Having thus ascertained the law which (to the best of our information) appears to govern the influence of climate upon European life in India, we are justified in inferring that the same law operates in other countries. Indeed the conclusion is inevitable until it shall have been shewn, in any particular instance, that some other law prevails. Our requirements, therefore, for determining the true law of mortality among Europeans residing abroad are limited to the determination of the values of the single constant representing the extra risk for the respective climates. And herein lies the great importance of the discovery of general laws, viz., that by their aid we are enabled to economize our facts, and, by bringing them all to bear upon a single point, compel them, as it were, to yield us information which we should otherwise be unable to obtain. As an instance of this I may refer to the Tables published in Vol. 7, p 134, of this *Journal,* embodying a considerable collection of facts relating to foreign risks. These facts, although insufficient to serve for the construction of independent Tables of mortality for different climates, are yet numerous enough to enable us to determine, with a tolerable degree of accuracy, the single constant required for the adjustment of the Home Table to each case; and I may perhaps on some future occasion ask the indulgence of your readers for an analysis of the experience referred to.

I am, Sir,

Your very obedient servant,

10, *King Street, Cheapside,*　　　　　　　W. M. MAKEHAM.
　　27th February, 1868.

DEMONSTRATION OF A FORMULA FOR INTERPOLATION.

To the Editor of the Assurance Magazine.

Sir,—I have been asked on more than one occasion how the formulæ were obtained which I used in graduating the mortality among the males of the peerage (*Assurance Magazine,* vol. xii., p. 221); and as the question may not be without interest to some readers of the *Journal,* I will now state the method.

I had noticed the following resemblance between two sets of expressions. If a series u_0, u_5, u_{10}, &c. is differenced, and Δu_0, $\Delta^2 u_0$, . . . be the initial terms of each order of differences; $\Delta^4 u_0$ being constant; then, as is shown on page 23 of the current volume of the *Journal,*

$$\delta u_0 = \frac{1}{5}\,\Delta u_0 - \frac{2}{5^2}\,\Delta^2 u_0 + \frac{6}{5^3}\,\Delta^3 u_0 - \frac{21}{5^4}\,\Delta^4 u_0 \quad (1)$$

$$\delta^2 u_0 = \qquad\quad \frac{1}{5^2}\,\Delta^2 u_0 - \frac{4}{5^3}\,\Delta^3 u_0 + \frac{16}{5^4}\,\Delta^4 u_0 \quad (2)$$

$$\delta^3 u_0 = \qquad\qquad\qquad\quad \frac{1}{5^3}\,\Delta^3 u_0 - \frac{6}{5^4}\,\Delta^4 u_0 \quad (3)$$

$$\delta^4 u_0 = \qquad\qquad\qquad\qquad\qquad\quad \frac{1}{5^4}\,\Delta^4 u_0 \quad (4)$$

Also, if Σ_1 be the sum of the first five terms $u_0 + \ldots + u_4$ of a series, Σ_2 the sum of the next five terms u_5 to u_9, and so on; and $\Delta\Sigma_1$, $\Delta^2\Sigma_1$, $\Delta^3\Sigma_1$, the differences of these sums, their third difference being constant instead of the fourth, it may be proved by subtracting from one another the values of Σ_1, Σ_2, &c. in terms of u_0, δu_0, $\delta^2 u_0$, &c. that

$$\delta u_0 = \frac{1}{5^2}\Delta\Sigma_1 - \frac{4}{5^3}\Delta^2\Sigma_1 + \frac{16}{5^4}\Delta^3\Sigma_1 \qquad (5)$$

$$\delta^2 u_0 = \qquad\qquad \frac{1}{5^3}\Delta^2\Sigma_1 - \frac{6}{5^4}\Delta^3\Sigma_1 \qquad (6)$$

$$\delta^3 u_0 = \qquad\qquad\qquad\qquad \frac{1}{5^4}\Delta^3\Sigma_1 \qquad (7)$$

