continuous setting are classical theories viewed as continuous with the discrete metric, and we see that this notion lacks the machinery necessary to prove the reverse direction of the Baldwin-Lachlan characterization of uncountable categoricity.

In an effort to better understand minimality in continuous logic, we also introduce a continuous notion of dp-minimality, and provide a few equivalent characterizations. We use these to show that the theory of infinite dimensional Hilbert spaces is a natural example of a dp-minimal continuous theory.

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Paul K. Gorbow, ***Self-similarity in the Foundations***, University of Gothenburg, Sweden, 2018. Supervised by Ali Enayat (primary), Peter LeFanu Lumsdaine, and Zachiri McKenzie (secondary). MSC: 03C62, 03E30, 03E35, 03E55, 03H99, 03G30. Keywords: set theory, ZF, KP, NF, NFU, nonstandard model, self-embedding, topos, algebraic set theory, endofunctor.

Abstract

This thesis concerns embeddings and self-embeddings of foundational structures in both set theory and category theory. In the first part, rank-initial self-embeddings of countable nonstandard models of set theory are comprehensively studied. In the second part, the theory of topoi is reformulated so as to accommodate a special self-embedding. This results in the formulation of first-order theories corresponding by equiconsistency to intuitionistic and classical variants of Quine's New Foundations. These research tracks are connected in that the techniques of the first part can be used to construct an array of classical models of the theory formulated in the second part.

The first part of the work on models of set theory consists in establishing a refined version of Friedman's theorem on the existence of embeddings between countable nonstandard models of a fragment of ZF, and an analogue of a theorem of Gaifman to the effect that certain countable models of set theory can be elementarily end-extended to a model with many automorphisms whose sets of fixed points equal the original model. The second part of the work on set theory consists in combining these two results into a technical machinery, yielding several results about nonstandard models of set theory relating such notions as self-embeddings, their sets of fixed points, strong rank-cuts, and set theories of different strengths.

In particular, back-and-forth constructions are carried out to establish various generalizations and refinements of Friedman's theorem on the existence of rank-initial embeddings between countable nonstandard models of the fragment $\mathrm{KP}^{\mathcal{P}} + \Sigma_1^{\mathcal{P}}$-Separation of ZF; and Gaifman's technique of iterated ultrapowers is employed to show that any countable model of GBC + "the class of ordinals is weakly compact" can be elementarily rank-end-extended to models with well-behaved automorphisms whose sets of fixed points equal the original model. These theoretical developments are then utilized to prove various results relating self-embeddings, automorphisms, their sets of fixed points, standard systems, strong rank-cuts, and set theories of different strengths. Here is one example:

Theorem. *Suppose that* $\mathcal{M} \models \mathrm{KP}^{\mathcal{P}} + \Sigma_1^{\mathcal{P}}\text{-Separation} + \text{Choice}$ *is countable and nonstandard. The following are equivalent*:

(a) *There is a strong rank-cut of* $\mathcal{M}$ *that is isomorphic to* $\mathcal{M}$.
(b) $\mathcal{M}$ *expands to* $(\mathcal{M}, \mathcal{A}) \models \mathrm{GBC} +$ *"the class of ordinals is weakly compact"*.

The second part of the thesis consists in the formulation of a novel categorical set theory, $\mathrm{ML}_{\mathsf{Cat}}$, which is proved to be equiconsistent to New Foundations (NF), and which can be modulated to correspond to intuitionistic or classical NF, with or without atoms. NF is a set theory that rescues the intuition behind naive set theory, by imposing a so called stratification constraint on the formulae featuring in the comprehension schema.

