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The less-is-more effect: Predictions and tests

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Abstract

In inductive inference, a strong prediction is the less-is-more effect: Less information can lead to more accuracy. For the task of inferring which one of two objects has a higher value on a numerical criterion, there exist necessary and sufficient conditions under which the effect is predicted, assuming that recognition memory is perfect. Based on a simple model of imperfect recognition memory, I derive a more general characterization of the less-is-more effect, which shows the important role of the probabilities of hits and false alarms for predicting the effect. From this characterization, it follows that the less-is-more effect can be predicted even if heuristics (enabled when little information is available) have relatively low accuracy; this result contradicts current explanations of the effect. A new effect, the below-chance less-is-more effect, is also predicted. Even though the less-is-more effect is predicted to occur frequently, its average magnitude is predicted to be small, as has been found empirically. Finally, I show that current empirical tests of less-is-more-effect predictions have methodological problems and propose a new method. I conclude by examining the assumptions of the imperfect-recognition-memory model used here and of other models in the literature, and by speculating about future research.

Keywords: less-is-more effect, recognition heuristic, recognition memory.

1 A strong prediction: The less-is-more effect

In psychology's quest for general laws, the *effort-accuracy* tradeoff (Garrett, 1922; Hick, 1952) is a top candidate: The claim is that a person cannot put less effort in a task and increase accuracy. Because it is widely accepted, the tradeoff provides an opportunity for theory development. If a theory implies that the effort-accuracy tradeoff can be violated, this is a strong prediction. By strong prediction of a theory I mean a prediction that does not follow from most competing theories (see Trafimow, 2003). A strong prediction provides for informative tests. It is unlikely that the data will confirm the prediction, but if they do, support for the theory will increase greatly. For example, Busemeyer (1993) provided support for decision field theory by showing that it is consistent with violations of the speed-accuracy tradeoff.

The speed-accuracy tradeoff is one instantiation of the effort-accuracy tradeoff in which the effort a person puts into performing a task is measured by the time she uses. Effort can also be measured by the amount of other resources that are expended, such as information or computation. In this paper, I consider violations of the tradeoff between information and accuracy. This violation is particularly interesting because it invites us to sometimes throw away information. This invitation flies in the face of epistemic responsibility, a maxim cherished by philosophers and practitioners of science (Bishop, 2000). Some violations of the information-accuracy tradeoff, where information refers to recognition, have been predicted and observed in tasks of inductive inference, and are collectively referred to as the *less-is-more* effect (Gigerenzer, Todd, & the ABC research group, 1999).

Here, I make two kinds of contributions to the study of the less-is-more effect: to the theoretical predictions of the effect, and to the empirical tests of the predictions. Specifically, in the next section, I define the inference task and the less-is-more effect, and present Goldstein and Gigerenzer's (2002) characterization of the less-is-more effect for the case of perfect memory. In Section 3, for imperfect memory, I derive a more general characterization of the conditions under which the less-is-more effect is predicted. The predictions are illustrated numerically with parameter estimates from the recognition memory literature. Furthermore, I discuss implications for theoretical explanations of the effect. A new type of less-is-more effect is also predicted. In the next section, I discuss methodological issues in empirically testing less-is-more effect predictions, and present a method for doing so by using data from recognition experiments. In Section 5, I conclude by examining the assumptions of the imperfect-memory model used here and of other models in the literature, and by speculating about future research.

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2 A characterization of the less-is-more effect: Perfect memory

2.1 The inference task and the less-is-more effect

I first define the inference task: There exist N objects that are ranked, without ties, according to a numerical criterion. For example, the objects may be cities and the criterion may be population. A pair of objects is randomly sampled and the task is to infer which object has the higher criterion value. When recognition memory is perfect, a person's information is modeled by the number of objects she recognizes, n, where $0 \leq n \leq N$. The person's accuracy is the probability of a correct inference and it is a function of her information, $Pr(n)$. I next define the less-is-more effect.

Definition 1. When recognition memory is perfect, the less-is-more effect occurs if and only if there exist n_1 and n_2 such that $n_1 < n_2$ and $Pr(n_1) > Pr(n_2)$.

The less-is-more effect may at first appear to be impossible: Of course, whatever processing can be done with less information (recognizing n_1 objects), could also be done when adding information (recognizing the same n_1 objects and an extra $n_2 - n_1$ objects); so, how can less information lead to more accuracy? The catch is that different amounts of information may lead to different processing. For example, when the population of two cities is compared, recognizing the name of one of the two cities enables the *recognition heuristic* (Goldstein & Gigerenzer, 2002):

"If one object is recognized and the other object is not, the recognized object is inferred to have the higher criterion value."

The recognition heuristic cannot be used when both objects are recognized so that some other inference rule has to be used, such as a linear rule with unit weights (Dawes & Corrigan, 1974). The processing done with less information may be more accurate than the processing done with more information. For example, the recognition heuristic uses one cue (recognition) and there are conditions under which using one cue is more accurate than summing many cues (Hogarth & Karelaia, 2005; Katsikopoulos & Martignon, 2006; this point has also been made specifically for the recognition cue by Davis-Stober, Dana, & Budescu, 2010).

To test empirically the occurrence of the less-is-more effect, one can compare the accuracy of (i) two agents (individuals or groups) who recognize different amounts of objects out of the same set (e.g., American cities), or (ii) the same agent who recognizes different amounts of objects in two different sets (e.g., American and German cities).

Pachur (in press) reviewed a number of studies and concluded that there is some evidence for the less-is-more effect in some experiments, but not in others. I agree with this conclusion: Goldstein and Gigerenzer (2002) had about a dozen American and German students infer which one of San Diego or San Antonio is more populous, and found that the Germans were more accurate (1.0 vs. .67). They also found that 52 American students were equally accurate (.71) in making 100 population comparisons of German, or American cities. Reimer and Katsikopoulos (2004) had three-member groups of German students perform 105 population comparisons of American cities. Out of seven pairs of groups, the groups who recognized fewer cities were more accurate in five cases, by .04 on the average. Pohl (2006) had 60 German students compare the populations of 11 German, 11 Italian, and 11 Belgian cities. The students recognized more German than Italian cities and more Italian than Belgian cities, and their accuracies had the same order: .78 for German, .76 for Italian, and .75 for Belgian cities. Pachur and Biele (2007) asked laypeople and soccer experts to forecast the winners of matches in the 2004 European national-teams soccer tournament. In 16 matches, experts were more accurate than laypeople (.77 vs. .65). The correlation between the accuracy of 79 laypeople and the number of teams they recognized was positive (.34).

I believe that all of these tests of less-is-more-effect predictions have methodological problems and I will make this point and suggest a new method in Section 4.

2.2 The perfect-memory model

Goldstein and Gigerenzer (2002) derived an equation for $Pr(n)$ based on a model of how a person makes inferences, which I now describe. The first assumption of the *perfect-memory* model is the following:

Assumption 1. The person recognizes all objects she has experienced, and does not recognize any object she has not experienced.

It is easy to criticize this assumption as it is known that recognition memory is not perfect (Shepard, 1967). Pleskac (2007) and Smithson (2010) proposed models of imperfect recognition memory, and I will discuss them together with a new model, in Section 3.

The following three assumptions specify the inference rules used for the different amounts of recognition information that may be available:

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Assumption 2. If the person does not recognize any of the objects in the pair, she uses guessing to infer which object has the higher criterion value.

