# THE ROTATION OF THE MARS PLANET

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## 1. INTRODUCTION

In a non-inertial reference frame, of which the motion is given, the differential equations describing the rotation of a planet around its center of mass can be derived either under the form of Euler type equations, or from certain relations between angular velocity vectors or even angles. Unfortunately these equations are not well adapted to an analytical integration. Nevertheless, with some simplifications, Ward (1973), and Christensen (1977), by considering the motion of the spin axis in a non-inertial reference frame attached to the moving orbital plane, have found very large periodic variations in the obliquity of the Mars planet over a period of the order 1.2 $10^{+5}$ years, which would be of great importance to the climatic history of the planet.
These oscillations cannot be attributed to the relative motion with respect to the orbit, but actually they follow geometrically from a resonance-type phenomenon which occurs basically in an inertial space.

On these basis and for the purpose of making a comprehensive investigation of the precession and nutations of the Mars planet, we have choosen to conduct it in an inertial system of coordinates axis (R).

## 2. METHOD

We have used the Euler dynamical equations with usual notations. Assuming that $\mathbf{r}$ remains constant, the method of variation of parameters leads to solve the differential systems :

$$\begin{cases} \dot{f} = J\ (L \cos st + M \sin st) \\ \dot{g} = J\ (L \sin st - M \cos st) \end{cases} \tag{1}$$

$$\begin{cases} \dot{\psi} = [f \sin(\varphi + st) - g \cos(\varphi + st)] / \sin\theta \\ \dot{\theta} = f \cos(\varphi + st) + g \sin(\varphi + st) \end{cases} \tag{2}$$



*R. L. Duncombe (ed.), Dynamics of the Solar System, 145–150.*

with $J = \frac{A+B}{2AB}$ and $s = \frac{1}{2}\left(\frac{C-A}{B} + \frac{C-B}{A}\right) r$

L and M can be derived from the expansion of the gravitational potential energy U of the present bodies (Borderies, 1977)under the form :

$$L = [\,Y\ (U)\ + X\ (U)\,]\ /2i, \quad M = [\,Y\ (U) - X\ (U)\,]\ /2$$

with $i^2 = -1$, and where the operators X and Y satisfy the following properties :

$$X\ (E^{m'}_{1m}) = (1 - m'+1)\ E^{m'-1}_{1m}$$

$$Y\ (E^{m'}_{1m}) = (1 + m'+1)\ E^{m'+1}_{1m}$$

$E^{m'}_{1m}$ is a Euler function.

These properties allow us to explicit completely the systems (1) and (2) in the most general case. In particular this has been done when the acting bodies, assumed to be punctual, were the Sun, Jupiter and the Earth. In the case of solar torques, a model (6,6) of harmonic coefficients has been considered for Mars. For nutations due to Jupiter and the Earth we have limited ourselves to $C_{20}$. The systems (1) and (2) are solved analytically by successive approximations. To zero order and with the secular part of U we obtain the precession. For the approximation to first order we take $\theta$ constant, $\psi$ and $\varphi$ linear, and mean orbital elements among which I and $\Omega$ have been derived from Brouwer and Van Woerkom (1951). For the next approximation, the main periodic variations are brought in the right hand side members of the equations.

## 3. INFLUENCE OF THE SUN

The precession rate produced by solar torques and then the nutations due to the direct effect of the Sun combined with the secular motions of Mars orbit have been computed by the above-mentionned method, and for any couple of gravity coefficients ($C_{1m'}$, $S_{1m'}$). With the current value of $\theta$ and a mean value for I, it is found that $\dot{\psi} = -711''$. per century.

As it was expected, potential coefficients other than $C_{20}$ produce nutations of much smaller amplitudes and tesseral harmonics cause short-periodic oscillations.

