Waves & Oscillations in the Solar Atmosphere: Heating and Magneto-Seismology
Proceedings IAU Symposium No. 247, 2007
R. Erdélyi & C. A. Mendoza-Briceño, eds.

doi:10.1017/S1743921308014798

Coronal loop seismology using multiple transverse loop oscillation harmonics

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Abstract. TRACE observations (23/11/1998 06:35:57-06:48:43UT) in the 171 Å bandpass of an active region are studied. Coronal loop oscillations are observed after a violent disruption of the equilibrium. The oscillation properties are studied to give seismological estimates of physical quantities, such as the density scale height. A loop segment is traced during the oscillation, and the resulting time series is analysed for periodicities. In the loop segment displacement, two periods are found: 435.6 ± 4.5 s and 242.7 ± 6.4 s, consistent with the periods of the fundamental and 2nd harmonic fast kink oscillation. The small uncertainties allow us to estimate the density scale height in the loop to be 109 Mm, which is about double the estimated hydrostatical value of 50 Mm. The eigenfunction is used to do spatial coronal seismology, but that method does not give any conclusive results.

Keywords. waves, methods: data analysis, Sun: atmosphere, Sun: Corona, Sun: oscillations

1. Introduction

In the last decade, a wealth of oscillatory phenomena has been discovered in the solar corona (for an overview, see Nakariakov & Verwichte 2005; Banerjee *et al.* 2007). These oscillatory phenomena are a tool to do MHD coronal seismology with (Roberts *et al.* 1984).

Of particular interest are transverse coronal loop oscillations, i.e. rapidly damped oscillations of coronal loops displacing the loop axis. In recent years, they have received a lot of attention from observers and modellers alike. The oscillations are believed to be fast magnetosonic kink modes.

These oscillatory events were first observed by Aschwanden *et al.* (1999); Nakariakov *et al.* (1999). Later on, oscillations in a sample of 17 loops were studied and analysed by Schrijver *et al.* (2002); Aschwanden *et al.* (2002). Recently, Verwichte *et al.* (2004) found two events with signatures of 2 different periodicities in a single loop in an arcade. Wang & Solanki (2004) found oscillations with a vertical polarisation, and Li & Gan (2006) observed oscillations in shrinking loops.

Fast magnetosonic kink oscillations were first studied analytically by Zaitsev & Stepanov (1975). The dispersion relation for these modes was later independently derived by Edwin & Roberts (1983).

Because of inherent problems with the observational study of the solar corona (line-of-sight integration and the high density contrast with the photosphere), it is practically impossible to measure the coronal density and the coronal magnetic field. Some attempts to measure the density were undertaken (Aschwanden *et al.* 2003) and the average value of the magnetic field was estimated by Lin *et al.* (2000, 2004), but the error bars on the results are large.

It is, however, possible to measure the density and magnetic field by doing coronal seismology (Uchida 1970; Roberts *et al.* 1984). Coronal seismology can be achieved by studying oscillations in the corona and comparing the observed properties with models. By adjusting the model, restrictions on physical quantities in the corona can be obtained.

The transverse oscillations are an excellent tool to do coronal seismology with. The coronal dissipative coeffecients were estimated by Nakariakov *et al.* (1999) and the local magnetic field in the oscillating loop was calculated by Nakariakov & Ofman (2001). Andries *et al.* (2005a) used the double periodicity measured by Verwichte *et al.* (2004) to measure the density scale height in the corona. Verwichte *et al.* (2006) determined the radial density structure in oscillation coronal loops, and Arregui *et al.* (2007) found a lower bound for the internal Alfvén transit time.
Since then, a lot of interest has gone out to multiple periodicities in the same structure. The influence of the density stratification in the corona on the ratio of the period of the fundamental and the second harmonic was studied by McEwan *et al.* (2006); Dymova & Ruderman (2006). Unfortunately, the only measurement of such a double periodicity (Verwichte *et al.* 2004) had very large errors and could not be used to confidently establish the density scale height. More recently, De Moortel & Brady (2007) observed a loop mainly oscillating as a 2nd harmonic, but also showing a periodicity consistent with the fundamental mode.

In this paper, we report on the high accuracy measurement of a double periodicity in a single loop. The two periods, together with the spatial structure of the oscillation, are used to determine coronal loop parameters which are difficult to measure, such as the density scale height in coronal loops.

The results presented in this proceeding are explained in greater detail in Van Doorsselaere *et al.* (2007).