Assumption 3. If the person recognizes one object, she uses the recognition heuristic.

Assumption 4. If the person recognizes both objects, she uses an inference rule other than guessing or the recognition heuristic-this family of rules is labeled as knowledge.

Based on Assumptions (2)–(4), if the person recognizes n of the N objects, she uses guessing, recognition heuristic, and knowledge with the following respective probabilities:

$$\begin{aligned} g(n) &= (N-n)(N-n-1)/N(N-1) \\ r(n) &= 2n(N-n)/N(N-1) \\ k(n) &= n(n-1)/N(N-1) \end{aligned} \quad (1)$$

The last assumption of the perfect-memory model is the following:

Assumption 5. The accuracy of the recognition heuristic,α, and the accuracy of knowledge, β, are constant across n (and α, $\beta > \frac{1}{2}$).

Smithson (2010) pointed out that Assumption 5 could be potentially violated, and constructed plausible examples where this is the case.[1] Pachur and Biele (2007) reported that the correlation between α and n was .19 and the correlation between β and n was .22. Across ten experiments, Pachur (in press) reported that the average of the absolute value of the correlation between α and n was .27 and the average of the absolute value of the correlation between β and n was .18. In Section 4, I will argue that the estimates of α and β used in these studies are incorrect, and the reported correlations should be interpreted with caution.

From Assumption 5 and Equation (1), it follows that the accuracy of a person who recognizes n objects is given as follows:

$$Pr(n) = g(n)(\tfrac{1}{2}) + r(n)\alpha + k(n)\beta \quad (2)$$

[1]There are more criticisms of the perfect-memory model. For example, Dougherty, Franco-Watkins, and Thomas (2008) have suggested that the notion of familiarity should be incorporated in the model and that recognition is not an all-or-none variable.

Example 1. Take $N = 100$. If $\alpha = .8$ and $\beta = .6$, then $Pr(50) = .68$ and $Pr(100) = .6$, and a less-is-more effect is predicted. If $\alpha = .8$ and $\beta = .8$, then $Pr(50) = .73$ and $Pr(100) = .8$, and more generally $Pr(n_1) < Pr(n_2)$ for all $n_1 < n_2$, and the effect is not predicted.

It is straightforward to study Equation (2) and derive the following characterization of the less-is-more effect (Goldstein & Gigerenzer, 2002, p. 79):

Result 1. For the perfect-memory model, the less-is-more effect is predicted if and only if $\alpha > \beta$.[2]

Remark 1. More specifically, it holds that if $\alpha > \beta$, there exists $n*$ such that $Pr(n*) > Pr(N)$: There is a person with an intermediate amount of information who is more accurate than the person with all information. And, if $\alpha \leq \beta$, then $Pr(N) \geq Pr(n)$ for all $n < N$, meaning that the person with all information makes the most accurate inferences.

3 A characterization of the less-is-more effect: Imperfect memory

The inference task is the same with that of Section 2: There exist N objects that are ranked, without ties, according to a numerical criterion; for example, the objects may be cities and the criterion may be population. A pair of objects is randomly sampled and the person has to infer which object has the higher criterion value. The person's information is modeled by the number of objects she has *experienced*, n_e, where $0 \leq n_e \leq N$. The person's accuracy is the probability of a correct inference and it is a function of her information, $Pr(n_e)$. We have the following definition of the less-is-more effect.

Definition 2. When recognition memory is imperfect, the less-is-more effect occurs if and only if there exist $n_{e,1}$ and $n_{e,2}$ such that $n_{e,1} < n_{e,2}$ and $Pr(n_{e,1}) > Pr(n_{e,2})$.

I next propose a simple model of how people make inferences when recognition memory is imperfect.

3.1 An imperfect-memory model

In this model, an experienced object that is recognized is called a *hit*; an experienced object that is not recognized

[2]This result also holds for groups that decide by the simple majority rule if the members (i) have the same n, α , and β , and (ii) recognize objects and make inferences independently of each other (Reimer & Katsikopoulos, 2004, p. 1126).

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is called a *miss*; a non-experienced object that is recognized is called a *false alarm*; and, a non-experienced object that is not recognized is called a *correct rejection*. A main assumption of my imperfect-memory model is the following:

Assumption 6. Each experienced object has probability h of being a hit, and each non-experienced object has probability f of being a false alarm.

Assumption 1 of the perfect-memory model can be obtained from Assumption 6 by setting $h = 1$ and $f = 0$. The differences between the other assumptions of the imperfect- and perfect-memory models are due to the fact that the role played by the recognition cue when memory is perfect is played by the experience cue when memory is imperfect. A person with imperfect memory cannot use the experience cue (they have access only to the recognition cue), but let us still define the *experience heuristic*:

"If one object is experienced and the other object is not, the experienced object is inferred to have the higher criterion value."

The experience heuristic is a convenient device for analyzing the less-is-more effect. The following is assumed:

Assumption 7. The accuracy of the experience heuristic, A, and the accuracy of knowledge when both objects are experienced, B, are constant across n_e (and A, $B > \frac{1}{2}$).

Assumption 7 generalizes Assumption 5 of the perfect-memory model because when memory is perfect it holds that $A = \alpha$ and $B = \beta$.

Now I specify the processing used for the different amounts of experience information that may be available. There are three types of pairs of objects that can be sampled, according to whether the number of experienced objects in the pair equals zero, one, or two. As in (1), these types occur with respective probabilities $g(n_e)$, $r(n_e)$, and $k(n_e)$. As Pleskac (2007) pointed out, accuracy depends not only on which of guessing, experience heuristic, or experience-based knowledge is used, but also on if each object in the pair is a hit, miss, false alarm, or correct rejection.

For example, assume that object 1 is experienced and object 2 is not experienced. There are four possibilities: (i) object 1 is a hit and object 2 is a correct rejection, (ii) object 1 is a hit and object 2 is a false alarm, (iii) object 1 is a miss and object 2 is a correct rejection, and (iv) object 1 is a miss and object 2 is a false alarm. In (i), the agent uses the recognition heuristic to make an inference because she recognizes only object 1. Because in this case the recognition and experience cues have the same value on both objects, it is as if the agent used the experience heuristic, and her accuracy equals A[3].

Similarly, Pleskac (2007, p. 384-385) showed that accuracy equals $1-A$ in (iv), $\frac{1}{2}$ in (iii), and $zA+(1-z)(\frac{1}{2})$ in (ii) (z is the probability that an experienced object has at least one positive value on a cue other than experience). Similarly, when the agent has experienced both objects, it can be shown that accuracy equals B if both objects are hits and $\frac{1}{2}$ otherwise. When none of the two objects is experienced, it can be shown that accuracy always equals $\frac{1}{2}$.

Pleskac interpreted h as the proportion of experienced objects that are recognized and f as the proportion of non-experienced objects that are recognized, and did not derive a simple equation for $Pr(n_e)$. When h and f are interpreted as probabilities, it is straightforward to do so according to the logic above, as follows:

$$\begin{aligned}Pr(n_e) = r(n_e)[&h(1-f)A + hf(zA + (1-z)(\tfrac{1}{2}))+\\ &(1-h)(1-f)(\tfrac{1}{2}) + (1-h)f(1-A)]+\\ &k(n_e)[h^2B + (1-h^2)(\tfrac{1}{2})] + g(n_e)(\tfrac{1}{2}).\end{aligned}$$

Rewriting this, we get the main equation of the imperfect-memory model:

$$Pr(n_e) = g(n_e)(\tfrac{1}{2}) + r(n_e)\alpha_e + k(n_e)\beta_e, \qquad (3)$$

where $\alpha_e = (h - f + hfz)A + (1 - h + f - hfz)(\frac{1}{2})$, and $\beta_e = h^2B + (1 - h^2)(\frac{1}{2})$.