For according to the ordinary formula for the sum of any number of terms of a series,

$$\Sigma_1 = 5u_0 \; + \frac{5.4}{1.2}\delta u_0 + \frac{5.4\;3}{1.2.3}\delta^2 u_0 + \frac{5.4.3.2}{1.2.3.4}\delta^3 u_0$$

$$\Sigma_2 = 10u_0 + \frac{10.9}{1.2}\delta u_0 + \frac{10.9.8}{1.2.3}\delta^2 u_0 + \frac{10.9.8.7}{1.2.3.4}\delta^3 u_0 - \Sigma_1$$

$$\Sigma_3 = 15u_0 + \frac{15\cdot14}{1.2}\delta u_0 + \frac{15.14.13}{1.2.3}\delta^2 u_0 + \frac{15.14.13.12}{1.2.3.4}\delta^3 u_0 - (\Sigma_1 + \Sigma_2)$$

$$\Sigma_4 = 20u_0 + \frac{20.19}{1.2}\delta u_0 + \frac{20.19.18}{1.2.3}\delta^2 u_0 + \frac{20.19.18.17}{1.2.3.4}\delta^3 u_0 - (\Sigma_1 + \Sigma_2 + \Sigma_3)$$

$$\therefore \Sigma_1 = 5u_0 + 10\delta u_0 + 10\delta^2 u_0 \; + 5\delta^3 u_0$$
$$\Sigma_2 = 5u_0 + 35\delta u_0 + 110\delta^2 u_0 + 205\delta^3 u_0$$
$$\Sigma_3 = 5u_0 + 60\delta u_0 + 335\delta^2 u_0 + 1155\delta^3 u_0$$
$$\Sigma_4 = 5u_0 + 85\delta u_0 + 685\delta^2 u_0 + 3480\delta^3 u_0$$

If these quantities are differenced, the first column of differences will be

$$25\delta u_0 + 100\delta^2 u_0 + 200\delta^3 u_0 = \Delta\Sigma_1 \qquad (8)$$
$$25\delta u_0 + 225\delta^2 u_0 + 950\delta^3 u_0$$
$$25\delta u_0 + 350\delta^2 u_0 + 2325\delta^3 u_0$$

the second column

$$125\delta^2 u_0 + 750\delta^3 u_0 = \Delta^2\Sigma_1 \qquad (9)$$
$$125\delta^2 u_0 + 1375\delta^3 u_0$$

and the last difference

$$625\delta^3 u_0 = \Delta^3\Sigma_1$$

$$\therefore \delta^3 u_0 = \frac{1}{5^4}\Delta^3\Sigma_1, \text{ as stated in (7).}$$

Substituting this value of $\delta^3 u_0$ in (9) we have

$$\Delta^2\Sigma_1 = 125\delta^2 u_0 + \frac{750}{5^4}\Delta^3\Sigma_1$$

$$\therefore \delta^2 u_0 = \frac{1}{5^3}\Delta^2\Sigma_1 - \frac{6}{5^4}\Delta^3\Sigma_1 \qquad (6)$$

and substituting these values of $\delta^2 u_0$ and $\delta^3 u_0$ in (8) we have

$$\Delta\Sigma_1 = 25\delta u_0 + \frac{100}{5^3}\Delta^2\Sigma_1 - \frac{600}{5^4}\Delta^3\Sigma_1 + \frac{200}{5^4}\Delta^3\Sigma_1$$

$$= 25\delta u_0 + \frac{100}{5^3}\Delta^2\Sigma_1 - \frac{400}{5^4}\Delta^3\Sigma_1$$

$$\therefore\ \delta u_0 = \frac{1}{5^2}\Delta\Sigma_1 - \frac{4}{5^3}\Delta^2\Sigma_1 + \frac{16}{5^4}\Delta^3\Sigma_1 \tag{5}$$