2. Basic properties of the event

We use the 171 Å observations of the Transition Region And Coronal Explorer (TRACE) satellite (Handy *et al.* 1999) to study the oscillations in an active region on the 23rd of November 1998, from 06:35:57 to 06:48:43UT. The time series has an average cadence time of 33 s.
In the images preceding the studied time series (and in Fig. 1, left), it is observed that a low lying loop is violently disrupted (indicated by C in Fig. 1, left). After this violent event, a cloud of material is seen to escape the loop system (shown by B in Fig. 1, left), pushing aside the overlying loops (A in Fig. 1, left). Disturbed out of their equilibrium, the overlying loops start to oscillate in the wake of the escaping material.

Loop A is our main loop of interest. It has also been studied by Aschwanden *et al.* (2002) (case 3a). They estimated the loop length to be 390 Mm. We measure the initial loop length to be 440 Mm long. During the 24 frames of the time sequence (765 s), the loop shortens significantly to 365 Mm. These measured lengths are compatible with the previously reported values.

The loop exhibits a more involved oscillatory pattern than expected for a fundamental mode. During the oscillation, a "knot" is formed near the loop top (see Fig. 1, right). If the oscillation would be a fundamental mode, the form of the loop would be maintained (Schrijver *et al.* 2002). The deformation of the loop shape clearly shows that higher harmonics must be involved. On the other hand, the loop axis is still displaced, and no density oscillations are observed. This indicates that the fast kink mode is observed, but that a combination of longitudinal mode numbers is excited. The longitudinal structure of the oscillation suggests that the 2nd harmonic kink oscillation is observed.

https://doi.org/10.1017/S1743921308014798 Published online by Cambridge University Press

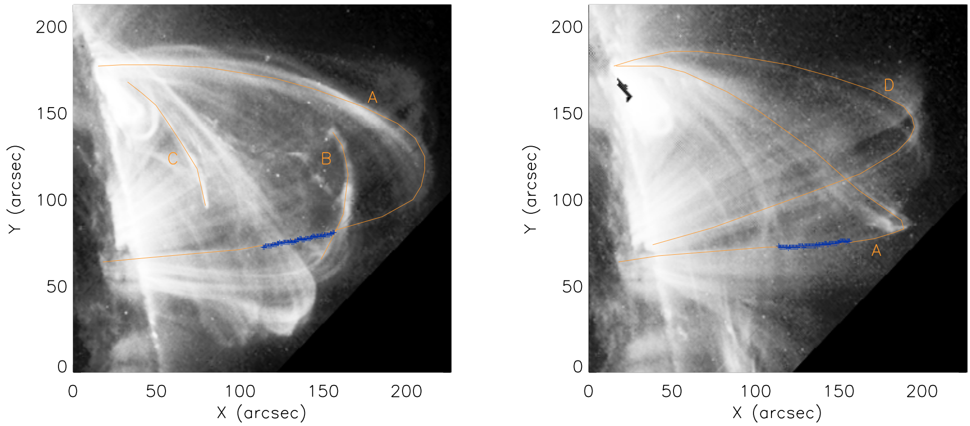

Figure 1. left panel: The starting frame of the studied time sequence. A indicates the oscillating loop studied in this paper. B points at the material expelled by the triggering loop (C). right panel: The 14th frame of the studied time sequence. A indicates the oscillating loop studied in this paper. D shows another oscillating loop in the same loop complex.

3. Analysis of the event

3.1. *Determination of the oscillation characteristics*

To facilitate the analysis, the images are rotated by 45° in the anti-clockwise direction. By doing this, the loop is almost aligned parallel with the X-axis (see Fig. 1). A disadvantage of this rotation is that the resolution reduces by a factor of $\sqrt{2}$, because no interpolation is performed and only pixels with an even sum of indices are retained.
For a fixed horizontal coordinate (x), the vertical position of the loop (y) is estimated throughout the duration of the oscillation. This procedure is repeated for 60 positions along the loop leg. The estimated perturbations $y(x,t)$ are indicated by a blue line on Fig. 1.

