is equal to:

$$\rho = \frac{\int_{\varepsilon_0}^{\infty} \varepsilon^{-\frac{\varepsilon}{\Theta}} \sqrt{\varepsilon}\, d\varepsilon}{\int_{0}^{\infty} \varepsilon^{-\frac{\varepsilon}{\Theta}} \sqrt{\varepsilon}\, d\varepsilon} \quad , \tag{7}$$

where ε_0 is the escape energy, and Θ is equal to two thirds of the average kinetic energy, i.e.

$$\Theta = \frac{2}{3}\,\frac{T}{N} = \frac{1}{3}\,\frac{U}{N} \quad . \tag{8}$$

On the other hand the mean value of ε_0 , i.e. the escape energy, is equal

$$\varepsilon_0 = \overline{\sum_k} \frac{Gm^2}{r_{ik}} = \frac{2U}{N} \quad , \tag{9}$$

where the line over the sum indicates an average over i. Comparing (8) with (9) we find:

$$\varepsilon_0 = 6\Theta \ .$$

Substituting in (7), we obtain the approximation:

$$P \cong \frac{e^{-\varepsilon_0/\Theta}\ \Theta\sqrt{\varepsilon_0}}{\frac{1}{2}\sqrt{\pi}\ \Theta^{3/2}} = 2e^{-6}\sqrt{\frac{6}{\pi}} \quad ,$$

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i.e. one hundredth of the total number of stars should escape the cluster in a relaxation time. Therefore the dissipation time of the cluster should be of the order of several billion years.

This result was obtained for a cluster consisting of stars with equal mass. Therefore these numbers are applicable only to stars of the cluster with masses close to the average stellar mass in the cluster. For stars with masses two to three times smaller than the average, the escape time will be of the order of a few hundred million years. It is known that open clusters have few dwarfs. Perhaps the poverty in dwarfs shows the position of the cluster on its evolutionary path.

If we assume that the open clusters we observe are different stages in the evolution of one and the same cluster, in as much as the stars escaping from the cluster carry out positive kinetic energy, the total cluster energy

$$H = T - U \tag{10}$$

should decrease with the transition from richer to poorer clusters.

If we substitute (3) in (10), we find:

$$H = \frac{1}{2} U \quad . \tag{11}$$

Therefore under the above assumption, U should increase. The data in the article of Orlova shows that such an increase of U with the decrease of N is not observed.

Another possible hypothesis is that all clusters were formed approximately at the same epoch (perhaps even at the epoch of the formation of the galaxy itself). Then the evolution of rich clusters with a large diameter should be slower. Among other things, these rich and large clusters should contain a higher percentage of dwarfs. It seems to the author that this result is backed by observations. E.g. the clusters h and χ Persei are rich and contain a high percentage of dwarfs at the same time. On the other hand, a number of poor clusters have hardly any dwarfs.

Hence, it becomes clear that to make further conclusions it is of great interest to determine not only the luminosity function for different clusters, but also the total energy H, which according to (11) could be determined from the absolute value of the potential energy.