ASTRONOMICAL CONSTANTS. A SURVEY OF DETERMINED VALUES (¹)

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RÉSUMÉ. — Les auteurs donnent les principales relations liant les constantes fondamentales à celles qui en sont dérivées sous forme analytique et sous forme de tableaux de nombres. Puis, ils citent et commentent les résultats des déterminations des paramètres définissant la figure et le champ de gravité de la Terre, de la parallaxe solaire, de la constante de l'équation de la Lune, des masses des planètes et des constantes de l'aberration, de la nutation et de la précession. Ils donnent une importante bibliographie.

ABSTRACT. — The authors give the main relations between fundamental and derived constants in analytical and numerical forms. Then, they enumerate and comment results of determinations of parameters of the figure and gravity field of the Earth, solar parallax, constant of lunar equation, the masses of planets, the constants of aberration, of nutation and of precession and give an extensive list of references.

ZUSAMMENFASSUNG. — Verff. geben die hauptsächlichen Beziehungen zwischen den fundamentalen und den abgeleiteten Konstanten in analytischer und numerischer Form. Die Ergebnisse der Bestimmungen der folgenden Konstanten werden sodann zusammengestellt und kommentiert : Parameter der Figur und des Schwerefeldes der Erde; Sonnenparallaxe; Konstante der Mondgleichung; Planetenmassen; Aberrations-, Nutations- und Präzessionskonstante. Am Schluss wird eine ausführliche Bibliographie gegeben.

Резюме. — Авторы дают важнейшие зависимости между фундаментальными и производными постоянными в виде аналитических формул и таблиц. Они цитируют и комментируют результаты определений параметров определяющих форму Земли и её гравитационное поле, солнечного параллакса, постоянной уравнения Луны, масс планет и постоянных аберрации, прецессии и нутации. Авторы дают значительную библиографию.

(¹) Prepared at the request of the Organizing Committee of I. A. U.-Symposium No. 21.

Introduction. — A survey of values of the astronomical constants is presented as they resulted from various determinations. It is intended to serve as a basis for comparisons and for discussion, and to facilitate conclusions on future improvements. It is not intended to be a complete historical survey. In general, determinations have been omitted which can no longer be regarded as being of interest in a discussion on possible improvements of the conventional values. In some cases, however, determined values have been included because the method of determination may still deserve some interest. The selection depends to some extent on personal judgement; in doubtful cases the decision has been made in favour of inclusion.

Determinations of various astronomical constants are listed in several recent articles; mention may be made of the tables by A. Kopff [1], K. A. Kulikov [2], and by M. W. Makemson *et al.* [3].

In recent investigations, several authors have attempted to assign weights to the various determinations in order to arrive at " most likely " values of certain constants by averaging the values according to these weights. The results of such compilations are not included in this survey. Accordingly, we have refrained from making any personal comment that could be regarded as a valuation of the determinations on the basis of our own judgement. Comments on the various values refer to facts in connection with the applied methods of determination. At the end of the tables remarks are listed in numerical order, the numbers referring to the corresponding numbers in the tables.

The errors of the determinations are cited as probable errors (p. e.) or as mean errors (m. e.), the latter are " standard deviations ".

Notations. — We use the following notations :

c, vacuum velocity of light;

k, constant of aberration;

$\pi_\odot$, solar parallax;

$n_\oplus$, sidereal mean motion of the Earth (per second);

$n_☾$, sidereal mean motion of the Moon (per second);

$\varphi_\oplus$, eccentricity angle of the Earth's orbit;

G, gravitational constant;

M, mass of the Earth;

μ, mass of the Moon in units of the Earth's mass;

$m_{\oplus+☾}$, mass of the system Earth-Moon in units of the Sun's mass;

$\sin \pi_☾$, sine of the Moon's parallax defined by $\frac{R}{a_☾}$ with $n_☾^2 \bar{a}_☾^3 = GM(1+\mu)$ and $\frac{\bar{a}_☾}{a_☾} = 1 + \nu_1 = 1.000907681$, *cf.* W. de Sitter [47];

L,	constant of lunar equation;
P,	coefficient of the principal term in the Moon's parallactic inequality;
U,	Earth's gravitational potential;
r,	distance from the Earth's centre;
β,	geocentric latitude;
R,	equatorial radius of the Earth;
$\frac{1}{f}$,	flattening of the Earth.

Velocity of light. — The mean value of the determinations of the vacuum velocity of light as derived by E. Bergstrand [4] is $c = 299\,793.0 \pm 0.3$ km/s. The value adopted by the I. U. G. G. is $c = 299\,792.5 \pm 0.1$ km/s, *cf.* K. D. Froome [5].

Relations between fundamental and derived constants. — Some of the fundamental and derived constants are related to each other by the following equations

$$(1) \qquad k\pi_{\odot}c = (\text{radian})''(1+\nu_2)\,\mathrm{R}\,n_{\oplus}\sec\varphi_{\oplus} \qquad \text{or} \qquad k\pi_{\odot} = \mathrm{C}_1.$$

$$(2) \qquad \sin^3\pi_{☾} = \frac{\mathrm{R}^3 n_{☾}^2}{\mathrm{GM}(1+\mu)}(1+\nu_1)^3,$$

$$(3) \qquad \pi_{\odot}^3 = \left(\frac{\sin\pi_{☾}}{1+\nu_1}\right)^3 \left(\frac{n_{\oplus}}{n_{☾}}\right)^2 \frac{m_{\oplus+☾}}{1+m_{\oplus+☾}}(1+\nu_1)^3 \qquad \text{or} \qquad \pi_{\odot}\left(1+m^{-1}_{\oplus+☾}\right)^{\frac{1}{3}} = \mathrm{C}_2,$$

$$(4) \qquad 1+m^{-1}_{\oplus+☾} = \left(\frac{\mathrm{C}_2}{\mathrm{C}_1}\right)^3 k^3 \qquad \text{[from equations (1) and (3)]}.$$

$$(5) \qquad \mathrm{L} = \frac{\mu}{1+\mu}\,\frac{\pi_{\odot}}{\sin\pi_{☾}},$$

$$(6) \qquad \mathrm{P} = 49.853''.2\,\frac{1-\mu}{1+\mu}\,\frac{\pi_{\odot}}{\sin\pi_{☾}},$$

$$(7) \qquad \mu^{-1} = 4.13744\,\frac{\mathrm{P}}{\mathrm{L}} + 1 \qquad \text{[from equations (5) and (6)]}.$$

with ν_1 and ν_2 as defined by W. de Sitter [47].