Equation (3) says that the inference task with imperfect recognition memory can be viewed as an inference task with perfect recognition memory where (i) the accuracy of the recognition heuristic α_e equals a linear combination of the accuracy of the experience heuristic A and $\frac{1}{2}$, and (ii) the accuracy of recognition-based knowledge β_e equals a linear combination of the accuracy of experience-based knowledge B and $\frac{1}{2}$. It holds that α_e and β_e are well defined in the sense that they lie between 0 and 1.

Example 2. Take $N = 100$. If $A = .8$, $B = .8$, $h = .64$, $f = .02$, and $z = .5$, then $Pr(70) = .64$ and $Pr(100) = .62$, and a less-is-more effect is predicted.

[3]Note that for this and the following claims to hold, it has to be assumed that the experience and recognition cues are conditionally independent given the criterion. This means that if the criterion value of an object is known, the probability that the object has been experienced does not change depending on whether the object will be recognized or not. Pleskac (2007) justifies this assumption by saying that the value of A reflects the true correlation between environmental mediators and the criterion.

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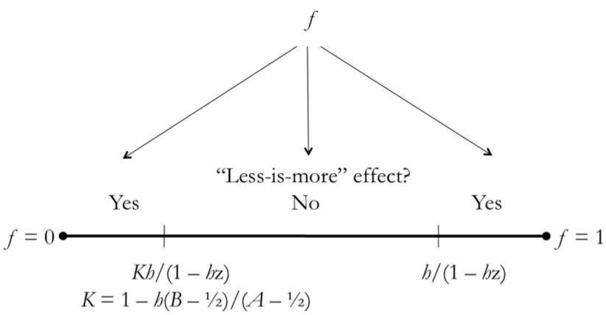
Figure 1: The conditions under which the less-is-more effect is and is not predicted for the imperfect-memory model (A is the accuracy of the experience heuristic, B is the accuracy of experience-based knowledge, h is the probability of a hit, f is the probability of a false alarm, z is the probability that an experienced object has at least one positive value on a cue other than experience). For simplicity, the figure was drawn assuming that $Kh/(1-hz) > 0$ and $h/(1-hz) < 1$, but this is not necessarily the case.



More generally, Figure 1 depicts graphically the conditions under which the less-is-more effect is predicted, and Result 2 states and proves this characterization.

Result 2. For the imperfect-memory model, the less-is-more effect is predicted if and only if either (i) $f < Kh/(1-hz)$, where $K = 1 - h(B - \frac{1}{2})/(A - \frac{1}{2})$, or (ii) $f > h/(1-hz)$.

Proof. Note that if $h - f + hfz > 0$, (3) implies $\alpha_e > \frac{1}{2}$. Also, note that it always holds that $\beta_e > \frac{1}{2}$.

Assume $h - f + hfz > 0$. Result 1 applies and the less-is-more effect is predicted if and only if $\alpha_e > \beta e$. The condition $\alpha_e > \beta_e$ can be rewritten as $f < Kh/(1-hz)$, where $K = 1 - h(B - \frac{1}{2})/(A - \frac{1}{2})$. The condition $h - f + hfz > 0$ is equivalent to $f < h/(1-hz)$, which, because $K < 1$, is weaker than $f < Kh/(1-hz)$. Thus, if $f < Kh/(1-hz)$, the less-is-more effect is predicted.

The above reasoning also implies that if $Kh/(1-hz) \leq f < h/(1-hz)$, the less-is-more effect is not predicted. Also, if $f = h/(1-hz)$, then $\alpha_e = 0$, and the effect is not predicted.

Assume $f > h/(1-hz)$. Then, it holds that $\alpha_e < \frac{1}{2}$ and Result 1 does not apply. The second derivative of $Pr(n_e)$ equals $(1-2\alpha_e) + 2(\beta_e - \alpha_e)$, which is strictly greater than zero because $\alpha_e < \frac{1}{2}$ and $\alpha_e < \beta_e$. Thus, $Pr(n_e)$ is convex, and a less-is-more effect is predicted.

Remark 2. Result 2 generalizes Result 1: For a perfect memory, it holds that $h = 1$, $f = 0$, $A = \alpha$, and $B = \beta$. For $h = 1$ and $f = 0$, condition (ii) is always violated. From $h = 1$ and $f = 0$, condition (i) reduces to $K > 0$, which, because $h = 1$, is equivalent to $A > B$, or $\alpha > \beta$.

Example 2 (continued). Consistently with Result 2, if $A = .8$, $B = .8$, $h = .64$, $f = .02$, and $z = .5$, condition (i) is satisfied: $f = .02 < .34 = Kh/(1-hz)$, and a less-is-more effect is predicted. If $A = .8$, $B = .8$, $h = .64$, $f = .64$, and $z = .5$, then conditions (i) and (ii) in Result 2 are violated and a less-is-more effect is not predicted. If $A = .8$, $B = .8$, $h = .37$, $f = .64$, and $z = .5$, then condition (ii) in Result 2 is satisfied: $f = .64 > .45 = h/(1-hz)$, and a less-is-more effect is predicted; for example, $Pr(0) = .5 > .48 = Pr(30)$. In paragraph 3.3, I discuss the meaning of this example in more detail. But first I differentiate between the two types of less-is-more effects in Result 2.

3.2 Full-experience and below-chance less-is-more effects

The predicted less-is-more effect is qualitatively different in each one of the two conditions in Result 2. If (i) is satisfied, $Pr(n_e)$ has a maximum and there exists a n_e* such that $Pr(n_e*) > Pr(N)$, whereas if (ii) is satisfied, $Pr(n_e)$ has a minimum and there exists a n_e* such that $Pr(0) > Pr(n_e*)$[4]. In other words, in (i), there is a person with an intermediate amount of experience who is more accurate than the person with all experience, and in (ii), there is a person with no experience who is more accurate than a person with an intermediate amount of experience. I call the former a *full-experience* less-is-more effect and the latter a *below-chance* less-is-more-effect. The effects studied in the literature so far were of the full-experience (or full-recognition) type, interpreted as saying that there are beneficial degrees of ignorance (Schooler & Hertwig, 2005). The below-chance effect says that there could be a benefit in complete ignorance.

Result 2 tells us that the less-is-more effect is predicted, but not its magnitude. To get a first sense of this, I ran a computer simulation varying A and B from .55 to 1 in increments of .05, and h, f, and z from .05 to .95 in increments of .05 (and $N = 100$). There were $10^2 20^3 = 800,000$ combinations of parameter values. For each combination, I checked whether the less-is-more effect was predicted, and, if yes, of which type. The frequency of a less-is-more effect type equals the proportion of parameter combinations for which it is predicted. For the combinations where an effect is predicted, two additional indexes were calculated.

[4] In both cases, the optimum of $Pr(n_e)$ is achieved at $n_e* = [(1-2\alpha_e)N + \beta_e - \frac{1}{2}]/[(1-2\alpha_e) + 2(\beta_e - \alpha_e)]$, and it holds that $0 < n_e* < N$.