The noteworthy result is the resonant nutation with the coefficient $C_{20}$ and the argument $\Omega - \psi$, which is explained by the fact that the small factor $\dot{\Omega} - \dot{\psi}$ occurs in the denominator of the expressions of the amplitudes. This sets the problem of the mean value $\theta_0$ of $\theta$.
As yet we have not made this determination. However we have computed the nutations of Mars for several values of $\theta_0$, ranging from 20° to 30° and with a mean value Io of I. The obtained amplitudes for the resonant

nutations are listed in table 1.

| $\theta_o$ | $B_1$ | $A_1$ |
|---|---|---|
| 20° | 8° 39' 49" | - 3° 38' 5" |
| 22° | 7° 19' 43" | - 3° 32' 19" |
| 24° | 6° 11' 19" | - 3° 26' 12" |
| 26° | 5° 12' 9" | - 3° 19' 46" |
| 28° | 4° 20' 29" | - 3° 13' 5" |
| 30° | 3° 34' 59" | - 3° 6' 11" |

Table 1 - Amplitudes $A_1$ and $B_1$ of the resonant nutation on $\theta$ and $\psi$ respectively for several values of $\theta_o$

For the next approximation, the main variations of $\theta$, $\psi$ and I are brought in the right-hand side members of the equations which are then expanded as Taylor series to first order around mean values.

The nutations resulting from coupling with the variations $\Delta\theta = A_o \cos V_o$ and $\Delta\psi = B_o \sin V_o$, and depending on $C_{20}$, are shown in Table 2. The very important motion in longitude with argument $\Omega - \psi$ is the effect of the large variation of $\theta$ on the node of Mars equator with respect to the xy - plane of (R). The largest terms correspond to the arguments $\Omega - \psi$, $2\Omega - 2\psi$ and $3\Omega - 3\psi$. As expected, it is found that the other nutations have small amplitudes.

| T (x $10^4$ years) | | First Iteration | Second Iteration | |
|---|---|---|---|---|
| | | | $V_o = \Omega - \Psi$ | $V_o = 2\Omega - 2\Psi$ |
| 16.86 | $A_\Psi$ | 5° 3' 58" | 4° 40' 21" | 2" |
| | $A_\theta$ | - 3° 18' 47" | 27" | - 4" |
| 8.43 | $A_\Psi$ | 3' 6" | 18' 13" | 1' 5" |
| | $A_\theta$ | - 1' 32" | - 5' 49" | 0 |
| 5.62 | $A_\Psi$ | | 5" | 6" |
| | $A_\theta$ | | - 2" | - 2" |

Table 2 - Amplitudes $A_\psi$ and $A_\theta$ of the nutations with arguments $\Omega - \psi$, $2\Omega - 2\psi$, $3\Omega - 3\psi$. T is the period.

The long periodic behaviour of Mars orbit inclination can be known from the theory of the secular variations of the orbital elements of the principal planets by Brouwer and Van Woerkom (1951). In this theory secular variations of the orbital plane of a planet are expressed as :

$$P = \sin I \sin \Omega = \sum_{j=1}^{8} N_j \sin(-s'_j t + e_j) \qquad (3)$$

$$Q = \sin I \cos \Omega = \sum_{j=1} N_j \cos(-s'_j t + e_j) \qquad (4)$$

Where I and $\Omega$ are referred to the ecliptic and equinox of 1950.0 . A Fourier analysis of the function I (t) has been achieved and has shown that the larger variations can be fairly well represented by 5 superimposed sinusoïds of which we have determined the amplitudes and phases by a least-squares process. The obtained values are given in Table 3. For mean value of I we have found Io = 3°.57

| K | $H_K$ (ARC SECOND/ YEAR) | $M_K$ (DEGREES) | $D_K$ (DEGREES) |
|---|---|---|---|
| 1 | 18.744 | 0.67 | 32 |
| 2 | 8.100 | 0.39 | 170 |
| 3 | 6.990 | 0.24 | 307 |
| 4 | 2.221 | 0.17 | 259 |
| 5 | 1.110 | 1.66 | 221 |

Table 3. Frequencies ($h_k$) and corresponding amplitudes ($M_k$) and phase constants ($d_k$) for the inclination of Mars.