On recommendation of I. A. U.-Commission No. 7 the Earth's potential U shall be written in axially symmetric cases

$$(8) \qquad \mathrm{U} = \frac{\mathrm{GM}}{r}\left[1 - \sum_{n=2}^{\infty} \mathrm{J}_n \left(\frac{\mathrm{R}}{r}\right)^n \mathrm{P}_n(\sin\beta)\right].$$

cf. Y. Hagihara [6]; for other forms and relations *cf.* D. Brouwer [7].

Adopting Froome's value of c, the data of the *World Geodetic System* ($\mathrm{GM} = 3.98602 \times 10^{20}\ \mathrm{cm}^3.\mathrm{s}^{-2}$), and the values $n_{\oplus}$ and $n_{☾}$ given by

https://doi.org/10.1017/S0074180900105017 Published online by Cambridge University Press

G. M. Clemence [8], the equations (1) to (6) yield the following relations (k, $\pi_\odot$, $\sin \pi_☾$ in seconds of arc)

$$(9) \qquad k\pi_\odot = 180.2413,$$

$$(10) \quad \begin{cases} \sin\pi_☾ = 3\,422''.495 \text{ for } \mu^{-1} = 81.53 & \text{cor. to } a_☾ = 384\,395.4 \text{ km} \\ \quad\;\;\, = 3\,422.461 \qquad\quad\;\; = 81.33 & \qquad\qquad\; = 384\,399.2 \text{ km} \end{cases}$$

($\mu^{-1} = 81.53$ Conventional value,
$= 81.33$ Chosen for easy interpolation only).

For comparison the observed values of $a_☾$ may be reported according to the reinterpretation by I. Fischer [9] :

384 415 } Meridian observations of Mösting A from 1905 to 1910 by
384 413 } Christie and Gill; two systems of weighting;
384 401 : Occultations; O'Keefe and Anderson;
384 400 : Radar echoes; Yaplee *et al.*

$$(11) \quad \begin{cases} \pi_\odot = 607''.0461 \\ \quad\;\; = 607''.0401 \end{cases} \left(\frac{m_{\oplus+☾}}{1+m_{\oplus+☾}}\right)^{\frac{1}{3}} \quad \begin{matrix} \text{for} & \mu^{-1} = 81.53, \\ \text{»} & \text{»} = 81.33; \end{matrix}$$

$$(12) \quad \begin{cases} 1 + m^{-1}_{\oplus+☾} = 38.20339\, k^3 \\ \qquad\qquad\; = 38.20225\, k^3 \end{cases} \quad \begin{matrix} \text{for} & \mu^{-1} = 81.53, \\ \text{»} & \text{»} = 81.33; \end{matrix}$$

$$(13) \qquad 1 + \mu^{-1} = 60.268\,\pi_\odot\, L^{-1};$$

$$(14) \quad \begin{cases} P = 14.56633 \\ \quad = 14.56648 \end{cases} \left(\frac{1-\mu}{1+\mu}\right)\pi_\odot \quad \begin{matrix} \text{for } \sin\pi_☾ \text{ belonging to} & \mu^{-1} = 81.53, \\ \text{»} & \text{»} = 81.33. \end{matrix}$$

Tables I to III have been computed as auxiliary tables to illustrate the equations (9) and (11) to (13).

TABLE I.

Constant of aberration, solar parallax, and reciprocal of mass Earth-Moon

[Equations (9) and (12)].

k.	$\pi_\odot$.	$m^{-1}_{\oplus+☾}$. $\mu^{-1} = 81.53$.	$\mu^{-1} = 81.33$.
20.45	8.8138	326 724	326 714
46	8094	327 203	327 193
47	8051	327 683	327 673
48	8008	328 164	328 154
49	7965	328 645	328 635
50	7923	329 126	329 116
51	7880	329 608	329 598
52	7837	330 090	330 080
20.53	8.7794	330 573	330 563

$m^{-1}_{\oplus+☾} = 329\,390$ (ephemerides) belongs to
$k = 20''.5055$ and $20''.5057$ respectively.

https://doi.org/10.1017/S0074180900105017 Published online by Cambridge University Press

TABLE II.

Solar parallax and reciprocal of mass Earth-Moon

[Equation (11)].

$\pi_{\odot}$.	$m^{-1}_{\oplus+\mathbb{C}}$ $\mu^{-1} = 81{,}53$.	$\mu^{-1} = 81{,}33$.
8.788	329 605	329 595
790	329 380	329 370
792	329 155	329 145
794	328 931	328 921
796	328 706	328 697
798	328 482	328 472
800	328 258	328 248
802	328 035	328 025
804	327 811	327 801
8.806	327 588	327 578

$m^{-1}_{\oplus+\mathbb{C}} = 329\ 390$ (ephemerides) belongs to

$\pi_{\odot} = 8''.7899$ and $8''.7898$ respectively.

TABLE III.

Lunar equation and Moon's mass [equation (13)].

L.	μ^{-1}. $\pi_{\odot} = 8''.79$.	$\pi_{\odot} = 8''.80$.
6''.415	81.581	81.675
420	516	610
425	452	546
430	388	482
435	324	418
440	260	354
445	196	290
450	133	226
6.455	81.069	81.162

Figure and external gravity field of the Earth. — The following currently used values of the Earth's equatorial radius R and flattening $\frac{1}{f}$ shall be reported :

R (m).	$\frac{1}{f}$.	Source.
6 378 388	297	International Ellipsoid; *cf.* K. Jung [10]
6 378 245	298.3	Reference ellipsoid used in U.S.S.R.; *cf.* A.A. Isotov [11]
6 378 163 ± 21	298.24 ± 0.01	*World Geodetic System* derived by W. M. Kaula [12]; nearly identical with the *Astrogeodetic World Datum* of I. Fischer [9]

Strictly speaking, R is the mean equatorial radius. The ellipticity β_{22} of the equator defined as

$$\beta_{22} = \frac{R_{Max} - R_{Min}}{R}$$

has been derived, together with the longitude λ_{22} of the major axis of the equator, from the second sectorial harmonic in the Earth's potential. In table IV determinations of β_{22} and λ_{22} are given. λ_{22} is measured positive to the East.