Table 1: The frequency, average prevalence, and average magnitude of the full-experience and below-chance less-is-more effects (see text for definitions), for the imperfect-memory model (varying A and B from .55 to 1 in increments of .05, and h, f, and z from .05 to .95 in increments of .05; and $N = 100$), and for the perfect-memory model ($h = 1$ and $f = 0$).

	Frequency (Full Exp.)	Frequency (B.Chance)	Avg. Prev. (Full Exp.)	Avg. Prev. (B.Chance)	Avg. Mag. (Full Exp.)	Avg. Mag. (B.Chance)
Imperfect memory	29%	37%	23%	36%	.01	.01
Perfect memory	50%	0%	23%	0%	.02	.00

The *prevalence* of a less-is-more effect equals the proportion of pairs ($n_{e,1}$, $n_{e,2}$) such that $0 \leq n_{e,1} < n_{e,2} \leq N$ and $Pr(n_{e,1}) > Pr(n_{e,2})$ (Reimer & Katsikopoulos, 2004). I report the average, across all parameter combinations, prevalence of the two less-is-more-effect types.

The *magnitude* of a less-is-more effect equals the average value of $Pr(n_{e,1}) - Pr(n_{e,2})$ across all pairs ($n_{e,1}$, $n_{e,2}$) such that $0 \leq n_{e,1} < n_{e,2} \leq N$ and $Pr(n_{e,1}) > Pr(n_{e,2})$. I report the average, across all parameter combinations, magnitude of the two less-is-more-effect types.

The simulation was run for both imperfect- and perfect-memory (where $h = 1$ and $f = 0$) models. The results are provided in Table 1. Before I discuss the results, I emphasize that the simulation assumes that all combinations of parameters are equally likely. This assumption is unlikely to be true, and is made because of the absence of knowledge about which parameter combinations are more likely than others.

The first result of the simulation is that the imperfect-memory model predicts a less-is-more effect often, abo3ut two-thirds of the time, 29% for the full-experience effect plus 37% for the below-chance effect. The perfect-memory model cannot predict a below-chance effect, and it predicts a lower frequency of the less-is-more effect (50%) than the imperfect-memory model. The second result is that, according to the imperfect-memory model, the below-chance effect is predicted to have higher average prevalence than the full-experience effect. Note also that the distribution of the prevalence of the below-chance effect is skewed: almost 50% of the prevalence values are higher than 45% (the prevalence distribution was close to uniform for the full-experience effect). The third result of the simulation is that the average magnitude of both less-is-more-effect types is small (.01 or .02); for example, in the imperfect-memory model, only about 5% of the predicted less-is-more effects have a magnitude higher than .05.[5] This result is consistent with conclusions from empirical research (Pohl, 2006; Pachur & Biele, 2007; Pachur, in press). I would like to emphasize, however, that even small differences in accuracy could be important in the "real world", as, for example, in business contexts.

[5]For both effects, maximum magnitude equals .25, and maximum prevalence equals 50%.

3.3 The accurate-heuristics explanation

In the predictions of the less-is-more effect in Example 2, we had $A = B$. This seems curious because a necessary and sufficient condition for predicting the effect when memory is perfect is $\alpha > \beta$ (see Result 1 and Example 1). In fact, Pleskac (2007) has argued that the condition $A > B$ is necessary for the less-is-more effect when memory is imperfect.

The conditions $\alpha > \beta$ or $A > B$ express that a heuristic (recognition or experience), is more accurate than the knowledge used when more information is available. If $\alpha > \beta$ or $A > B$, the less-is-more effect can be explained as follows: "Less information can make more likely the use of a heuristic, which is more accurate than knowledge." This is an explanation commonly, albeit implicitly, proposed for the effect (e.g., Hertwig & Todd, 2003, Theses 2 & 3), and I call it the *accurate-heuristics* explanation.

Result 2 speaks against the accurate-heuristics explanation because it shows that $A > B$ is neither necessary nor sufficient for predicting the less-is-more effect. The condition for the full-experience effect, $f < Kh/(1 - hz)$, can be interpreted as indicating a small f and a medium h, where f and h can compensate for $A \leq B$.[6] The condition for the below-chance effect, $f > h/(1 - hz)$, is independent of A and B, and can be interpreted as indicating a large f and a small h[7] (simu-

[6]All else being equal, increasing A and decreasing B does make this condition easier to satisfy because it increases $K = 1 - h(B - \frac{1}{2})/(A - \frac{1}{2})$.

[7]A sketch of the role of misses and false alarms for predicting the less-is-more effects has as follows: For the full-experience effect, as the number of objects experienced increases from some to all, there are more pairs of objects where both objects are misses (h is medium), and guessing is used inappropriately instead of the experience heuristic or knowledge. For the below-chance effect, as the number of objects experienced increases from zero to some, there are more pairs of objects where one object is a miss (h is small, but nonzero) and the other object is a false alarm (f is large) and the experience heuristic is used inappropriately.

lations showed that z has a minor influence on both conditions).

In sum, Result 2 shows that it is the imperfections of memory (probabilities of misses and false alarms) that seem to drive the less-is-more effect, rather than whether the enabled heuristic (the experience heuristic) is relatively accurate or not.

I illustrate the evidence against the accurate-heuristics explanation by using parameter values from the recognition memory literature. Jacoby, Woloshyn, and Kelley (1989) provide estimates for h and f for the recognition of names.[8] In each of two experiments, a condition of full attention and a condition of divided attention were run. In the divided-attention condition, participants were distracted by having to listen long strings of numbers and identify target sequences. In the full-attention condition, the average value of h was $(.65 + .63)/2 = .64$, and I used this value as an estimate of a high probability of a hit. In the divided-attention condition, the average h was $(.43 + .30)/2 = .37$, and this is my estimate of a low hit probability.

As an estimate of a low probability of a false alarm, I used the average value of f in the full-attention condition, $(.04 + 0)/2 = .02$. I did not, however, use the average value of f in the divided-attention condition as an estimate of a high probability of a false alarm. This value $(.11)$ does not seem to represent situations where recognition accuracy approaches below-chance performance (Roediger, 1996). Koutstaal and Schacter (1997) argue that high probabilities of false alarms can occur when non-experienced items are "…broadly consistent with the conceptual or perceptual features of things that were studied, largely matching the overall themes or predominant categories of earlier encountered words" (p. 555). For example, false recognition rates as high as $.84$ have been reported (Roediger & McDermott, 1995), with false recognition rates approaching the level of true recognition rates (Koutstaal & Schacter, 1997). I chose a value of $.64$ (equal to the high estimate of h in the Jacoby et al. experiments) as a high estimate of f. This choice is ad-hoc and serves to numerically illustrate the below-chance less-is-more effect. If .11 were chosen as a high estimate for f, then a full-experience effect would be predicted instead of a below-chance effect (see right graph of lower panel in Figure 2 below).

To allow comparison with the perfect-memory case where predictions were illustrated for $\alpha = 8$, I set $A = .8$; B was set to $.75$, $.8$, and $.85$. Results were robust across z, so I set $z = .5$. In Figure 2, there are illustrations of predicting and not predicting the less-is-more effect. In the two graphs of the upper panel, the full-experience effect is predicted: Accuracy is maximized at some amount of experience n_e*, between 60 and 80, that is smaller than the full amount of experience $N = 100$. In the left graph of the lower panel, no less-is-more effect is predicted. In the right graph of the lower panel, the below-chance effect is predicted: Accuracy is higher at no experience $n_{e,1} = 0$ than at a larger $n_{e,2} > 0$, until $n_{e,2}$ equals approximately 50.

