

## FIELD STRENGTH VS. TEMPERATURE RELATION AND THE STRUCTURE OF SUNSPOTS

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**ABSTRACT** The relationship between the magnetic vector and the temperature of a large symmetric sunspot is studied on the basis of 1.56  $\mu\text{m}$  spectra. From this relation we estimate the shape of the  $\tau = 1$  surface, i.e. the Wilson depression, as a function of radial distance in the sunspot. We find that the Wilson depression is relatively small throughout the penumbra and changes by 200–500 km at the umbral boundary. We also estimate the magnitude of magnetic gradients and curvature forces.

Keywords: Solar Magnetic Fields — Sunspots — MHD

### 1. INTRODUCTION

Alfvén (1943) first predicted a relationship between temperature,  $T$ , and magnetic field strength,  $B$  (cf. Maltby 1977). In the umbra the predicted relation was observationally confirmed and Martínez Pillet and Vázquez (1992) derived an estimate of the average umbral Wilson depression,  $Z_W$ , from such observations. Using 1.56  $\mu\text{m}$  spectra Kopp and Rabin (1992) extended the  $B$  vs.  $T$  relation to the whole sunspot. Here we extend the observed relation to include the magnetic inclination angle to the vertical,  $\gamma'$ , and also improve and extend the interpretation, which allows us to estimate  $Z_W$  throughout the sunspot.

### 2. OBSERVED RELATIONSHIPS

The data are composed of Stokes  $I$  and  $V$  spectra of the Landé  $g = 3$ , Fe I line at 1.5648  $\mu\text{m}$ , obtained in a relatively symmetric, mature sunspot near solar disc centre.  $B$  and  $\gamma'$  were determined by fitting the observed profiles with synthetic profiles (Solanki et al. 1992).

The continuum intensity,  $I_c$ , is converted into temperature at unit continuum optical depth,  $T(\tau_{1.6} = 1)$ , using the Eddington-Barbier relation. Figure 1

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shows the resulting  $B$  vs.  $T$  relationship, which is very similar to the one seen by Kopp & Rabin (1992). The plateau at the umbral boundary implies that  $T$  changes rapidly there, while  $B$  does not.

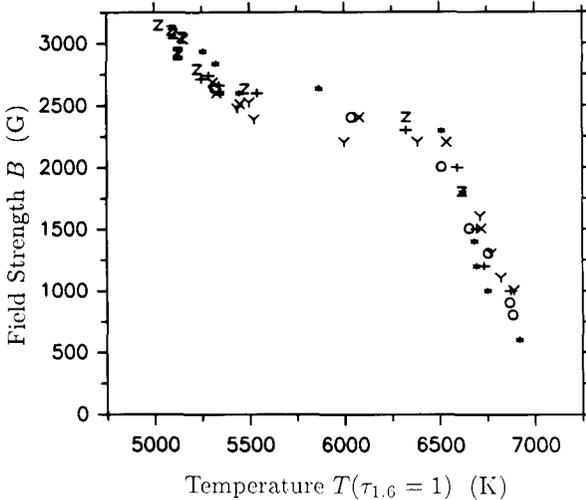


Fig. 1.  $B$  vs.  $T(\tau_{1.6} = 1)$ . Symbols represent different halves of 3 sunspot slices.

The  $\gamma'$  values show a remarkably linear dependence on  $T$  (Fig. 2). The umbral boundary is only identified by the increased scatter and the lower density of points. Thus, although  $B$  does not change when passing from the umbra to the penumbra, the magnetic orientation becomes rapidly more horizontal.

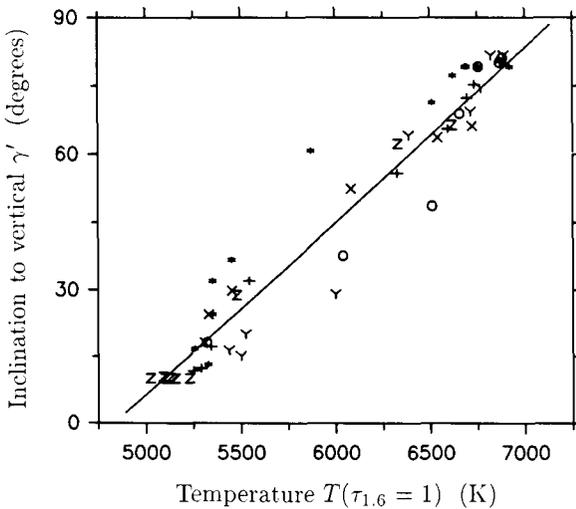
### 3. INTERPRETATION

To interpret the  $T$  vs.  $B$  relationship we radially integrate the radial component of the magnetohydrostatic (MHS) force-balance equation in cylindrical symmetry. This step gives a relationship between the vertical magnetic component,  $B_z$ , the horizontal component,  $B_r$ , and gas pressure,  $P$ :

$$\begin{aligned}
 P_0(z) - P(r, z) &= \frac{1}{8\pi} \left( B_z^2(r, z) + 2 \int_r^a B_z(r', z) \frac{\partial B_r(r', z)}{\partial z} dr' \right) \\
 &= B_z^2(r, z)/8\pi + F_c(r, z)/8\pi.
 \end{aligned}
 \tag{1}$$

Subscript 0 refers to the quiet sun,  $r$  and  $z$  denote the radial and vertical coordinate, respectively, and  $a$  is the radial distance of a point outside the sunspot.  $F_c$  represents the curvature integral. Note that Eq. (1) is valid only for a constant  $z$ .