Determinations of the numerical coefficients J_n in the Earth's gravitational potential as defined by equation (8) are given in table V.

TABLE IV.

Ellipticity of the equator and longitude of its major axis.

No.	$\beta_{22} \times 10^5$.	λ_{22}	m. e.	Author.	Observed objects.
1.....	1.1 ± 0.5	—36°	± 16°	Kaula [13]	*Vanguard* 1, 1958
2. ...	3.21 ± 0.29	—33.15	± 0.53	Izsak [14]	*Vanguard* 2 and 3, 1960
3.....	1.39 ± 0.18	—37.5	± 3.8	Kozai [15]	*Sputnik* 3, 1958 *Vanguard* 2 and 3, 1960
4.....	2.4 ± 0.5	—11	± 6	Newton [17]	*Transit* 4 A, 1961

Remarks to table IV :

1. From 2 241 observations over 385 days by Minitrack; analysis of the potential from variations of elements.

2. Derived from long-periodic variations of the elements.

   The result from 187 photoreduced Baker-Nunn observations of *Vanguard* 2 is 3.47 ± 0.28, — 33°.5 ± 1°.9.

   The result from 216 photoreduced Baker-Nunn observations of *Vanguard* 3 is 2.88 ± 0.32, — 32°.3 ± 3°.1.

3. From photoreduced Baker-Nunn observations; analysis of the gravitational potential from long-periodic perturbations in the orbital elements.

4. From the along-track discrepancies in the orbit, derived from transit Doppler data.

TABLE V.

The Earth's external gravity field (all values $\times 10^{-6}$).

No.	J_2.	J_3.	J_4.	J_5.	J_6.	J_7.	J_8.		Author.
1.	+1082.53±0.06	−2.4 ±0.3	−1.70±0.06	−0.05±0.07	-	-	-	m.e.	O' Keefe *et al.* [18]
2.	1082.79±0.15	-	−1.4 ±0.2	-	+0.9 ±0.8	-	-	m.e.	King-Hele [19]
3.	1083.15±0.20	-	−1.4 ±0.3	-	+0.7 ±0.6	-	-	m.e.	Smith [20]
4.	-	−2.36±0.14	-	−0.19±0.10	-	−0.28±0.11	-	m.e.	Newton *et al.* [21]
5.	1082.7	−2.5	−1.7	+0.3	+0.7	−0.6	+0.1	-	Michielsen [22]
6.	1082.61±0.06	−2.05±0.10	−1.43±0.06	+0.08±0.11	+0.20±0.05	-	-	m.e.	Kaula [12]
7.	1082.19±0.03	−2.29±0.02	−2.13±0.04	−0.23±0.02	-	-	-	p.e.	Kozai [16]
8.	+1082.79±0.05	−2.44±0.08	−1.09±0.20	0.0 ±0.2	+0.73±0.20	−0.45±0.1	-	m.e.	King-Hele *et al.* [23]

Remarks to table V :

1. From published elements of *Vanguard* 1 for 37 epochs in 1958.
2. From the motion of the node of *Sputnik* 2, *Vanguard* 1, *Explorer* 7.
3. From the motion of the node of *Sputnik* 3, *Vanguard* 2, *Transit* 1 B rocket and *Tiros* 1.
4. From variations in the perigee distance of *Vanguard* 1, *Transit* 1 B and *Transit* 2 A.
5. From *Vanguard* 1, *Transit* 1 B rocket and *Sputnik* 4.
6. Coefficients of the World Geodetic System, derived from *Sputnik* 2 [24] and *Vanguard* 1 (*cf.* No. 1).
7. From long-periodic inequalities and secular motions in the elements of *Vanguard* 1, *Vanguard* 3 and *Explorer* 7.
8. J_{2n} from the motion of 7 satellites with inclinations between 28° and 97°; J_3, J_5, J_7 are mean values from other determinations; the above values for J_{2n} are obtained by restricting n to 3, the complete results are the following :

$$10^6 J_2 = +1082.78 \pm 0.05, \qquad 10^6 J_8 = +0.24 \pm 0.2,$$
$$10^6 J_4 = -\ 0.78 \pm 0.2, \qquad 10^6 J_{10} = -0.50 \pm 0.2,$$
$$10^6 J_6 = +\ 0.70 \pm 0.1, \qquad 10^6 J_{12} = +0.28 \pm 0.2.$$

J_2 is almost unaffected by the evaluation of higher harmonics, whereas J_4 suffered a sharp change when J_{10} was included. Mean values of several determinations by other authors are :

$$10^6 J_2 = +1082.7 \pm 0.2,$$
$$10^6 J_4 = -\ 1.6 \pm 0.2,$$
$$10^6 J_6 = +\ 0.7 \pm 0.2.$$

Solar parallax. — The solar parallax is defined as the angle subtended by the Earth's equatorial radius at a distance of one astronomical unit. The astronomical unit (a. u.) is the unit of distance, so defined that the mean angular motion of a planet of negligible mass, moving in the gravitational field of the Sun alone, would be 0.017 202 09895 radian per ephemeris day precisely. According to this definition, the mean distance of the Earth from the Sun, calculated by Kepler's Third Law from the observed mean motion and adopted mass Earth-Moon is 1.000000236 a. u.

The conventional value of the solar parallax adopted for the ephemerides by the Paris Conference in 1896 (Conférence internationale des Étoiles fondamentales de 1896, Gauthier-Villars, Paris) is $\pi_{\odot} = 8''.80$. In table VI determinations are listed which result in values of the solar parallax from one or the other method of observation.

Table VI.

Solar parallax.

