SPREADING OF EXCEPTIONAL CLAIMS BY MEANS OF AN INTERNAL STOP LOSS COVER

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I

Assurance companies usually divide their portfolio into a series of tariff groups. This division is necessary for a comparison between the claims and the available premiums in the different tariff groups. By comparing these figures the company is able to verify if the tariff is appropriate or if a modification in line with the empirical data is to be recommended. Apart from this technical argument companies are sometimes compelled to make such a division because of their constitution (e.g. mutual companies) or because it is prescribed by the supervisory authorities. Companies underwriting in various countries are under the obligation, as a rule, to keep special accounts for different countries and currencies. All these reasons lead to a division of the whole business into more or less independent subdivisions which quite obviously have not as much capacity to equalize the risk fluctuations as the whole portfolio.

In every tariff group an exceptional fluctuation may occur either from a large individual claim or from claims under number of policies. This leads to a substantial loss in the relevant profit and loss statement at the end of the year. If this debit balance is carried forward to the new account every year, the account of such a tariff group may show a loss over a number of years. It is obvious that a company will endeavour to avoid such an awkward situation.

This sort of problem is usually dealt with by means of reinsurance and, to some extent this will be the right solution. However if every tariff group should be able to meet its own liabilities, a rather extended reinsurance is necessary. The capacity of the whole portfolio then would be of no importance, which suggests that a solution through reinsurance alone is not entirely satisfactory.

A more suitable way would seem to consist in a method in which the whole portfolio would act as reinsurer for all tariff groups. This would ensure that the capacity of the whole portfolio would be utilised. Nevertheless peak risks in the portfolio have to be reinsured all the same. The details of this method will be demonstrated in section II and its practical application in section III.

II

To avoid a loss in a particular tariff group owing to an exceptional claim either by amount or by accumulation, an internal reinsurance cover must minimize both of these chance effects. A Stop Loss cover is an optimal solution in this respect. It will be recalled that this reinsurance covers the risk of a year's total of claims exceeding a preassigned sum, e.g. k % of the total of premiums. On the other hand, the tariff group in question has to pay a certain reinsurance premium. If the method of premium calculation is to be equitable the reinsurance premium must be equal to the expected value of the excess claims plus an appropriate security loading.

The company will earmark for this purpose a certain fund which is debited with all excess claims falling due on the occurrence of a claim and credited with all Stop Loss premiums from the different tariff groups. It goes without saying that this arrangement has to start with a sufficient risk reserve from the very beginning. Moreover the Stop Loss premiums have to be determined according to the risk with a sufficient security loading in addition. A closer consideration of these necessities leads to forth the following:

A. The method of Stop Loss premium calculation is well known from collective risk theory. If $f(y, P)$ denotes the frequency function of the totality of claims y in a given year and the net premium P (P being equal to the expected value of claims) and if the preassigned limit for the Stop Loss premium $\pi\ (kP, P)$ is kP, the following equation holds:

$$\pi\ (kP, P) = \int_{kP}^{\infty} (y - kP)\, f(y, P)\, dy \qquad (1)$$

For numerical computations of Stop Loss premiums the method of Esscher (see e.g. [1]*)) is generally applied. The application of Esscher's method depends on the skewness of the frequency

*) Numbers in [ ] refer to the list of references.

function $f(y,P)$ and the accuracy of the results derived from that method diminishes more and more with increasing skewness. In such cases the direct numerical evaluation of the integral in equation (1) seems the only way to proceed.

A particular problem consists in determining the parameters of the frequency function $f(y, P)$ which are usually not known and which may only be found through laborious estimation proceedings. An approach to this problem consists in analysing the observed values for the number and the amount of claims. As a rule this sort of investigation will ultimately lead to satisfactory results, the estimate of parameters for determining the frequency function $f(y,P)$ will generally improve with the years. At the outset however more or less rough estimations may not be avoidable. These estimations have to be based on a thorough knowledge of the characteristic properties of the relevant kind of insurance. It is recommended that cautious hypotheses be adopted, in particular with regard to the skewness of the relevant frequency function.

B. Even if the difficulties in the aforementioned para. A are disregarded and if the Stop Loss premiums in the different tariff groups correspond to the expected value of the excess claims, the system of an internal Stop Loss cover may fail if extensive claims occur in the early years. It is therefore necessary that the fund for the internal Stop Loss cover has an initial risk reserve at its disposal and that every tariff group has to contribute an appropriate security margin. Under the assumption that the claim records in the different tariff groups are mutually independent, collective risk theory provides a procedure—the so-called ruin problem—to determine the initial risk reserve and the appropriate security loading. The starting point is the ruin probability function $\psi(u)$ which indicates the probability that the risk reserve of initial amount u to which the Stop Loss premiums are added and from which the excess claims are deducted, will ever become negative.