To analyse the perturbations, Gaussian noise with a standard deviation of 1.41 arcsec (i.e. 2 pixels in the rotated image) is added to $y(x,t)$. Then, for a fixed vertical slit (i.e. a fixed x position), the resulting noisy data is fitted with the function

$$A \sin\left(\frac{2\pi t}{P} + \phi\right) \exp\left(-t/\tau\right) + C + Dt, \tag{3.1}$$

where A is the amplitude of the oscillation, P the period, ϕ the phase, τ the damping time and C and D describe the average intercept and global shift of the loop. For each fixed x slit, a set of $\{A, P, \phi, \tau, C, D\}$ is obtained.
The above procedure is repeated 200 times with a varying noise. For each slit, a statistical distribution of parameters is obtained. From this distribution, the mean and the variance can be computed. We thus obtain the mean values and errors of $\{A, P, \phi, \tau, C, D\}$ for a fixed x value.
Fig. 2 shows the dependency of the mean parameters and their errors on x. It can be established from Fig. 2 that, for the pixels lying higher up the loop ($x \in [135.1'', 156.3'']$), a consistent value for the period and phase is obtained. This suggests that the fitting identifies the same oscillation throughout that part of the loop. By averaging over the top part of the loop, a more precise estimate of the oscillation properties can be made. The errors on the oscillation properties are reduced drastically by taking the mean over 31 points. For the considered interval, an average period of 435.6 ± 4.5 s is found, and

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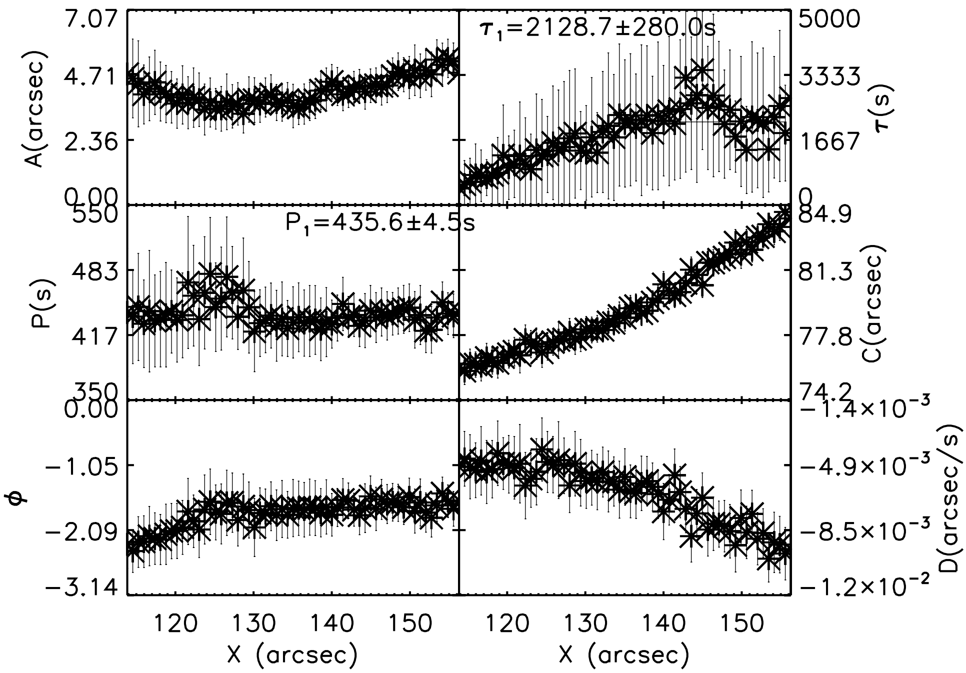

Figure 2. From top to bottom: (left) A, P, ϕ, (right) τ, C, D for the fit of the original signal of 60 pixels along the loop leg.

an average damping time of 2129 ± 280 s. The value of the period is compatible with the previous estimates for this event (see Aschwanden *et al.* 2002, case 3a): $P = 522$ s, the value for the damping time is almost double the previously estimated value: $\tau = 1200$ s.

As a next step in the analysis, for each x position, the best-fitted function is subtracted from the original signal. As above, the obtained residu and additional Gaussian noise is fitted with Eq. 3.1. Again, a statistical ensemble is found for each x position.
The dependence on x of the mean fitting parameters and their errors in the residu signal is shown in Fig. 3. Again a consistent oscillation is detected in the top part of the loop. The average period found in the residu is 242.7 ± 6.4 s, and the average damping time 872 ± 221 s.

3.2. *Coronal loop seismology*

The ratio of periods of these two oscillations can be calculated to be $P_1/P_2 = 1.795 \pm 0.051$. This value significantly deviates from 2.

Long-wavelength kink modes of a magnetic cylinder are known to be weakly dispersive (e.g. Nakariakov & Verwichte 2005). The P_1/P_2 ratio caused by the dispersion in an unstratified long loop is within a few percent of 2 (see Fig. 2 in McEwan *et al.* 2006). The observed deviation of P_1/P_2 from 2 is too large to be explained by dispersion only.