No.	$\pi_\odot$	p. e.	Author.	Method and observations.
1...	8″.8036	±0″.0046	Gill [25]	Trig. Victoria, Iris, Sappho (1888-1889), heliometer obs.
2...	8.807	±0.0027	Hinks [26]	Trig. Eros (1900-1901) phot. RA
	8.806	±0.004	» [28]	» » » micromet. RA
3...	8.790	±0.001	Spencer Jones [29]	Trig. Eros (1930-1931)
4...	8.805	±0.005	» » [30]	Occult. Moon (1880-1922)
	8.796	±0.004	» » [31]	» » (1753-1908)
5...	8.7925	±0.0030	Brouwer [32]	Occult. Moon (1932-1942)
	8.7981	±0.0026	» [32]	Meridian obs. Moon (1925-1948)
6...	8.803	±0.004	Spencer Jones [33]	Radial vel. 21 stars (1906-1912)
7...	8.805	±0.007	Adams [34]	Radial vel. α *Boo* (1937-1940)
8...	8.7988	±0.0006	Witt [35]	Dynamic. Eros (1893-1931)
9...	8.79835	0.00039	Rabe [36]	Dynamic. Eros (1926-1945)
10...	8.7974	±0.0008	Mc Guire *et al.* [37]	Radar *Pioneer* V (1960)
11...	8.7943	±0.0003	Thomson *et al.* [38]	Radar Jodr. Bank, Venus (1961)
12...	8.79439	±0.00005	Kotelnikov [39]	Radar U.S.S.R., Venus (1961)
13...	8.79460	–	Maron *et al.* [40]	Radar Moorestown (RCA), Venus (1961)
14...	8.794491	±0.000024	Pettengill *et al.* [41]	Radar Millstone (MIT), Venus (1961)
15...	8.794098	±0.000015	Muhleman *et al.* [42]	Radar Goldstone (JPL), Venus (1961)

Remarks to table VI :

1. The individual results from the three planets are

Victoria........................	8″.8013 ±	0″.0061
Iris...........................	8.8120 ±	0.0090
Sappho........................	8.7981 ±	0.0114

With weights defined as the reciprocals of the squares of the probable errors Gill obtains 8″.8036 ± 0″.0046. On the assumption that the value determined from Iris may be affected by the colour of this minor planet, Gill computes the weighted mean of the results from Victoria and Sappho (8″.8006 ± 0″.0054) and combines it with 8″.8036 ± 0″.0046 to the weighted mean value 8″.802 ± 0″.005.

3. The determination of right ascensions results in the weighted mean value 8″.7900 ± 0″.0013 of the separate determinations at 16 different instruments. By combining all observational material, after eliminating systematic differences in α between the various instruments, the value 8″.7875 ± 0″.0009 is obtained. The first solution is independent of systematic corrections to the α-systems with different instruments. The determination of declinations, all observations combined, results in 8″.7907 ± 0″.0011.

4. From the observations 1880-1922 (at Cape) follows

$$P = 125''.15 \pm 0''.06$$

From the observations 1753-1908 (at various observatories) follows $P = 125''.023 \pm 0''.033$. H. Spencer Jones combines the results, and derives a mean value $\pi_\odot = 8''.799 \pm 0''.003$ (based on a semi-diameter of the Moon of $932''.70$).

5. The occultations 1932-1942 yield $P = 124''.969 \pm 0''.042$, the meridian observations 1925-1948 $P = 125''.050 \pm 0''.037$. In both cases $\pi_\odot$ is based on $\mu^{-1} = 81.53$.

8, 9. The method results in the determination of the mass of the Earth-Moon system. Summarizing all results obtained from the discussion of the motion of Eros, we have :

Observations.	Author.	$m^{-1}_{\oplus+☾}$	p. e.
1893-1907.............	G. Witt [43]	328 659	± 82
1893-1914.............	E. Noteboom [44]	328 370	± 68
1893-1931.............	G. Witt [35]	328 390	± 69
1926-1945.............	E. Rabe [36]	328 452	± 43

The value given in the paper of Witt [43] is $327\,920 \pm 143$ (standard deviation). Later Witt communicated to Noteboom that this result was influenced by a mistake, and that the corrected value is $328\,659 \pm 123$ (m. e.) (*cf.* Noteboom [44], p. 169).

11. Radar operated at 408 Mc/s, 1961 Apr. 8-25, yielding the light-time of 499.011 ± 0.017 s for one a. u. and the parallax with conventional values of *a* and *c*.

12. Radar in U. S. S. R. operated at 700 Mc/s, according to indirect information (Pettengill *et al.* [41], p. 186) yielding

$$1 \text{ a. u.} = 149\,599\,500 \pm 800 \text{ km.}$$

13. Radar experiments (1961 March 21, Apr. 7 and 8),

$$1 \text{ a. u.} = 149\,596\,000 \pm ? \text{ km.}$$

14. Radar operated at 440 Mc/s, 1961 March 6-May 18, yielding 1 a. u. $= 149\,597\,850 \pm 400$ km. The parallax is based on

$$R = 6\,378\,388 \text{ m.}$$

There is a systematic variation of the determined values of the astronomical unit from March 6 to May 18 within a range of 600 km. W. Priester *et al.* [45] found a correlation between this variation and the solar 20 cm flux during the same period of time as measured at the Heinrich-Hertz-Institut, Berlin-Adlershof. Independently, W. B. Smith [46] discussing the MIT-radar measurements states : " Certain inconsistencies in the data on range, Doppler shift, and Doppler spectral width cast some doubt on the assumptions of free-space propagation and the model of a uniformly rough spherical surface for the planetary target ".

15. Radar operated at 2 388 Mc/s, 1961 March 10-May 10, yielding 1 a. u. = 149 598 845 ± 250 km. The parallax is based on

$$R = 6\,378\,145 \text{ m}.$$

Constant of lunar equation. — The variation of the geocentric positions of a planet due to the motion of the Earth about the centre of gravity of the Earth-Moon system (barycentre) is a function of the constant L of the lunar equation. The motion about the barycentre also gives rise to a periodic term, the lunar inequality in the Sun's longitude. The numerical coefficient of this term is related to L by

$$L_s = 1.00450\ L \qquad (cf.\ \text{W. de Sitter [47]}).$$

From the quantity L and the solar parallax the mass of the Moon is derived according to equation (5).