I now discuss other less-is-more effect predictions in the literature. Smithson (2010) analyzed a perfect- and an imperfect-memory model where knowledge consists of one cue. This implies that A and B are not necessarily constant across n_e, contradicting Assumption 7; on the other hand, Smithson modeled h and f as probabilities constant across n_e, agreeing with Assumption 6. He showed that the less-is-more effect can be predicted even if $\alpha \leq \beta$ or $A \leq B$. These results speak against the accurate-heuristics explanation. More specifically, Smithson also showed that the prediction of the less-is-more effect is largely influenced by aspects of memory such as the order in which objects are experienced and recognized and not so much on whether, or not, the experience and recognition heuristics are more accurate than recognition-based- or experience-based knowledge.

How can one reconcile Smithson's and my results with Pleskac's (2007) conclusion that $A > B$ is necessary for the less-is-more effect when memory is imperfect? There are at least two ways. First, Pleskac studied a different model of imperfect memory from Smithson's and the model presented here. In Pleskac's model, recognition memory is assumed to be a Bayesian signal detection process. As a result of this assumption, the false alarm and hit rates are not constant across n_e, thus contradicting Assumption 6 (which both models of Smithson and myself satisfy). On the other hand, in Pleskac's model A and B are independent of n_e, thus agreeing with Assumption 7 (which Smithson's model does not satisfy). Second, Pleskac (2007) studied his model via simulations, and it could be that some predictions of the less-is-more effect, where it was the case that $A \leq B$, were not identified.

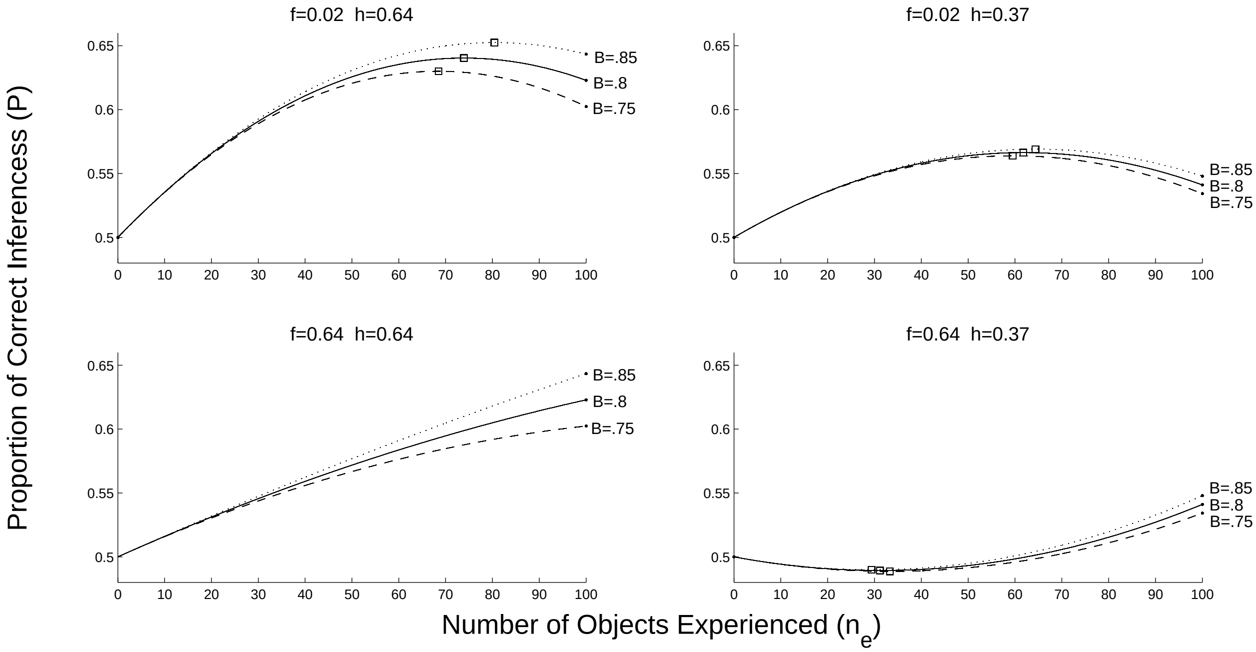
This concludes the discussion of the theoretical predictions of the less-is-more effect. In the next section, I develop a method for testing them empirically.

4 Testing less-is-more-effect predictions: Methodological problems and a new method

In a task of forecasting which one of two national soccer teams in the 2004 European championship would win

[8] I acknowledge that estimates of h and f are often influenced by experimental factors such as payoff structure and instructions (Broeder & Schuetz, 2009), but I ignore this issue in order to use estimates that have some basis in empirical research.

Figure 2: Illustrations of conditions under which the less-is-more effect is and is not predicted. In all graphs, $N = 100$, $z = .5$ (results are robust across different values of z), $A = .8$, and B equals $.75$, $.8$, or $.85$. In the two graphs of the upper panel, a full-experience effect is predicted for $f = .02$ and $h = .64$ or $.37$ (the squares denote maximum accuracy). In the left graph of the lower panel, no less-is-more effect is predicted for $f = h = .64$. In the right graph of the lower panel, a below-chance effect is predicted for $f = .64$ and $h = .37$ (the squares denote minimum accuracy).



a match, Pachur and Biele (2007) did not observe the less-is-more effect even though "…the conditions for a less-is-more effect specified by Goldstein and Gigerenzer were fulfilled" (p. 99). Pohl (2006, Exp. 3) drew the same conclusion in a city-population-comparison task. The condition these authors mean is $\alpha > \beta$ (both values were averages across participants). Are these conclusions justified?

I do not think so, for at least two reasons. First, $\alpha > \beta$ is not the condition that should be tested. Second, even if it were, the estimates of and used are incorrect. Both complaints rely on the assumption that recognition memory is imperfect. If this assumption is granted, according to the analyses presented here one could test $f < Kh/(1 - hz)$ or $f > h/(1 - hz)$, but not $\alpha > \beta$. Furthermore, the quantities used as estimates of α and β are estimates of complicated functions that involve A, B, h, f, n, and N. In the next paragraph I prove and illustrate this fact, and in the paragraph after that I use it to construct a new method for testing less-is-more-effect predictions.

4.1 What do estimates of α and β in the literature (α_{Lit} and β_{Lit}) measure?

Following Goldstein and Gigerenzer (2002), the α_{Lit} of a participant in an experiment has been estimated by the proportion of pairs where she recognized only one object, and in which the recognized object had a higher criterion value; and β_{Lit} has been estimated by the proportion of pairs where the participant recognized both objects, and in which she correctly inferred which object had a higher criterion value (the definitions are formalized in the result that follows). Result 3 shows what these estimates measure.

Result 3. For the imperfect-memory model, (i) $\alpha_{Lit} = (p - q)A + pqB + (1 - p)(1 + q)(\frac{1}{2})$; (ii) $\beta_{Lit} = p^2B + (1 - p)(1 + 3p)(\frac{1}{2})$, where $p = he/[he + f(1 - e)]$, $q = (1-h)e/[(1-h)e+(1-f)(1-e)]$, $e = (r-f)/(h-f)$, and $r = n/N$.

