

papers, by Thibault, Sklar and Clerc and Hartmann, take us into the area of dynamical systems and consider such matters as invariant curves and flows.

Inevitably in a collection of this nature, the styles differ and the level of mathematical background required to appreciate the contents also varies considerably from author to author. A few misprints were detected and the English language takes a battering on odd occasions, notably in the appearance of words such as "modellized". In conclusion, it can be said that this is a book for dipping into, with something for everyone. If the reader's appetite for a particular topic is whetted particularly, it should be possible to delve further by using the references cited liberally at the end of many of the the contributions.

ADAM C. McBRIDE

BARNES, B. A., MURPHY, G. J., SMYTH, M. R. F. and WEST, T. T., *Riesz and Fredholm theory in Banach algebras* (Research Notes in Mathematics 67, Pitman, 1982), 123 pp, £7.95.

Of the various classes of Banach space operators which have been singled out for special study, the compact operators are arguably the most important and best understood. The classical Riesz–Schauder theory shows that their spectral structure parallels that of finite matrices while, from an algebraic point of view, they form a natural ideal in the algebra of all bounded linear operators, the corresponding quotient algebra being the so-called Calkin algebra. Closely related to the compact operators are the Fredholm operators which, although originally defined spatially, may alternatively be characterized as having invertible images in the Calkin algebra. Another cognate class of operators, less central but still of interest, is that consisting of the Riesz operators. They may be defined either as having the same spectral structure as compact operators or, equivalently, as having images in the Calkin algebra with zero spectrum.

The aim of the present monograph is to examine how the ideas of Fredholm and Riesz theory can be developed in the context of a general Banach algebra A . This strategy is to find appropriate definitions for Fredholm and Riesz elements of A and then show that the results of operator theory have analogues in this algebraic setting. The first difficulty encountered in such a programme is that there is no satisfactory definition of a compact element in a general Banach algebra. However, by first considering invertibility modulo the socle when A is semi-simple and then extending to the general case, it is possible to obtain a sensible definition of a Fredholm element of A . Counterparts to the main theorems of classical Fredholm theory, including the index and punctured neighbourhood theorems, can now be proved. After developing their version of algebraic Fredholm theory, the authors turn to Riesz theory in Banach algebras. Elementary results can be obtained by considering elements with zero spectrum modulo an arbitrary closed ideal K but, to take things further, additional restrictions on K are needed. Not surprisingly, these ideas have some special features when A is a C^* -algebra. In that case, it is possible to give satisfactory definitions of finite rank and compact elements and, reflecting the situation for operators on Hilbert space, each Riesz element can be decomposed as the sum of a compact and a quasinilpotent element. Whether such a decomposition is always possible for Riesz operators on Banach spaces is the main problem left open in the subject at an operator-theoretic level. Unfortunately, the algebraic techniques developed here do not appear to shed much light on it.

The book has been well organized and each chapter ends with an extensive section of notes and comments. The authors have made a point of giving plenty of examples to illustrate their results, many of which appear here for the first time, and they include a chapter of applications. Finally, although the main aim is to develop the theory at an algebraic level, the book begins with an account of the necessary background material from operator theory. This in itself is a good survey, which in fact contains both classical results and some interesting new material.

T. A. GILLESPIE

SMYTH, K. T., *Primer of modern analysis* (Springer-Verlag, 2nd ed. 1983), xv + 446 pp. DM 97.

The original version of this book was published by Bogden & Quigley in 1971; this slightly extended version now appears in the Springer series "Undergraduate Texts in Mathematics". It is

an interesting book with its own individual flavour. It was inspired by an honours course at the University of Wisconsin; the author does not say how long that course was but it was evidently not short. In fact the book is in three parts which implicitly concedes that it might well be used in three different courses. Indeed the third part would not be out of place in a graduate program, whether in USA or UK. It is a book about theory—it aims to motivate and to explain theory with as much involvement from the reader as possible. Almost all the exercises are geared to theoretical aspects. The student who wishes a training in analytic techniques for solving specific problems should seek elsewhere.

Part I is intended for an honours calculus course; it covers differentiation and integration in one variable, continuous functions, Taylor's formula and sequence and series. To quote the author: "Part I begins with a half intuitive—half rigorous discussion of applications, chosen to arouse interest and to show the need for a precise and general theory, and then develops this theory for functions of one variable". The student may be forgiven for wondering where half rigour becomes whole rigour, and the author himself does concede the difficulty of encapsulating in a book the "spontaneous, creative disorder that characterizes an exciting course". For such an individual book one may be permitted some individual criticisms. I regret that for differentiation the author has chosen to emphasize quotients instead of linear approximation. I regret that he has expounded the Riemann integral and not the elementary integral obtained via uniform limits of step functions. He does give a solid treatment of Taylor's formula, in particular of the uniqueness theorem, but it is a pity that the elegant derivation of the formula by repeated integration by parts is relegated to a late exercise.

While one may have reservations about Part I, such reservations evaporate in Part II. This gives an excellent and well integrated account of metric spaces (essentially \mathbb{R}^n and function spaces), paths in \mathbb{R}^n , the requisite linear algebra, functions of several variables up to the implicit function theorem, manifolds (very well expounded) and higher derivatives. Part III begins with an efficient account of Lebesgue integration in \mathbb{R}^n , based on outer measures and includes some interesting applications (for example, multiple series and Sard's theorem). The remaining chapters become increasingly technical—differentiation of regular Borel measures, the problem of surface area, Brouwer topological degree, and finally extension theorems for various classes of functions, in particular with a view to applications to partial differential equations.

All in all, despite one's criticisms, the book is something of a tour de force. As usual it has been produced very well by Springer with ample spaces in the margins for the student to make his own annotations!

J. DUNCAN

BLASCHKE, WILHELM, *Gesammelte Werke*, Band 1 ed. by W. BURAU, S. S. CHERN *et al.* (Thales-Verlag, Essen 1982), pp. 365.

Wilhelm Blaschke was born in Graz on 13th September 1885. He died on 17th March 1962, and during his long life made an international reputation as one of the most influential geometers in Germany. From 1906 he studied at Vienna where he was influenced by Wirtinger. He then studied at Bonn, at Pisa with L. Bianchi, and at Göttingen with F. Klein and D. Hilbert who together had perhaps the greatest influence on his subsequent career.

In 1910 he was appointed Privat-Dozent in Bonn, and in 1913 he became professor at the Technische Hochschule in Prague. In 1915 he was appointed professor at the University of Königsberg and in 1919 he was appointed to the University of Tübingen. Later the same year he went to Hamburg to take charge of a new mathematics institute. He devoted much of his life to making Hamburg a centre for mathematical research and teaching. Hamburg was his main location until 1953. Throughout that period he was regarded as the best known German geometer. In 1931/32 he was guest professor at the Universities of Stanford and Chicago. Moreover, he visited India, China, Japan, the Soviet Union, South America and all states in Europe. His favourite place abroad was Pisa. In 1953 and 1955 he was professor at Istanbul.