TABLE VII.

Constant of lunar equation.

$L = 6''.425$ (conventional value),
$L_s = 6.454$ (» »).

No.	L.	p. e.	Author.	Method and observations.	
1.........	6″.456	± 0″.012	Newcomb [48]	Sun	(1801-1864)
2.........	6.450	± 0.009	Morgan, Scott [49]	»	(1900-1937)
3.........	6.414	± 0.009	Gill [23]	Victoria	(1889)
4.........	6.4305	± 0.0031	Hinks [27]	Eros	(1900-1901)
5.........	6.4390	± 0.0015	Spencer Jones [29]	»	(1930-1931)
6.........	6.4378	± 0.0012	Jeffreys [50]	»	(1930-1931)
7.........	6.4356	± 0.0028	Rabe [36]	»	(1930-1931)
8.........	6.4428	± 0.0014	Delano [51]	»	(1930-1931)

Remarks to table VII :

1. From $L_s = 6''.485 \pm 0''.018$ (m. e.).
2. From $L_s = 6''.479$; $\mu^{-1} \doteq 81.26$ for $\pi_\odot = 8''.803$.
3. Determination of $\frac{\mu}{1+\mu}\pi_\odot = 0.10642 \pm 0.00014$;

$$\mu^{-1} = 81.693 \pm 0.082 \text{ for } \pi_\odot = 8''.800.$$

Empirical corrections of the form of a cubic in time are applied to the ephemeris.

4. L $= 6''.425 \times (1.00085 \pm 0.00048)$; $\mu^{-1} = 81.530 \pm 0.047$ for $\pi_{\odot} = 8''.806$ (the p. e. includes the uncertainty due to the p. e. of $\pi_{\odot}$). The error of the ephemeris is determined graphically in various ways. L is a mean value of 8 solutions.

5. L $= 6''.4305 \times (1.00133 \pm 0.00023)$;

$$\mu^{-1} = 81.271 \pm 0.021 \text{ for } \pi_{\odot} = 8''.790 \pm 0''.001.$$

L is determined from short arcs in applying five analytic and graphical methods for eliminating the variation due to errors in the elements of the orbit and to the inhomogeneity of star places.

6. L $= 6''.4305 \times (1.00114 \pm 0.00028$ m. e.) resulting from a revision of Jones' work in using a more sophisticated statistical method for the elimination of the fluctuation from short stretches of the residuals.

7. L $= 6''.4305 \times (1.00080 \pm 0.00043)$. The residuals in the co-ordinates of Eros for 20 normal places 1930-1931 (10 days apart), resulting after the completion of the general solution (1926-1945), determine the quantity L and the coefficients of a smoothing cubic in time. Remaining residuals still show periodic fluctuations.

This values of L combined with Jeffreys' value to L $= 6''.437 \pm 0''.002$ gives $\mu^{-1} = 81.375 \pm 0.026$ for $\pi_{\odot} = 8''.79835$.

8. L $= 6''.4305 \times (1.00191 \pm 0.00021)$;

$$\mu^{-1} = 81.222 \pm 0.027 \quad \text{for } \pi_{\odot} = 8''.790 \pm 0''.001.$$

An improvement of the orbit was made to reduce the deviations of the path of Eros from Witt's ephemeris. L is determined from the remaining residuals.

Another value of L, namely L $= 6''.4305 \times (1.00195 \pm 0.00026) = 6''.4430 \pm 0''.0017$, $\mu^{-1} = 81.219 \pm 0.030$ for $\pi_{\odot} = 8''.790 \pm 0''.001$ is obtained, if Stracke's correction to the mean motion (instead of Delano's) is applied.

Masses of the planets. — In the following table the results of new determinations of masses of the major planets including their satellites are given together with the conventional values used in the computation of ephemerides.

TABLE VIII.

Reciprocal masses of the planets.

No.	m^{-1}.	p. e.	Author.	Method and observations.	
Mercury :	6 000 000	(conventional value).			
1........	6 120 000	± 43 000	Rabe [36]	Eros	(1926-1945
2........	5 970 000	± 455 000	Duncombe [52]	Venus	(1750-1949
3........	5 980 000	± 170 000	Makover, Bokhan [53]	Comet Encke	(1898-1954

TABLE VIII (*continued*).

No.	m^{-1}.	p. e.	Author.	Method and observations.	
Venus : 408 000 (conventional value).					
4........	404 700	± 800	Spencer Jones [54]	Sun	(1836-1923)
5........	407 000	± 500	Morgan, Scott [49]	Sun	(1900-1937)
6........	409 300	± 1 400	Clemence [55]	Mercury	(1765-1937)
7........	408 645	± 208	Rabe [36]	Eros	(1926-1945)
Earth-Moon : 329 390 (conventional value).					
8........	328 390	± 69	Witt [35]	Eros	(1893-1931)
9........	328 452	± 43	Rabe [36]	Eros	(1926-1945)
Mars : 3 093 500 (conventional value).					
10........	3 088 000	± 5 000	van den Bosch [56]	Satellites	(1877-1909)
11........	3 110 000	± 7 700	Rabe [36]	Eros	(1926-1945)
12........	3 088 000	± 3 000	Clemence [57]	Satellites	(since 1877
Jupiter : 1 047.355 (conventional value).					
13........	1 047.40	± 0.03	de Sitter [58]	Satellites, min. planets	
14........	1 047.4	± 0.4	D. K. Kulikov [59]	Jupiter VIII	(1908-1946)
15........	1 047.39	± 0.03	Clemence [57]	Newcomb's material	
Saturn : 3 501.6 (conventional value).					
16........	3 497.64	± 0.27	Hertz [60]	Jupiter	(1884-1948)
17........	3 494.8	± 1.3	Jeffreys [61]	Satellites	(1924-1937)
18........	3 499.7	± 0.4	Clemence [62]	Jupiter	(1779-1941)
Uranus : 22 869 (conventional value).					
19........	22 934	± 6	Harris	Satellites	
Neptune : 19 314 (*) (conventional value).					
20........	19 094	± 22	Gaillot [63]	Uranus	(1690-1903)
21........	18 889	± 62	van Biesbroeck [64]	Nereid	(1949-1955)
Pluto : 360 000 (conventional value).					
22........	400 000	± 40 000	Brouwer [65]	Uranus, Neptune	(1712-1941)

(*) In Newcomb's Tables of Uranus and in the " integration orbits " of Jupiter to Pluto; the value 19 700 in the other tables.