By applying Eq. (1) to the observations we determine  $Z_W$  throughout the sunspot. For known  $B_z$ ,  $P(r, z)$  and  $F_c$  a unique  $Z_W$  results from the fact that horizontal force balance is only satisfied for a given  $P_0$ , itself a unique function



**Fig. 2.**  $\gamma'$  vs.  $T(\tau_{1.6} = 1)$ . The straight line is a least-squares fit to the data.

of  $z$ .  $B_z$  is known from the observations and  $P(r, Z_W)$  may be determined from the measured  $T(r, z)$  with the help of atmospheric models, e.g. those of Kurucz (1991). For simplicity, we initially assume  $F_c(z) = 0$ , determine  $Z_W(r)$  from Eq. (1) and only then try to gauge the effects of a non-vanishing  $F_c$  on  $Z_W$ .

#### 4. RESULTS

In Fig. 3 we plot  $z(\tau_{1.6} = 1) = -Z_W$  vs.  $r/r_p$ , the radial distance from the centre of the spot, normalized to the outer penumbral radius,  $r_p$ . The umbral edge,  $r_u$ , lies at  $r/r_p = 0.4-0.5$ .  $Z_W$  appears to change mainly near  $r_u$ .

We estimate  $F_c$  and  $\partial\gamma'/\partial z$  by comparing the  $Z_W$  plotted in Fig. 4 with  $Z_W$  obtained from Wilson-effect measurements made near the limb. In the umbra the Wilson effect gives  $Z_W = 600 \pm 200$  km (Gokhale & Zwaan 1972), while Maltby (1977) has argued that  $Z_W > 500$  km, based on empirical models. Combining these two constraints we obtain in the umbra:

$$3.5 \times 10^5 \text{ dyn cm}^{-2} \lesssim F_c/8\pi \lesssim 1.6 \times 10^6 \text{ dyn cm}^{-2}. \quad (8)$$

Therefore, in the umbra the curvature integral is of a similar magnitude as the gas and magnetic pressure terms (cf. Martínez Pillet & Vázquez 1992).  $F_c(r, z)$  can also be rewritten in terms of  $\partial\gamma'/\partial z$  and we can set a lower limit on the average  $\partial\gamma'/\partial z$  in the penumbra by requiring that the  $\tau = 1$  level in the inner penumbra must not lie higher than in the outer penumbra, as dictated by Wilson-effect measurements. An upper limit is set by requiring that  $Z_W \lesssim 800$  km in the umbra. These conditions give

$$-6 \times 10^{-3} \text{ }^\circ/\text{km} \lesssim \langle \partial\gamma'/\partial z \rangle_{\text{penumbra}} \lesssim 0.04 \text{ }^\circ/\text{km}. \quad (13)$$

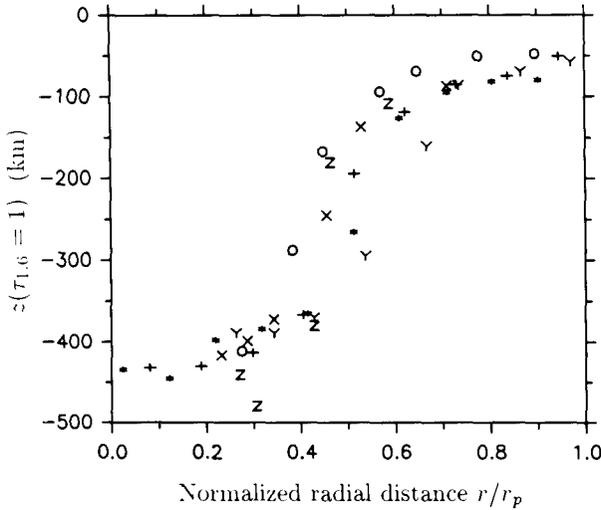


Fig. 3.  $z(\tau_{1.6} = 1)$  vs.  $r/r_p$ .

Note that already for  $\langle \partial\gamma'/\partial z \rangle_{penumbra} \lesssim -0.01^\circ/km$  a static equilibrium cannot be maintained. Finally, by analysing the possible effects of  $T$ ,  $B$  and  $F_c$  on  $Z_W$  at the umbral boundary, we find that  $Z_W$  jumps by 200–500 km there.

## 5. CONCLUSIONS

We have investigated the relationship between  $T$ ,  $B$  and  $\gamma'$  using spectra at 1.56  $\mu m$ . The  $B(T)$  relationship found by Kopp & Rabin (1992) is confirmed. In addition, a linear  $\gamma'(T)$  relation is found. From these relations we estimate that the Wilson depression,  $Z_W$ , at the sunspot boundary is 40–60 km and appears to increase only slowly towards the umbra. At the umbral boundary  $Z_W$  increases by another 200–500 km. This qualitative radial dependence of  $Z_W$  agrees well with the picture obtained from the Wilson effect (Wilson & McIntosh 1969, Wittmann & Schröter 1969). Finally, we have also constrained the curvature integral and  $\partial\gamma'/\partial z$ .

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