Now it will be seen that the coefficients of $\Delta\Sigma_1$, $\Delta^2\Sigma_1$, &c. in (5) are the same as those of $\Delta^2 u_0$, $\Delta^3 u_0$, &c. in (2); and the same identity of coefficients occurs in (3) and (6) and in (4) and (7). As this identity could not be accidental I proceeded to try substitution of coefficients on a series involving a larger number of differences. The method of obtaining δu_0, &c. from the differences (Δu_0, $\Delta^2 u_0$, &c.) of every hth term of the original series is given by Mr. Neison in his Contributions to Vital Statistics, and by means of it I found the values of $\delta^2 u_0$, $\delta^3 u_0$, &c. in terms of $\Delta^2 u_0$, $\Delta^3 u_0$, &c., putting $h = 10$, and supposing the sixth difference constant; the value of δu_0 not being required. These are as follows:—

$$\delta^2 u_0 = \frac{1}{10^2}\Delta^2 u_0 - \frac{9}{10^3}\Delta^3 u_0 + \frac{77\cdot25}{10^4}\Delta^4 u_0 - \frac{669\cdot75}{10^5}\Delta^5 u_0 + \frac{5895\cdot225}{10^6}\Delta^6 u_0$$

$$\delta^3 u_0 = \qquad\quad \frac{1}{10^3}\Delta^3 u_0 - \frac{13\cdot5}{10^4}\Delta^4 u_0 + \frac{146\cdot25}{10^5}\Delta^5 u_0 - \frac{1480\cdot5}{10^6}\Delta^6 u_0$$

$$\delta^4 u_0 = \qquad\qquad\qquad \frac{1}{10^4}\Delta^4 u_0 - \frac{18}{10^5}\Delta^5 u_0 + \frac{235\cdot5}{10^6}\Delta^6 u_0$$

$$\delta^5 u_0 = \qquad\qquad\qquad\qquad\qquad \frac{1}{10^5}\Delta^5 u_0 - \frac{22\cdot5}{10^6}\Delta^6 u_0$$

$$\delta^6 u_0 = \qquad\qquad\qquad\qquad\qquad\qquad\qquad \frac{1}{10^6}\Delta^6 u_0$$

Then reducing the indices of $\delta^2 u_0$, $\delta^3 u_0$, &c., and of $\Delta^2 u_0$, $\Delta^3 u_0$, &c. by unity, and substituting Σ_1 for u_0, these being the changes necessary to convert the formulæ (2), (3) and (4) into (5), (6) and (7), I arrived at the following formulæ, which proved correct in the using.

$$\delta^1 u_0 = \cdot01\Delta\Sigma_1 - \cdot009\Delta^2\Sigma_1 + \cdot007725\Delta^3\Sigma_1 - \cdot0066975\Delta^4\Sigma_1 + \cdot005895225\Delta^5\Sigma_1$$
$$\delta^2 u_0 = \qquad \cdot001\Delta^2\Sigma_1 - \cdot00135\Delta^3\Sigma_1 + \cdot0014625\Delta^4\Sigma_1 - \cdot0014805\Delta^5\Sigma_1$$
$$\delta^3 u_0 = \qquad\qquad \cdot0001\Delta^3\Sigma_1 - \cdot00018\Delta^4\Sigma_1 + \cdot0002355\Delta^5\Sigma_1$$
$$\delta^4 u_0 = \qquad\qquad\qquad\qquad \cdot00001\Delta^4\Sigma_1 - \cdot0000225\Delta^5\Sigma_1$$
$$\delta^5 u_0 = \qquad\qquad\qquad\qquad\qquad\qquad \cdot000001\Delta^5\Sigma_1$$

In vol. xii., page 221, however, the u_0 and Σ_1 are omitted: δ stands there for δu_0 and Δ for $\Delta\Sigma_1$.

Yours obediently,

21, *Fleet Street*,　　　　　　　　　　　　G. W. BERRIDGE.
　　4th March, 1868.