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An oscillation $\Delta I = M \cos v_1$ of the inclination (with $v_1 = ht + c$) results in nutations depending on $C_{2o}$ and given in table 4. The largest amplitudes are obtained with the arguments $\Omega - \psi - v_1$, $\Omega - \psi + v_1$, $2\Omega - 2\psi - v_1$, $2\Omega - 2\psi + v_1$ and $v_1$. They are listed in this order for each value of h.

| H (ARC SECOND/ YEARS) | T (x $10^4$ YEARS) | $A_\psi$ | $A_\theta$ |
|---|---|---|---|
| 18.744 | 4.90 | 8' 15" | - 5' 24" |
| | 11.72 | - 19' 43" | 12' 54" |
| | 3.80 | 16" | - 8" |
| | 38.44 | - 2' 39" | 1' 19" |
| | 6.91 | - 5' 43" | 0 |
| 8.100 | 8.21 | 8' 3" | - 5' 16" |
| | 312.95 | - 5° 6' 34" | 3° 20' 29" |
| | 5.52 | 13" | - 7" |
| | 17.82 | 43" | - 21" |
| | 16.00 | - 7' 42" | 0 |
| 6.990 | 8.83 | 5' 19" | - 3' 29" |
| | 186.17 | 1° 52' 13" | - 1° 13' 23" |
| | 5.80 | 9" | - 4" |
| | 15.46 | 23" | - 11" |
| | 18.54 | - 5' 30" | 0 |
| 2.221 | 13.08 | 5' 35" | - 3' 39" |
| | 23.71 | 10' 8" | - 6' 37" |
| | 7.37 | 8" | - 4" |
| | 9.85 | 10" | - 5" |
| | 58.36 | - 12' 15" | 0 |
| 1.110 | 14.73 | 1° 1' 26" | - 40' 10" |
| | 19.71 | 1° 22' 10" | - 53' 44" |
| | 7.86 | 1' 21" | - 40" |
| | 9.09 | 1' 33" | - 46" |
| | 116.73 | - 3° 59' 13" | 0 |

Table 4 - Nutations inferred from periodic variations of I

The expressions for nutations depending on $C_{2o}$ and arising from the torque exerted on Mars by another planet have also been developped by the method of variation of parameters. These nutations have been computed in the case of the Earth and Jupiter. The dominant effects come from the latter planet though the amplitude remains very small ($\Delta\psi = 1''.1$, $\Delta\theta = 0''7$ for the largest, with period $1.2\ 10^6$ years)

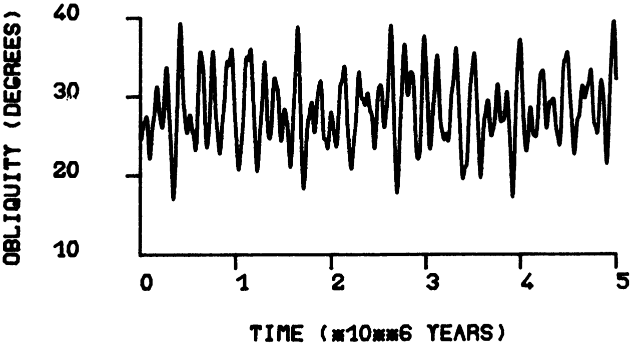


Fig. 1 Variations in the obliquity of Mars

## 4. CONCLUSION

Finally, we have plotted on fig.1 the behaviour of the obliquity $\epsilon$ with respect to the moving orbital plane of Mars. The periodic variations of $\epsilon$ look much more complex than those exhibited by Ward, due to the larger number of terms we took into account and to our more rigourous approach. Nevertheless, a term with an amplitude of around $10^\circ$ and period of $1.2\ 10^6$ years also exists in our results and we definitly think that such a phenomenon has now to be studied for its big impact on the planet climatic evolution.

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