The axioms of the categorical theory developed here express that its structures have an endofunctor, with certain coherence properties. By means of this endofunctor, an appropriate

axiom of power objects is formulated for the setting of NF. The most interesting direction of the equiconsistency result is established by interpreting the set theory in the category theory, through the machinery of categorical semantics, thus making essential use of the flexibility inherent in category theory. Moreover, $\mathrm{ML}_{\mathsf{Cat}}$ is connected to topos theory as follows:

THEOREM. *For any category* $\mathbf{C} \models \mathrm{ML}_{\mathsf{Cat}}$*, with endofunctor* $\mathbf{T}$*, the full subcategory on the fixed-points*[1] *of* $\mathbf{T}$ *is a topos.*

The relevance of this categorical work lies in that it provides a basis for studying the dynamics of NF within the realm of category theory. In particular, it opens up for constructions of categorical models of intuitionistic versions of NF, and for stratified approaches to type-theory. It may also be relevant for attempts to prove or simplify proofs of the consistency of classical NF.

[1] P. K. GORBOW, ***Self-similarity in the foundations***, Acta Philosophica Gothoburgensia, vol. 32, Ph.D. thesis, University of Gothenburg, 2018. Available at https://arxiv.org/abs/1806.11310.

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SAEIDEH BAHRAMI, ***Self-embeddings of Models of Peano Arithmetic***, Tarbiat Modares University, Iran, 2018. Supervised by Ali Enayat. MSC: 03C62, 03F30, 03H15. Keywords: Peano arithmetic, nonstandard model, self-embedding, fixed point, strong cut.

Abstract

This thesis investigates the behavior of *fixed points* of embeddings between countable nonstandard models of fragments $\mathrm{I}\Sigma_{n+1}$ of PA (Peano arithmetic), for $n \in \omega$. Our concentration is on studying the fixed points of embeddings of a countable nonstandard model $\mathcal{M}$ into a countable nonstandard model $\mathcal{N}$ whose images are proper initial segments of $\mathcal{N}$, namely *initial embeddings*. This area of research was initiated by Friedman [4] (who proved that every countable nonstandard model of PA is isomorphic to a proper initial segment of itself), and continued by a number of researchers, including Dimitracopoulos and Paris [3], Ressayre [6], and Tanaka [7]. Most of the results in the thesis appear in [1] and [2]. The following is a summary of the highlights of the thesis:

(1) For a given pair of countable nonstandard models $\mathcal{M}$ and $\mathcal{N}$ of $\mathrm{I}\Sigma_{n+1}$ which share a proper cut I that is closed under exponentiation, a sufficient condition for the existence of a Σ_n-elementary initial embedding j from $\mathcal{M}$ into $\mathcal{N}$ which fixes each element of I is that both $\mathcal{M}$ and $\mathcal{N}$ have the same I-standard system and $\mathrm{Th}_{\Sigma_{n+1}}(\mathcal{M}, i)_{i\in \mathrm{I}} \subseteq \mathrm{Th}_{\Sigma_{n+1}}(\mathcal{N}, i)_{i\in \mathrm{I}}$ (this is a generalization of a theorem by Hájek and Pudlák [5]). Moreover, if there exists such a Σ_n-elementary initial embedding, then there are continuum many of them with distinct images (this generalizes a theorem of Wilkie [8] that asserts that every countable nonstandard model of PA is isomorphic to continuum many initial segments of itself).

(2) For every countable nonstandard model $\mathcal{M}$ of $\mathrm{I}\Sigma_{n+1}$, a proper cut I of $\mathcal{M}$ is closed under exponentiation iff there exists a proper Σ_n-elementary (initial) self-embedding j of $\mathcal{M}$ such that I is the longest initial segment of fixed points of j. The $n = 0$ case of this result is also extended to models of the subsystem WKL_0 of second-order arithmetic, thereby leading to a generalization of Tanaka's self-embedding theorem [7].

[1] An object A of $\mathbf{C}$ is a *fixed-point* of the (axiomatized) endofunctor $\mathbf{T}$ of $\mathbf{C}$ if $\mathbf{T}A \cong A$ and this is witnessed by a certain natural isomorphism characterized by axioms of $\mathrm{ML}_{\mathsf{Cat}}$.