Remarks to table VIII :

1. Perturbations of Mercury on Eros.
2. Periodic perturbations of Mercury on Venus.
3. Perturbations of Mercury on Comet Encke.
4, 5. Periodic perturbations of Venus on Earth-Moon.
6. Periodic perturbations of Venus on Mercury.
7. Perturbations of Venus on Eros.
8, 9. Perturbations of Earth-Moon on Eros.
10. Result of the discussion of 27 determinations.
11. Perturbations of Mars on Eros.
12. No details published; the result includes more recent observational material than was available to van den Bosch.

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13. Heliometer observations of satellites and Newcomb's material after rejecting observations of comets and the older observations of satellites.
15. With revised weights; no details published.
16. Perturbations of Saturn on Jupiter by adjustment of the " integration orbit "; 3 996.98 ± 0.28 by adjustment of Hill's theory.
18. Perturbations of Saturn on Jupiter by adjustment of the " integration orbit ".
19. No details published (*cf.* Clemence [57]); based on photographic observations.
20. Perturbations of Neptune on Uranus.
22. 450 000 ± 90 000 from longitudes alone.

Constant of aberration. — Numerous determinations have been made by various methods, some of them with other objectives than that of a more accurate determination of the constant of aberration. A comprehensive collection of values of k derived from the closing errors in the latitude variation observations at the various stations are given by T. Hattori [66] together with critical comments on their significance.

In table IX determinations have been included which were intended by their authors to furnish an improved value of the constant. Not included in table IX are those values which result from the various determinations of the solar parallax according to equation (1).

TABLE IX.

Constant of aberration (1900).

(Conventional value 20″.47.)

No.	k.	p. e.	Author.	Method and observations.
	″	″		
1...	20.521	± 0.005	Chandler [67]	Observations prior to 1901
2...	20.475	± 0.010	Spencer Jones [33]	Radial veloc., 21 stars (1906-1908)
3...	20.514	± 0.008	Guinot [68]	Radial veloc., Venus (1956)
4...	20.489	± 0.003	Spencer Jones [69]	Greenwich Cookson FZT (1911-1936)
5...	20.5120	± 0.0021	K. A. Kulikov [70]	Pulkovo ZT, 28 000 obs. (1915-1929)
6...	20.4859	± 0.0063	Vashchilina [71]	Pulkovo Obs. W. Struve (1840-1842)
7...	20.522	–	Hattori [66]	ILS Mizusawa VZT (1942-1949)
	20.521	–		» » FZT
8...	20.511	± 0.005	Romanskaja [72]	Pulkovo ZT, 28 500 obs. (1929-1941)
9...	20.526	–	Fichera, Melchior [73]	ILS, 12 stations (1900-1954)
	20.514	–		
10...	20.528	± 0.008	Ševarlić [74]	Belgrad ZT (1949-1957)
11...	20.511	± 0.006	Guinot *et al.* [75]	Paris Astrolabe (1956-1961)
12...	20.479	± 0.008	Morgan [76[	U. S. Naval Obs. Transit (1908-1925)

Remarks to table IX :

1. Mean value from numerous observational sources.
3. 20″.513 ± 0″.011 from the combination of 39 spectra at evening elongations with 39 spectra at morning elongations of Venus.
   20″.515 ± 0″.012 from the combination of 74 spectra at morning elongations of Venus with 74 spectra of the Sun (1956-1957).
4. From a systematic difference between evening and morning observations of the form S sin ($\odot - \alpha$), interpreted as produced by an incorrect value of k; S = — 0″.0149 ± 0″.0024, Δk = + 0″.019 ± 0″.003.
6. 7 stars observed with the transit circle in the prime vertical.
7. From closing errors; list of values for k from closing errors at several stations, and remarks concerning their significance.
9. From published closing errors (951 000 observations), two procedures of weighting.
10. From closing errors.
11. 20″.509 ± 0″.006 from closing errors in latitude; 20″.520 ± 0″.019 from determinations of time averaged over yearly means deduced from the individual results for each pair of star groups, 20″.523 ± 0″.027 averaged over the results from 9 group pairs after rejecting three pairs with low weights.
12. From declinations of 17 polar stars.

Constant of nutation. — Table X contains all determinations of the constant published since 1920.

TABLE X.

Constant of nutation (1900)

(Conventional value 9″.21.)

No.	N.	Author.	Method and observations.
1...	9″.2069 ± 0″.0030 m. e.	Przybyllok [77]	ILS, 5 stations 83 000 obs. (1900-1915
2...	9.2066 ± 0.0055 p. e.	Jackson [78]	Greenwich Cookson FZT (1911-1929)
3...	9.2066 ± 0.0042 p. e.	Spencer Jones [79] and [69]	Greenwich Cookson FZT (1911-1936)
4...	9.206 ± 0.007 p. e.	H. R. Morgan [80]	Washington 9-inch trans. circle (19 1925).
5...	9.2108 ± 0.0019 m. e.	K. A. Kulikov [81]	Pulkovo ZT, 45 000 obs. (1904-1941)
6...	9.1985 ± 0.0051 m. e. 9.2073 ± 0.0041 » 9.1967 ± 0.0043 » 9.1955 ± 0.0034 »	Hattori [82]	ILS, 3 stations (1900-1935)
7...	9.1976 ± 0.0018 m. e.	Fedorov [83]	ILS, 3 stations (1900-1934)
8...	9.198 ± 0.002 m. e.	Fedorov [84]	ILS, 3 statioos 135 000 obs. (1900-19
9...	9.2055 ± 0.0047 m. e.	Romanskaja	Pulkovo ZT, 42 000 obs, (1915-1941)

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Remarks to table X :

1. The derived correction for each station has the form : constant term (interpreted as ΔN) + harmonic term in α, $[A \sin(\alpha + \Gamma)]$, the values of A lie between $0''.034$ and $0''.088$. In deriving the correction for each station the assumption was made that the variation of latitude is caused by pure polar motion.