The effect of the vertical density stratification ($\rho(z) = \rho_0 \exp(-z/H)$) on P_1/P_2 was studied by Andries *et al.* (2005a); McEwan *et al.* (2006). Andries *et al.* assumed vertical density stratification in the solar corona, and projected this dependency on a cylindrical loop. On the other hand, McEwan *et al.* took a exponential density stratification in the loop itself. Yet another approach was taken by Dymova & Ruderman (2006), who did the calculations for non-semi-circular loops in a vertically stratified atmosphere.

https://doi.org/10.1017/S1743921308014798 Published online by Cambridge University Press

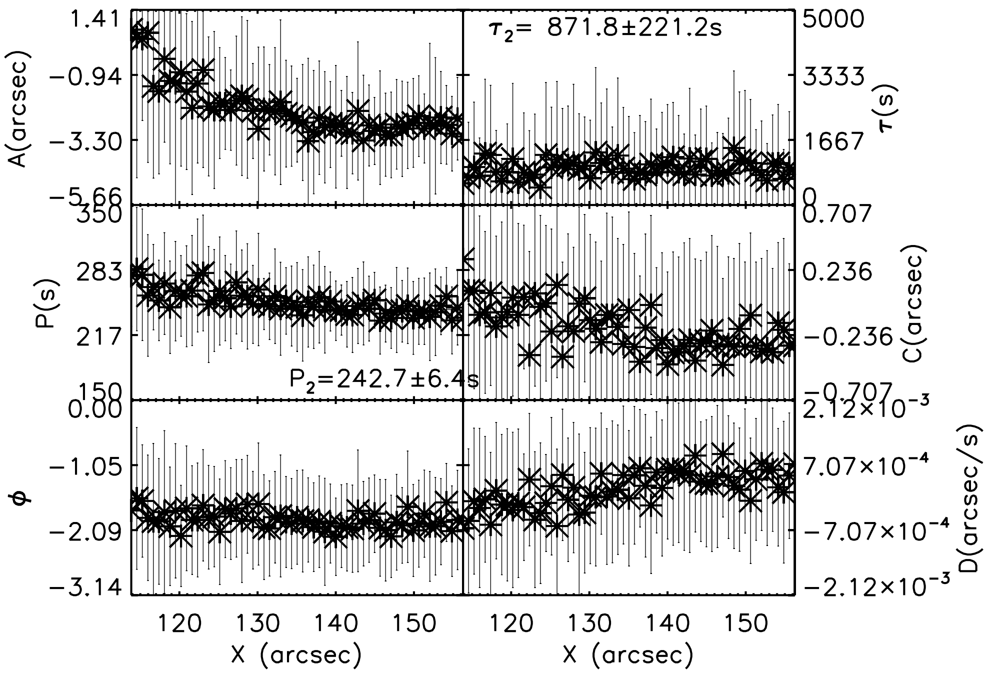

Figure 3. From top to bottom: (left) A, P, ϕ, (right) τ, C, D for the fit of the residu signal, obtained after subtraction of the fit in Fig. 2 from the original signal.

If it is assumed that the deviation of P_1/P_2 from 2 is solely caused by the vertical density stratification, a range of values for the relative density stratification $L/\pi H$ can be found (L is the loop length and H is the density scale height). Using the results of Andries *et al.* (2005a), we find a value of $L/\pi H = 1.17^{+0.28}_{-0.3}$.

To estimate the absolute value of the density scale height H, an estimate of the length of the loop L is needed. Before the event, the length of the loop is estimated to be 440 Mm. At the end of the observations, the loop has shortened to approximately 365 Mm. To estimate the density scale height, an average loop length of 400 Mm is assumed.
Using this average length, the estimated value $L/\pi H = 1.17$ yields a density scale height of $H = 109^{+37}_{-21}$ Mm. The estimates of the density scale height do not take into account the errors on the loop length. The errors on the loop length may be as large as 10%. This results in an error on the density scale height of up to 10%. For the currently estimated value for H, this would be approximately 10 Mm. This error is much less than the error induced by the uncertainties on the ratio of the periods.

The seismologically estimated value for $H = 109$ Mm is more than double the value of 50 Mm, expected in a hydrostatically stratified plasma with a temperature of 1 MK, corresponding to the observational bandpass 171Å. This is not abnormal, as the stratification inside coronal loops may exceed the hydrostatical value by a factor of 4 (see Aschwanden *et al.* 2000, 2001). Even stronger, Aschwanden *et al.* (2000) show in their Fig. 7 that the ratio of the density scale heights in coronal loops and the hydrostatic scale height exhibits an increasing trend for longer loops.

