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CORRECTION

MARKSTRÖM, K. (2007). Negative association does not imply log-concavity of the rank sequence. *J. Appl. Prob.* **44,** 1119–1121.

The counterexample given in the above paper does not exactly satisfy one of the inequalities in the definition of negative association. Here we present a new correct counterexample, still for $n = 3$, meaning that it is a smallest possible counterexample. However, while the new example breaks log-concavity, as desired, it is more well behaved, in the sense that it is unimodal.

We will describe our measure in terms of the probabilities for a binary string $x_1x_2x_3$, which can be seen as the characteristic function of a set in B_3 in the standard way.

Example 1. Define the measure

$$\mu(000) = \mu(\{111\}) = \tfrac{1}{100},$$
$$\mu(100) = \mu(010) = \tfrac{2}{100}, \qquad \mu(001) = \tfrac{4}{100},$$
$$\mu(110) = \mu(101) = \mu(011) = \tfrac{30}{100}.$$

Proposition 1. *The measure* μ *is negatively associated but its rank sequence is not log-concave.*

Proof. For $n = 3$, there are nine pairs of increasing events A and B for which we need to check the condition that $\mu(A \wedge B) \leq \mu(A)\mu(B)$. Due to the symmetries of the measure, the number of distinct pairs is smaller.

1. A pair of events of the form $x_i = 1$:

$$\begin{aligned}
p(x_1 = 1) = p(x_2 = 1) &= \tfrac{1}{100}(2 + 30 + 30 + 1) = \tfrac{63}{100},\\
p(x_3 = 1) &= \tfrac{1}{100}(4 + 30 + 30 + 1) = \tfrac{65}{100},\\
p(x_1 = 1 \wedge x_3 = 1) = p(x_2 = 1 \wedge x_3 = 1)& \\
&= \tfrac{1}{100}(30 + 1)\\
&= \tfrac{31}{100}\\
&\leq p(x_1 = 1)p(x_3 = 1)\\
&= \frac{63 \times 65}{100^2}\\
&\approx 0.4095,\\
p(x_1 = 1 \wedge x_2 = 1) &= \tfrac{1}{100}(30 + 1)\\
&= \tfrac{31}{100}\\
&\leq p(x_1 = 1)p(x_2 = 1)\\
&= \frac{63^2}{100^2}\\
&\approx 0.3969.
\end{aligned}$$

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2. A single variable and event of the form $(x_i = 1 \wedge x_j = 1)$. For i, j, k all distinct,

$$\begin{aligned} p((x_i = 1 \wedge x_j = 1) \wedge (x_k = 1)) &= p(x_1 = 1 \wedge x_2 = 1 \wedge x_3 = 1) \\ &= \tfrac{1}{100} \\ &\leq p((x_i = 1 \wedge x_j = 1))p(x_k = 1) \\ &= \frac{31 \times 63}{100^2} \\ &\approx 0.19. \end{aligned}$$

For $k = 3$, the last probability will be even larger.

3. The events $(x_1 = 1 \vee x_2 = 1)$ and $x_3 = 1$:

$$\begin{aligned} p(x_1 = 1 \vee x_2 = 1) &= \tfrac{1}{100}(2 + 2 + 30 + 30 + 30 + 1) = \tfrac{95}{100}, \\ p((x_1 = 1 \vee x_2 = 1) \wedge (p(x_3 = 1))) &= \tfrac{1}{100}(30 + 30 + 1) \\ &= \tfrac{61}{100} \\ &\leq p((x_1 = 1 \vee x_2 = 1))p(x_3 = 1) \\ &= \frac{95 \times 65}{100^2} \\ &\approx 0.6175. \end{aligned}$$

4. For distinct i, j, the events $(x_i = 1 \vee x_3 = 1)$ and $x_j = 1$:

$$\begin{aligned} p(x_1 = 1 \vee x_3 = 1) = p(x_2 = 1 \vee x_3 = 1) &= \tfrac{1}{100}(2 + 4 + 30 + 30 + 30 + 1) = \tfrac{97}{100}, \\ p((x_2 = 1 \vee x_3 = 1) \wedge (p(x_1 = 1))) = p((x_1 = 1 \vee x_3 = 1) \wedge (p(x_2 = 1))) & \\ &= \tfrac{1}{100}(30 + 30 + 1) \\ &= \tfrac{61}{100} \\ &\leq p((x_1 = 1 \vee x_3 = 1))p(x_2 = 1) \\ &= \frac{97 \times 3}{100^2} \\ &\approx 0.6111. \end{aligned}$$

This concludes the proof that the measure is negatively associated.

To see that the rank sequence is not log-concave, we note that the rank sequence is

$$(r_0, r_1, r_2, r_3) = \left(\tfrac{1}{100}, \tfrac{8}{100}, \tfrac{90}{100}, \tfrac{1}{100}\right),$$

and that $r_1^2 - r_0 r_2 = -\frac{26}{100} \not> 0$.

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