2. From differences of latitude variations derived from evening and morning observations, assuming the constancy of the diurnal variation of latitude.

3. The value published by Jones ($N = 9''.2134$) is erroneous, *cf.* G. M. Clemence [8]. Remark No. 2 also applies here. The amplitude of the harmonic term in α is $A = 0''.020$.

4. From 4 000 observations of 15 circumpolar stars.

5. Weighted mean from four determinations; indicated is a strong correlation between the values N and the right ascensions of the used star groups.

6. All observed values of latitude were reduced to the declination system of GC; stations Mizusawa, Carloforte, Ukiah. Four different values of N result from different selection of star pairs. The amplitudes A of the harmonic term in α lie between $0''.006$ and $0''.024$. A real change of N is suspected since a variation parallel to Chandler's period is indicated. The variation of N with α may be fortuitous; it may be the effect of a magnitude equation, since there is no random distribution of the magnitudes of the observed stars over α.

8. The main terms derived from 135 000 observations at Carloforte, Mizusawa, Ukiah, the fortnightly terms from 230 000 observations including those by H. R. Morgan and A. J. Orlov, the semi-annual terms from observations of two bright stars at Poltava. The detailed results for nutation are

in longitude :

Main term..........	$-6''.853 \sin \Omega$	$+0''.008 \cos \Omega$
Fortnightly term....	$-0.0866 \sin 2$ ☾	$+0.0019 \cos 2$ ☾
Semi-annual term...	$-0.533 \sin 2 \odot$	

in obliquity :

Main term..........	$+9''.198 \cos \Omega$	$-0.001 \sin \Omega$
Fortnightly term....	$+0.0894 \cos 2$ ☾	$+0.0019 \sin 2$ ☾
Semi-annual term..	$+0.578 \cos 2 \odot$	

(in the notation by E. W. Woolard [85] : ☾ $= F + \Omega$, $\odot = F - D + \Omega$).

No correction was found for the theoretical value of the ratio of the axes of the nutational ellipse.

9. No details published, cf. *Trans. I. A. U.*, XI A, p. 172.

The correction to Newcomb's general precession. — The conventional value of the general precession in longitude adopted at the Paris Conference in 1896 is the value p resulting from Newcomb's [86] investigation ($p = 5\,025''.64$ per tropical century, for 1900).

Precessional corrections have been determined either from proper motions of stars given in systems which have adopted Newcomb's precession, or from the motion of planets. From proper motions the corrections Δm and Δn to general precession in right ascension and declination are determined, where

$$(15) \qquad \Delta m = \Delta p_1 \cos \varepsilon - \Delta \lambda,$$

$$(16) \qquad \Delta n = \Delta p_1 \sin \varepsilon,$$

Δp_1, correction to lunisolar precession;

$\Delta \lambda$, correction to planetary precession;

ε, obliquity of the ecliptic.

The proper motions in right ascensions may require a constant correction Δe as a consequence of an error of Newcomb's motion of the equinox, so that the fictitious proper motions caused by the corrections are

$$(17) \qquad \mu_\alpha = \Delta k + \Delta n \sin \alpha \tan \delta,$$

$$(18) \qquad \mu_\delta = \Delta n \cos \alpha,$$

with

$$(19) \qquad \Delta k = \Delta m - \Delta e = \Delta n \cot \varepsilon - (\Delta \lambda + \Delta e).$$

Therefore, the proper motions of the stars provide Δp_1 and $\Delta \lambda + \Delta e$. since

$$(20) \qquad \Delta p = \Delta p_1 - \Delta \lambda \cos \varepsilon,$$

and since $\Delta \lambda \cos \varepsilon$ is small compared with Δp_1, it can be said that the proper motions permit to deduce a correction to the general precession in longitude.

In table XI the values Δp_1 and $\Delta \lambda + \Delta e$ are given per tropical century as they resulted from the various determinations. The errors listed in this table are probable errors.

TABLE XI.

Corrections to Newcomb's lunisolar precession and motion of the equinox.

No.	Δp_1.	$\Delta e + \Delta\lambda$.	Author.	System and material.
	'' ''	'' ''		
1...	+0.93 ± 0.20	+1.14 ± 0.20	Fotheringham [87]	PGC : 5413 PGC stars
2...	1.13 ± 0.13	1.17 ± 0.13	Oort [88]	GC : 5413 PGC stars and 771 distant stars
3...	0.96 ± 0.12	0.96 ± 0.12	Plaskett, Pearce [89]	GC : 717 O 5 — B 7 stars
4...	0.84 ± 0.09	1.13 ± 0.09	Pariisky, Ogrodnikov, Fessenkov [90]	Newcomb : Auwers-Bradley stars
5...	1.01 ± 0.10	1.07 ± 0.09	van de Kamp and Vyssotsky [91]	GC : McCormick faint stars
	0.98 ± 0.12	1.03 ± 0.11		μ_b of » » »
	1.34 ± 0.20	1.07 ± 0.20	»	Allegheny - Johannesburg reference stars
6...	1.19 ± 0.13	1.21 ± 0.13	Oort [92]	GC : McCormick faint stars
	0.89 ± 0.10	0.85 ± 0.10		results Nos. 1 — 3, comb. with indep $\Delta e + \Delta\lambda$.
7...	0.94 ± 0.04	1.10 ± 0.04	Wilson, Raymond [93]	GC : catalogue-material
8...	0.98 ± 0.26	1.02 ± 0.26	Gliese [94]	FK3 : μ_b of FK3 stars
9...	1.23 ± 0.15	1.30 ± 0.15	Ali [95]	GC : O-B stars of GC
10...	1.11 ± 0.07	1.14 ± 0.07	Oort [96]	GC : mean values of Nos. 2, 3, 7, 9
	1.01 ± 0.07	1.18 ± 0.07		FK3 : reduced to FK3 system
11...	1.51 ± 0.08	1.79 ± 0.08	Williams, Vyssotsky [97]	FK3 : faint stars
	1.31 ± 0.07	1.64 ± 0.07	and [98]	FK3 : bright and faint stars
12...	0.85 ± 0.35	—	Brouwer [99]	Merc., Earth secular motions
13...	0.88 ± 0.15	—	Clemence [100]	» » mot. of perihelia
14...	0.75 —	1.09 —	H. R. Morgan, Oort [101]	FK3/N 30 : Rediscussion of various determinations
15...	1.28 ± 0.20	1.21 ± 0.20	Gordon [102]	FK3 : 2 pos. catalogues
16...	0.65 ± 0.15	0.68 ± 0.15	Bakulin [103]	GC/FK3 : 220 supergiants
17...	0.76 ± 0.25	0.73 ± 0.07	Weaver, Morgan [104]	N 30 : 79 cepheids
18...	—	1.08 ± 0.07	Schilt [105]	N 30 : GC stars