Our estimate is an independent, seismological confirmation of the controversial results in Aschwanden *et al.* (2000). It may point out that coronal loops have a higher density scale height inside the loop when compared to the surrounding corona.

3.3. *Amplitude dependence on the vertical coordinate*

For a stratified medium, the fundamental mode is essentially coupled to the 3rd harmonic (see Andries *et al.* 2005b; Erdélyi & Verth 2007) without assuming a damping mechanism. The eigenfunction will thus be of the form

$$A(\sin(\theta) + a\sin(3\theta)), \tag{3.2}$$

where A is the global amplitude of the fundamental harmonic, θ is the arc length along the loop ($\theta \in [0, \pi]$) and the coupling parameter a depends of the importance of the density stratification, measured by $L/\pi H$. We expect the eigenfunction of the overtone to be

$$B\sin(2\theta), \tag{3.3}$$

where B is the amplitude of the second harmonics, assumed to be independent of A and solely determined by the form of the original perturbation. We have ignored the coupling effect to higher harmonics, and it thus will not be possible to measure the peak shift present in the eigenfunction (Verth *et al.* 2007).

If it is assumed that the line of sight is parallel to the loop baseline, the vertical height above the Sun z can be written as:

$$z = L/\pi \sin\theta, \qquad \text{leading to} \qquad \theta = \arcsin \pi z/L.$$

Substituting this formula in Eq. 3.2-3.3, we find the expected amplitude dependence on the vertical coordinate. For the fundamental mode, we obtain

$$A((1+3a)\frac{\pi z}{L} - 4a\left(\frac{\pi z}{L}\right)^3),$$

and for the 2nd harmonics, we find

$$2B\frac{\pi z}{L}\sqrt{1-\left(\frac{\pi z}{L}\right)^2}.$$

However, when fitting the function for the fundamental mode to the top panel of Fig. 2, and the function for the 2nd harmonics to the top panel of Fig. 3, no conclusive results are obtained. We find $A = 8.9 \pm 3.5\,\text{px}$, $a = 0.02 \pm 0.36$, $B = -4.1 \pm 0.5\,\text{px}$, $\pi z_0/L = 0.81 \pm 0.25$, $\pi z_1/L = 0.56 \pm 0.25$, where z_0 and z_1 indicate the position of the highest and lowest measured vertical slit, respectively. The value of a yields a range for $L/\pi H \in [0, 3.7]$ and thus reveals no extra information.

We know that the distance between z_0 and z_1 is exactly $30\,\text{px} = 15.3\,\text{Mm}$. We can thus calculate that $L \approx 192\,\text{Mm}$, in a range of $L \in [79\,\text{Mm}, \infty[$. The observationally estimated value for the length ($\approx 400\,\text{Mm}$) is higher than this seismological estimate, but lies within the exorbitant errorbars.

Using a more simple approach and neglecting all coupling and higher harmonics (i.e. take $B = 0$ and $a = 0$), the top panel of Fig. 2 can be fitted with a straight line. From this fit, it is found that the amplitude will be 0 for $x = 82.7\,\text{arcsec}$. However, from the observations, it can be estimated that the footpoint is situated approximately at $x = 17.7\,\text{arcsec}$. These two values do not agree at all.

4. Conclusions

In this article, TRACE 171Å observations of an active region were analysed. By tracing out a loop segment, two oscillation periods could be detected with high confidence: $435.6 \pm 4.5\,\text{s}$ and $242.7 \pm 6.4\,\text{s}$. Using these periods, and interpreting them as the fundamental and the 2nd harmonic oscillation, a value $P_1/P_2 = 1.795 \pm 0.051$ was found.

This value allowed us to establish $L/\pi H = 1.17$, and lead to a density scale height of $H = 109^{+37}_{-21}$ Mm. Such a value for the density scale height in the loop is significantly higher than the hydrostatically expected value (50 Mm). Our result seismologically confirms the results of Aschwanden *et al.* (2000), and suggests that the density scale height in coronal loops is much higher than that in the surrounding corona.

We used the eigenfunction to do spatial coronal seismology in order to obtain geometrical properties of the loop, but found that the observational errors are to large to achieve this. A loop length $L = 192$ Mm in a range of $[75\,\text{Mm}, \infty[$ was estimated. This value of L is not close to the observed loop length, but lies within the errorbars.

Acknowledgements

TVD would like to acknowledge the financial support of STFC and the travel support from the IAU.

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