Proof. Consider a participant. Let R be a recognized object and U an unrecognized object (both randomly sampled), with criterion values $C(R)$ and $C(U)$. Then, it holds that:

$$\alpha_{Lit} = Pr[C(R) > C(U)]. \tag{4}$$

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Using the logic preceding the derivation of Equation (3), it also holds that:

$$
\begin{aligned}
&Pr(C(R) > C(U) \mid \\
&\quad R \text{ is experienced}, U \text{ is experienced}) = B, \\
&Pr(C(R) > C(U) \mid \\
&\quad R \text{ is experienced}, U \text{ is not experienced}) = A, \\
&Pr(C(R) > C(U) \mid \\
&\quad R \text{ is not experienced}, U \text{ is experienced}) = 1 - A, \\
&Pr(C(R) > C(U) \mid \\
&\quad R \text{ is not experienced}, U \text{ is not experienced}) = \tfrac{1}{2}. \qquad (5)
\end{aligned}
$$

Let also:

$$
\begin{aligned}
p &= Pr(R \text{ is experienced}) \\
q &= Pr(U \text{ is experienced}). \qquad (6)
\end{aligned}
$$

Equations (4), (5), and (6)[9] imply $\alpha_{Lit} = pqB + p(1-q)A + (1-p)q(1-A) + (1-p)(1-q)(\frac{1}{2})$, and this can be rewritten as in part (i) of the result.

Let R and R' be recognized objects (both randomly sampled), with criterion values $C(R)$ and $C(R')$. Also, assume that the participant infers that $C(R) > C(R')$. Then it holds that:

$$\beta_{Lit} = Pr[C(R) > C(R')]. \qquad (7)$$

As in deriving (5), it holds that

$Pr(C(R) > C(R') \mid$
$\quad R$ is experienced, R' is experienced$) = B$

and

$Pr(C(R) > C(R') \mid$
$\quad R$ is not experienced, R' is not experienced$) = \frac{1}{2}$.

But we do not know

$Pr(C(R) > C(R') \mid$
$\quad R$ is experienced, , R' is not experienced$)$

and

$Pr(C(R) > C(R') \mid$
$\quad R$ is not experienced, R' is experienced$)$,

because we do not know which of R and R' has the higher criterion value. We can, however, reason that one of this probabilities equals A and the other one equals $1 - A$.

From the above and Equation (7), it follows that $\beta_{Lit} = p^2B + (1-p)^2(\frac{1}{2}) + 2p(1-p)(A + 1 - A)$, which can be rewritten as part (ii) of the result.

To complete the proof, I compute p and q. To do this, first let O be a randomly sampled object, and define $e = Pr(O \text{ is experienced})$. Let also $r = Pr(O \text{ is recognized})$, which, by definition, equals n/N. It holds that:

$$
\begin{aligned}
r &= Pr(O \text{ is recognized}) \\
&= Pr(O \text{ is recognized} \mid O \text{ is experienced}) \times \\
&\quad Pr(O \text{ is experienced}) + \\
&\quad Pr(O \text{ is recognized} \mid O \text{ is not experienced}) \times \\
&\quad Pr(O \text{ is not experienced}) \\
&= he + f(1-e).
\end{aligned}
$$

Solving the above equation for e, we get $e = (r - f)/(h - f)$.

Note that $p = Pr(O \text{ is experienced} \mid O \text{ is recognized})$. By Bayes' rule, this probability equals
$Pr(O \text{ is recognized} \mid O \text{ is experienced}) \times$
$Pr(O \text{ is experienced})/Pr(O \text{ is recognized})$,
which turns out to be $he/[he + f(1-e)]$.

Similarly, $q = Pr(U \text{ is experienced}) =$
$Pr(O \text{ is experienced} \mid O \text{ is not recognized})$, and this turns out to be $(1-h)e/[(1-h)e + (1-f)(1-e)]$.

Remark 3. Because the estimates used can be determined from different experiments, it may be that some estimates are not well defined. For example, if $h > r$, then $e > 1$.

Example 3. Result 3 says that the estimates of α and β used in the literature, α_{Lit} and β_{Lit}, are not straightforward to interpret. If memory is perfect, the estimates measure what they were intended to: $f = 0$ implies $p = 1$, and $\beta_{Lit} = B = \beta = \beta_e$; if also $h = 1$, then $q = 0$, and $\alpha_{Lit} = A = \alpha = \alpha_e$. But if memory is imperfect, α_{Lit} may differ from α_e, and β_{Lit} may differ from β_e. Numerical illustrations of the difference between α_{Lit} and α_e, and β_{Lit} and β_e, are given in Table 2.

In Table 2, I used the average n, α_{Lit}, and β_{Lit} for each one of 14 groups in an experiment by Reimer and Katsikopoulos (2004) where participants had to compare the populations of $N = 15$ American cities. I also set $h = .9$, $f = .1$, and $z = .5$[10]. To compute α_e and β_e, I followed three steps: First, I computed p and q as in Result 3. Second, I solved for A and B from (i) and (ii)

[9] It is also assumed that objects are experienced independently of each other.

[10] The choice of h and f was based on the constraint that h has to be higher than all r observed in the experiment, so that $e > 1$ (see Remark 3). This led to $h > .87$, which I rounded up to $h = .9$. This estimate is also higher than the observed r in other experiments on the population-comparison task with American cities and participants from Germany and German-speaking Switzerland (.53 in Hertwig et al., 2008 and .63 in Pachur et al., 2009; in Pohl's experiment that involved European cities, $r = .82$). For f, I chose a value of .1 as indicating very good recognition memory, as also does $h = .9$. I set $z = .5$ because results are robust across z.

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Table 2: Numerical illustration of the difference between α_{Lit} and α_e, and β_{Lit} and β_e, based on the average n, α_{Lit}, and β_{Lit} for each one of 14 groups in an experiment by Reimer and Katsikopoulos (2004), where participants had to compare the populations of $N = 15$ American cities. I also set $h = .9$, $f = .1$, and $z = .5$. For two groups, the estimates of α_e were not between 0 and 1. On the average, α_e was larger than α_{Lit} by .05, and β_e smaller than β_{Lit} by .04.

n	9	12	10.3	12	11.3	13	11.3	12.3	9.3	12.3	10.7	12	8	9.7
α_{Lit}	.79	.79	.78	.81	.88	.87	.72	.70	.68	.66	.79	.81	.77	.79
α_e	.81	.90	.80	.91	-	-	.75	.77	.68	.67	.83	.92	.79	.82
β_{Lit}	.60	.58	.64	.62	.64	.66	.61	.60	.62	.64	.58	.60	.53	.54
β_e	.54	.55	.59	.59	.60	.63	.57	.57	.56	.61	.54	.57	.45	.49

of Result 3, and $B = [\beta_{Lit} - (1-p)(1+3p)(\frac{1}{2})]/p^2$, $A = [\alpha_{Lit} - pqB - (1-p)(1+q)(\frac{1}{2})]/(p-q)$. Third, I used Equation (3) to compute α_e and β_e.

The difference between α_{Lit} and α_e, or β_{Lit} and β_e exceeded .01 in 24 out of 26 cases. The difference was as large as .11. On the average, α_e was larger than α_{Lit} by .05, and β_e was smaller than β_{Lit} by .04[11]. I also performed a sensitivity analysis, varying h from .9 to 1 in increments of .01 and f from .1 to 0 in decrements of .01. There were $(14)11^2 = 1694$ cases. On the average, α_e was larger than α_{Lit} by .04, and β_e was smaller than β_{Lit} by .02. One may expect these differences to increase if h and f are less indicative of a very good recognition memory than they were here.