Remarks to table XI :

1. Extension of the solution given by L. Boss (*Astron. J.*, vol. 26, 1910, p. 95 and 111) in assuming that the local stellar system rotates uniformly about an axis perpendicular to the galactic plane.

2. From proper motion components in galactic latitude. The distant stars are O-B 5, N and *c* stars and δ *Cephei* variables.

3. From proper motion components in galactic latitude.

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4. Rediscussion of Newcomb's determination on the basis of the material provided by Auwers-Bradley stars with known parallaxes and proper motions in Newcomb's N_1-and declination system. The effects of galactic rotation are taken into account, and differences resulting from μ_α and μ_δ are discussed.

5. The material consists of : *a.* relative proper motions from McCormick parallax plates with an average epoch interval of 12.6 years referred to the system of the GC; *b.* proper motions of Allegheny-Johannesburg reference stars.

6. The determination given in the first line is the result of a rediscussion of the McCormick proper motions (*cf.* No. 5). In this solution the cited corrections and the galactic constant B entered as unknowns.

   The values in the second line are the result of a least squares solution of the combined results Nos. 1-3 and 6 with a determination of $\Delta e + \Delta\lambda$, which is independent of proper motions.

7. The values are the result of an analysis of the GC proper motions with a small percentage of stars cancelled. They were obtained from a simultaneous solution for solar motion, precessional corrections and galactic rotation. They are mean values from solutions based on equatorial and galactic components of the proper motions.

8. Result of the FK 3 proper motions in the galactic zone. Separate solutions for the galactic zone in the northern and the southern hemisphere result in rather different values, the weighted mean of them is $\Delta p_1 = +1''.23 \pm 0''.20$, $\Delta e + \Delta\lambda = +1''.31 \pm 0''.20$.

9. Determination of Δn and Δk from μ_b of 668 O-B stars. The solar motion is inferred from radial velocities. Proper motions corrected for the precessional errors are used to determine A, B and L_0.

10. On the basis of the differences FK 3-GC published by A. Kopff (*Astron. Nachr.*, vol. 269, 1939, p. 160), the average values of the quantities Δk, Δn, A and B resulting from the determinations Nos. 2, 3, 7, 9 are reduced to the system of FK 3. The result is in seconds of arc per century :

	Δk.	m.e.	Δn.	m.e.	A.	m.e.	B.	m.e.
On GC...	-0.13	± 0.04	$+0.44$	± 0.04	$+0.26$	± 0.07	-0.30	± 0.05
On FK 3.	-0.26	–	$+0.40$	—	$+0.34$	—	-0.17	—

11. The results in the first line are from proper motions of about 70 000 stars of average photovisual magn. 11.2, obtained at McCormick and Cape and referred to the FK 3 system. The quantities in the second line result from a combination of the corrections obtained from GC proper motions reduced to FK 3 and from the McCormick-Cape material.

12. The cited value is deduced from the secular motions of Mercury and the Earth based on $\pi_\odot = 8''.790$. If $\pi_\odot = 8''.7984$, then the value is $+0''.64$.

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14. The weighted average values per century :

$$\Delta k = -0''.40 \pm 0''.04 \text{ m.e.}, \qquad \Delta n = +0''.49 \pm 0''.04 \text{ m.e.}$$

are obtained from the following determinations based on FK 3 : Williams, Vyssotsky [97] : Galactic zone; Williams, Vyssotsky [98] : GC stars $6^m.0$-$7^m.0$; Oort [96] : Galactic stars.

It is estimated that latitude variations before 1890 cause a periodic error of about $+ 0''.14 \cos\alpha$ in μ_δ of FK 3. If this amount is applied to Δn, then $\Delta n = + 0''.35 \pm 0''.04$ is inferred.

Since other solutions referred to the μ_α-system of N 30 give similar values, and since from μ_δ referred to N 30 values of Δn around $+ 0''.30$ are obtained, $\Delta n = + 0''.30$ is adopted as the most probable value ($\Delta p_1 = + 0''.75$). It is pointed out that the determination of Δe from observations of the Sun, Moon and planets ($\Delta e = 0''.2$) is not in accordance with the result from proper motions. On the assumption that the best available proper motion system is the average of FK 3 and N 30 corrected for precession as indicated, the following values of the galactic constants are deduced :

$$A = + 0''.43 \text{ per century} = + 0.020 \text{ km. s}^{-1}.\text{ps}^{-1} \pm 0.0020 \text{ (m.e.)},$$
$$B = - 0''.15 \quad \text{« \quad »} \quad = - 0.007 \text{ km. s}^{-1}.\text{ps}^{-1} \pm 0.0015 \text{ (m.e.)}.$$

15. Derived from proper motions near the equator. The motions are deduced from the positions in the catalogues of Schjellerup (1865.0) and Morin-Kondratiev (1900.0) which were reduced to the FK 3 system.

16. The proper motions are discussed on the basis of the systems of GC and FK 3. It is found that the system is of noticeable influence only on the declination of the solar apex and the galactic constant B.

17. Derived from a simultaneous solution for Δn and the angular velocity of galactic rotation ω_0. $\Delta\omega$ (R) has been introduced as known from radial velocities of cepheids.

18. From proper motions of GC reduced to N 30. Δe is derived with the assumed value $\Delta p_1 = + 0''.80$ under the assumption that the space velocity component perpendicular to the plane of the galaxy is, on the average, independent of the galactic longitude.

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