For the ruin function the following well known relation will hold:

$$\psi(u) \approx e^{-Ru} \tag{2}$$

where u is the initial risk reserve measured in terms of the mean

sum at risk and where the adjustment coeffcient R is defined as the largest real number which satisfies the condition

$$\int_{-\infty}^{+\infty} e^{-Rx} \; f(x) \; d \; x = 1$$

In formula (3) $f(x)$ is the frequency function of an excess claim x which may be found easily by truncating the frequency function $f(y,P)$. According to formula (3) the Stop Loss premium including a security factor can be determined as follows:

$$(1 + \lambda_{kP}) \; \pi \; (kP,P) = ln \{1 + \int_{kP}^{\infty} [e^{R(y-kP)} - 1] \; f(y,P) \; d \; y\}^{1/R}$$

For further details it is referred to paper [2].

In practical applications the tolerable maximum value for $\psi(u)$ is first fixed, indicating the acceptable probability limit for ruin of the risk reserve of the internal Stop Loss cover. If this probability is denoted by 10^{-s} the adjustment coefficient R for a given initial reserve u will be determined as

$$R = \frac{s}{u \,.\, \log \; e} \tag{5}$$

Using this value of R, the Stop Loss premium, including an appropriate security factor, can be derived for each tariff group involved.

It is important that the assumed stochastic independence between the groups with separate accounts is fulfilled. This assumption often holds true, especially where geographically wide-spread insurance regions are concerned. Nevertheless in some cases a certain dependence in the claim records may exist. In those cases, the Stop Loss premiums determined according to formula (4) will have to be increased in an appropriate way. Clues for the amount of the additional loading can be drawn from the observation of the pooled total of claims for the tariff groups between which a degree of contagion is presumed.

III

A Swiss Assurance Company has created such an internal Stop Loss cover for its foreign affiliates according to the rules set forth

in section II. The following schedule contains the records of the risk premiums charged and the excess claims paid out in 7 different tariff groups. For reasons of secrecy all figures are multiplied by a certain factor.

Stop Loss Premiums (π) *and Excess Claims* (S)
for an internal Stop Loss cover
in a practical case

Tariff group		Year 1	2	3	4	5	$\sum$
A	π	13	23	37	37	46	156
	S	0	23	13	0	0	36
	π-S	13	0	24	37	46	120
B	π	25	20	20	27	18	110
	S	0	0	0	27	0	27
	π-S	25	20	20	0	18	83
C	π	34	39	36	31	35	175
	S	463	0	0	176	0	639
	π-S	-429	39	36	-145	35	-464
D	π	18	17	8	5	4	52
	S	0	0	0	105	0	105
	π-S	18	17	8	-100	4	-53
E	π	30	39	42	45	48	204
	S	71	0	0	7	88	166
	π-S	-41	39	42	38	-40	38
F	π	35	39	44	101	86	305
	S	0	7	42	204	0	253
	π-S	35	32	2	-103	86	52
G	π	11	14	26	32	27	110
	S	0	0	0	0	0	0
	π-S	11	14	26	32	27	110
$\sum$	π	166	191	213	278	264	1112
	S	534	30	55	519	88	1226
	π-S	-368	161	158	-241	176	-114

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The following conclusions may be drawn from the forementioned figures:

1) The schedule contains seven tariff groups observed during 5 insurance years, i.e. 35 statements. Of these, 12 have shown an excess claim. In 6 cases these claims did not exceed the Stop Loss premium, whereas in the other 6 cases the difference $\pi - S$ leads to a debit balance. In 23 or 2/3 of all statements no excess claims have occurred.

2) Of the seven tariff groups under observation for 5 years two have caused a loss to the internal reinsurer, whereas the other five groups have yielded a profit. Tariff group C has caused the largest loss since the total amount of all excesses was greater than 3 1/2 times the total amount of all its Stop Loss premiums.

3) If the seven tariff groups in question are pooled, a loss for the internal reinsurance fund resulted in the first and in the fourth insurance year. The other years show a profit. The fact that in the event the first year ended with a substantial loss shows the importance of starting such an internal cover with a sufficient risk reserve. In the present case the risk reserve started with about 2 000 units.

4) The grand total of the 5 years considered for the seven tariff groups has resulted a loss of approximately 10 % of the total Stop Loss premiums received during these 5 years. This loss is mainly caused by the extremely high claim for tariff group *C* in the first year and which has been practically compensated by the other tariff groups over the 5 year period.

5) In conclusion the impression is gained that the Stop Loss premiums charged were appropriately determined, although their calculation was not very easy because of insufficient knowledge of the parameters.

The method described for an internal Stop Loss cover has lead in practice to satisfactory results. It gives the facility of observing the claim record in every tariff group. To avoid undesirable disturbances, the excess claims are currently replaced by the corresponding Stop Loss premiums, thus providing a situation from

which it can be decided whether the tariff is appropriate or not in the different tariff groups. It should be noted that all eventual excess claims are automatically covered through the internal Stop Loss cover a much more advantageous position as compared with the "solution" where a special settlement is required whenever a large claim occurs.

References

[1] H. Ammeter, Die Ermittlung der Risikogewinne im Versicherungswesen auf risikotheoretischer Grundlage.
Bulletin de l'Association des Actuaires suisses, vol. 57, fasc. 2, Berne 1957.

[2] Le problème de la ruine dans la couverture des excédents de sinistres.
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