Remark 4. Importantly, Result 3 implies that it is not straightforward to interpret the reported correlations between α_{Lit} and n, or β_{Lit} and n (Pachur & Biele, 2007; Pachur, in press). It is not clear that these correlations suggest substantial, or any, correlations between A and n, or B and n, because both α_{Lit} and β_{Lit} are complicated functions of n.

In fact, correlations between α_{Lit} and n, or β_{Lit} and n, are predicted even if A, B, h, and f are *constant* across n. For example, I computed the correlations between α_{Lit} and n, and β_{Lit} and n with the six parameter combinations used in the upper panel of Figure 2, where $A = .8$, $B = .75$, .8, or .85, $h = .64$ or .37, and $f = .02$ $(N = 100)$. For each n from 0 to 100, I used the equations in the statement of Result 3 in order to compute α_{Lit} and β_{Lit}, and then computed their correlations with n.[12]

[11] Interestingly, it was always the case that $\alpha_{Lit} \leq \alpha_e$ and $\beta_{Lit} > \beta_e$. This means that $\alpha_{Lit} > \beta_{Lit}$ would imply $\alpha_e > \beta_e$, which in turn would imply a less-is-more effect. So, it would end up being correct to claim that if $\alpha_{Lit} > \beta_{Lit}$, then the less-is-more effect is predicted. It should be studied further under what conditions does this situation occur.

[12] For some n, the estimates of α_{Lit} or β_{Lit} were not between 0 and 1. On the average, this happened for eight values of n per parameter combination. These cases were excluded from the computations of correlations.

As can be seen in Table 3, correlations can be very substantial, varying from $-.59$ to $.85$. The average of the absolute value of the correlation between β_{Lit} and n is $.65$ (two correlations were negative and four were positive) and between α_{Lit} and n is $.24$ (all six correlations were negative).

4.2 A method for testing less-is-more-effect predictions

Based on the points made in the previous paragraph, I claim that, unless it can be established that recognition memory was perfect in an experiment, a correct empirical test of the less-is-more effect in that experiment should not use the estimates of α and β in the literature (α_{Lit} and β_{Lit}).

There is one more problem with empirical tests of the less-is-more effect in the literature. The way some of these tests are carried out often provides no evidence for or against any theory of the conditions under which the less-is-more effect is predicted. To provide such evidence, it does not suffice to just check if an effect is found when the conditions in Results 1 or 2 are satisfied: Even if a condition for predicting the less-is-more effect holds, the prediction is not that all pairs of agents that have amounts of information n_1 and n_2 such that $n_1 < n_2$ would also have $Pr(n_1) < Pr(n_2)$. For example, in the upper left panel of Figure 2, the less-is-more effect is predicted but it is also predicted that an agent who has experienced 40 objects would be less accurate than an agent who has experienced 100 objects. One cannot conclude that "no evidence for the less-is-more effect is found even if a condition that is sufficient for the effect holds", in the case that a person who has experienced 40 objects is less accurate than another person who has experienced 100 objects.

For example, consider Pohl's (2006) Experiment 3, which he interpreted as providing "no evidence for a less-is-more effect" (p. 262). For 11 German and 11 Italian

Table 3: Numerical illustration of correlations between α_{Lit} and n, and Litβ_{Lit} and n, even when A, B, h, and f are constant ($N = 100$). The average of the absolute value of the correlation between α_{Lit} and n is .65, and between Litβ_{Lit} and n is .24.

A	B	h	f	α_{Lit} and n correlation	β_{Lit} and n correlation
.8	.75	.64	.02	–.35	–.40
.8	.80	.64	.02	.59	–.21
.8	.85	.64	.02	.78	–.02
.8	.75	.37	.02	–.59	–.46
.8	.80	.37	.02	.75	–.28
.8	.85	.37	.02	.83	–.08

cities, he measured the average, across participants, n, α_{Lit}, and β_{Lit}. Pohl observed that n was larger in German (11) than in Italian (9.5) cities, and $\alpha_{Lit} = .82$, $\beta_{Lit} = .74$ for Italian cities; for German cities, α_{Lit} was not defined and $\beta_{Lit} = .79$. The less-is-more effect was not found: The accuracy for German cities (.78) was higher than the accuracy for Italian (.76) cities. But this is not evidence against the theory that, under the condition $\alpha > \beta$, an effect is predicted: For the particular values of n, α_{Lit}, and β_{Lit}, applying Equation (2) does not predict a less-is-more effect, but rather that $Pr(n)$ is higher for German (.79) than for Italian cities (.76), as it was indeed found.

In order to determine whether there is evidence for the less-is-more effect or not from an experiment, Snook and Cullen (2006), and Pachur and his colleagues (Pachur & Biele, 2007; Pachur, in press) searched for non-monotonic trends in the best-fitting polynomial to all data points $\{n, Pr(n)\}$. This partly addresses the issue I raised above because typically a large number of data points are considered, and it is likely that among them there are some pairs for which the effect is predicted. Remaining problems are that (i) the average prevalence and magnitude of the full-experience less-is-more effect are predicted to be small (see Table 1), and even the best-fitting polynomials of idealized curves are basically monotonic,[13] and (ii) no out-of-sample-prediction criteria were used to identify these polynomials.

I now propose a method for testing the theoretical predictions of the less-is-more effect that avoids the issues discussed above and in the previous paragraph. The method consists of (i) computing the predicted accuracies of pairs of agents (individuals or groups), (ii) comparing the accuracies to determine whether a less-is-more effect is predicted or not, and (iii) checking the predictions against the observations. I first specify what data are assumed to be available.

Definition 2. A recognition experiment is one in which α_{Lit}, β_{Lit}, n, and N are available.

All experiments in the literature run to test the less-is-more effect are recognition experiments. The method uses the data of a recognition experiment for each participant plus the values of three more parameters, h, f, and z, which practically would often be assumed equal for all participants. It may be fine to set z to a fixed value, say .5 (because $Pr(n)$ is robust across z). Currently, estimates of h and f are not available from experiments run to test the less-is-more effect. This is so because these experiments used natural stimuli (e.g., cities) that participants had experienced outside the laboratory. Estimates of h and f have to be taken from the literature on tasks where experience is controlled, that are similar to the task on which the test is based (e.g., recognition of names, as in Jacoby et al., 1989).

The computation of the accuracy of an agent, $Pr(n_e)$, given α_{Lit}, β_{Lit}, n, N, h, f, and z, is a straightforward, albeit cumbersome, application of Result 3 and Equations (1) and (3):

Input: α_{Lit}, β_{Lit}, n, N, h, f, and z.

Step 1: $r = n/N, e = (r - f)/(h - f), p = he/[he + f(1 - e)]$, and $q = (1 - h)e/[(1 - h)e + (1 - f)(1 - e)]$.

Step 2: $B = [\beta_{Lit} - (1 - p)(1 + 3p)(\frac{1}{2})]/p^2$, and $A = [\alpha_{Lit} - pqB - (1 - p)(1 + q)(\frac{1}{2})]/(p - q)$.

Step 3: $\alpha_e = (h - f + hfz)A + (1 - h + f - hfz)(\frac{1}{2})$, and $\beta_e = h^2 B + (1 - h^2)(\frac{1}{2})$.

[13] For the six curves in the upper panel of Figure 2, the best-fitting polynomial is a quadratic function where the coefficients of n^2 are of the order of 10^{-4}, and the coefficients of n are about 100 times larger.

Step 4: $n_e = (n - Nf)/(h - f)$,

Step 5: $g(n_e) = (N - n_e)(N - n_e - 1)/N(N - 1)$, $r(n_e) = 2n_e(N - n_e)/N(N - 1)$, and $k(n_e) = n_e(n_e - 1)/N(N - 1)$.

Step 6: $Pr(n_e) = g(n_e)(\frac{1}{2}) + r(n_e)\alpha_e + k(n_e)\beta_e$.

Output: $Pr(n_e)$.

I now illustrate the method. I use data from an experiment by Reimer and Katsikopoulos (2004) where three-member groups had to perform a population-comparison task with $N = 15$ American cities. In this research, seven pairs of groups were identified so that (i) the variability in n, α_{Lit}, and β_{Lit} across the three group members was "small", (ii) the difference in the average α_{Lit} and β_{Lit} between the two groups in a pair was "small", and (iii) the difference in the average n between the two groups in a pair was "large" (Reimer & Katsikopoulos, 2004, pp. 1018-1019). I used the average n, α_{Lit}, and β_{Lit} for each one of 14 groups (the values are provided in Table 2). I also set $h = .9$, $f = .1$, and $z = .5$ (see comments in Example 3).

In the Reimer-and-Katsikopoulos experiment, the less-is-more effect was observed in five out of the seven pairs (1-5) and not observed in two pairs (6, 7). As can be seen in Table 4, according to the method, the theory of the less-is-more effect, as summarized in Result 2, correctly predicts the effect in four pairs (1, 2, 4) and correctly predicts that there would be no effect in both pairs 6 and 7[14] (for one pair (3), the estimates of α_e were not between 0 and 1, and the method was not applied). I also let h vary from .9 to 1 in increments of .01 and f vary from .1 to 0 in decrements of .01. The theory correctly predicted the effect in 402 out of 550 cases, correctly predicted no effect in 203 out of 242 cases, and was not applied in 55 cases. More generally, it should be studied further how robust the results of the method are when there is noise in the parameter estimates. This is a concern because of the nonlinear terms in various equations of the method.[15]

5 Conclusions

Strong predictions such as the less-is-more effect provide opportunities for theory development and deserve further study. This work made four contributions to the study of the less-is-more effect: Three refer to the theoretical predictions of the effect and one refers to how to empirically test these predictions.

First, I presented a characterization of the conditions under which the less-is-more effect is predicted (Result 2; this generalized the characterization of Goldstein & Gigerenzer, 2002). A main implication of the characterization is that it provides evidence against an implicit, but common explanation for the effect, the accurate-heuristics explanation (paragraph 3.3). The predictions were illustrated with parameter estimates from a recognition-memory experiment (Figure 2). Second, a new type of less-is-more effect, the below-chance effect, was predicted (paragraph 3.2). Third, I presented a simulation which showed that the less-is-more effect is predicted to be frequent, but its average magnitude is predicted to be small (Table 1), as it has been found empirically.

The fourth contribution was to point out some methodological issues in the empirical testing of less-is-more-effect predictions. In Result 3, it was shown that, granting that recognition memory is imperfect, the parameter estimates used in the tests are not measuring what they were intended to (Example 3). This result, however, allows constructing a new method for testing less-is-more-effect predictions (paragraph 4.2). Result 3 also has implications for the plausibility of the imperfect-memory model used here and of other models (Pleskac, 2007; Smithson, 2010). The main assumptions of my model (Assumptions 6 and 7) are independence assumptions: The parameters of the model (accuracies of the experience heuristic and knowledge, A and B, and probabilities of hits and false alarms, h and f) are constant across information (the number of objects experienced, n_e). These assumptions were made for mathematical convenience. Pleskac (2007) makes the independence assumption only for A and B and Smithson (in press) makes it only for h and f

Both authors have plausibility arguments against the independence assumptions they do not make: Pleskac appeals to a Bayesian signal detection model of recognition memory (for which he also cites studies that provide empirical support) that implies that h and f vary with n_e, and Smithson constructs counterexamples where α and β vary with n, and A and B vary with n_e.

These arguments have a lot to recommend them. But it is not clear how substantial the correlations have been found to be. Empirical evidence is provided by Pachur and colleagues (Pachur & Biele, 2007; Pachur, in press) who, across ten experiments, report average correlations between α and n, and between β and n, that range from .18 to .27. The problem is that the estimates of α and β used in these studies are not straightforward to interpret because they are estimates of complicated functions of

[14] The average of the absolute value of the difference between the observed $Pr(n)$ and predicted $Pr(n_e)$ is .06, and the average of the absolute value of the difference between the predicted and observed size of the effect (i.e., accuracy difference between the two groups in a pair) is .04.

[15] There may be room here for improving the quantitative predictions by optimizing h, f, and z, but the point was to illustrate how the method is applied.

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Table 4: Numerical results of the method with data from Reimer and Katsikopoulos (2004; seven pairs of groups compared populations of $N = 15$ American cities). The method used the n, α_{Lit}, and β_{Lit} of this study (see Table 2), and $h = .9$, $f = .1$, $z = .5$ (see comments in Example 3), to predict $Pr(ne)$. For one pair (3), the estimates of α_e were not between 0 and 1, and the method was not applied. The less-is-more-effect predictions were correct for five pairs (1, 2, 4, 6, and 7) and incorrect for one pair (5).

	Pair 1		Pair 2		Pair 3		Pair 4		Pair 5		Pair 6		Pair 7	
n	9	12	10.3	12	11.3	13	11.3	12.3	9.3	12.3	10.7	12	8	9.7
Observed $Pr(n)$	.83	.75	.73	.69	.78	.75	.67	.63	.66	.64	.67	.73	.56	.66
Predicted $Pr(n_e)$	.67	.64	.67	.66	-	-	.63	.60	.61	.61	.66	.66	.64	.65

A, B, h, f, N, and importantly, of n itself. In fact, I numerically showed (Table 3) that setting A, B, h, f, and N to be constant still led to correlations between the estimate of α used in the literature and n (.65, across six parameter combinations) and between the estimate of β used in the literature and n (.24). It seems that there is currently no definite evidence about which, if any, of the independence assumptions should be revised.

Unsatisfying at it is, my conclusion is that new and carefully controlled experiments are required in order to test the independence assumptions made here. Other empirical work I propose is to test the below-chance less-is-more effect. This effect requires below-chance recognition performance and it is not clear if it would be observed with natural and representative stimuli (Brunswik, 1954).

I end with a list of open questions for models of recognition memory that can be used in future theoretical work (Smithson, 2010, also has a list). A main question is which types of modeling assumptions are consistent with the enabling-of-accurate-heuristics explanation, and which are not. So far, we know that Pleskac's assumptions seem to be consistent with this explanation, whereas the assumptions of Smithson and myself are not. Thus, the independence of the probabilities of hits and false alarms from experience could be key here. Other concepts that could be modeled are cognitive architecture (Schooler & Hertwig, 2005), familiarity (Dougherty et al., 2008), and learning and forgetting (Smithson, 2010).

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