BOOK REVIEWS

DOWSON, H. R., *Spectral Theory of Linear Operators* (London Mathematical Society Monographs No. 12, Academic Press, 1978), xii + 422 pp., £20·00.

The structure of an arbitrary linear operator on a finite dimensional complex linear space is completely understood in the sense that the Jordan form provides a canonical description which determines the operator up to similarity. In contrast, the structure of a general bounded linear operator on an infinite dimensional Banach space remains almost a complete mystery. Progress has been made, however, in analysing certain special classes of operators, where some aspect of the finite dimensional situation has been retained, and it is the aim of the present monograph to describe some of these developments. The author is concerned for the most part with operators on Banach spaces, rather than in the Hilbert space theory where the geometry of the underlying space plays a more significant role. Also, he deals almost exclusively with single operators, rather than with algebraic systems of operators.

The main classes of operators discussed are compact, Riesz, prespectral, spectral and well-bounded operators. Of these, the first is classical, dating back to the pioneering work of Fredholm, Riesz and others during the early part of this century, whereas the remainder have all been introduced during the last twenty-five years or so. It would not be appropriate to embark here on any technical description of these operators. However, the common theme occurring throughout is the idea of the decomposition of the underlying Banach space into complementary subspaces corresponding to certain distinguished subsets of the spectrum of the operator under discussion. (The spectrum takes the place in infinite dimensions of the set of eigenvalues in finite dimensions.) For compact and Riesz operators, the distinguished subsets are points, for prespectral and spectral operators they are Borel sets in the complex plane, and for well-bounded operators they are intervals of the real axis. For each class of operators a version of the Jordan decomposition is obtained. There is also a brief chapter on hermitian operators on Banach spaces. As well as being of interest in their own right, these have proved useful in the study of prespectral and spectral operators.

Since there are many good accounts of the theory of compact operators, this section has been kept reasonably brief. There is little mention of the duality aspects of the theory, but Ringrose's work on superdiagonalisation and Hilden's proof of Lomonosov's celebrated invariant subspace theorem have been included. Much of the material discussed in the rest of the book has only appeared previously in research articles and, in writing a more unified account, the opportunity has been taken to include simplifications of earlier proofs. The book has been carefully written and is well organised, with useful "Notes and Comments" sections giving historical perspective and indicating more recent developments. Only a basic working knowledge of functional analysis is assumed and so it should be of great help to the beginning graduate student who wishes to learn about this branch of operator theory. It will also be an excellent reference and source of information for those already working in the area.

T. A. GILLESPIE

REID, C., *Courant in Göttingen and New York. The story of an improbable mathematician* (Springer-Verlag, New York, 1976), 314 pp.

This book is a sequel to the outstandingly successful book on Hilbert by the same author which was published in 1970. Originally intended to be a collection of reminiscences rather than a biography of Courant, who was Hilbert's assistant in Göttingen and who later followed Klein as the dominant figure in the mathematical school there, the present book was published against the advice of some of Courant's friends who, because of Courant's ambivalent nature, would have preferred no biography

https://doi.org/10.1017/S0013091500003898 Published online by Cambridge University Press

to appear until perhaps 50 years after his death. This attitude is easily understood on a perusal of the book, for the reader is left in no doubt whatever that, although Courant was a remarkable man who was frequently brilliant and on many occasions kind and considerate, he was often perverse and decidedly objectionable. For instance, while in Göttingen, he arranged at his own expense a ski-ing holiday for a student who was recovering from a nervous breakdown, and in America, where he went as a refugee in 1934 and where he became Director of the institute which bears his name, he created minor posts for young people in need and paid their salaries entirely out of his own pocket. On other occasions, however, Courant seemed to have had a very definite bias in favour of himself, and there were instances when he was adversely criticised by fellow mathematicians for the lack of credit which he gave to his associates. Again, his calculating nature and his utter subservience to people, endowed with wealth but with few other attractive attributes, did not endear him to his colleagues.

In gathering material for her book, the author must have travelled far and wide tracking down associates of Courant and making tape recordings of their reminiscences. As a result, a higher proportion than is usual of the text is in direct speech, but this strengthens the authority of the narrative. The book, which consists of 28 chapters each dealing with a short period of Courant's life, begins with a short account of his pre-Göttingen days; then, after dealing with his association with Hilbert and with both the building up of the mathematical school in Göttingen and its dissolution after the advent of the Nazis in 1933, the book concludes with an account, which takes up almost half of the entire text, of Courant's creation of the institute in New York and of his advocacy of applied mathematics. Courant's original mathematical work, through which as has been said Dirichlet's principle runs like a thread, and his books are mentioned at appropriate places.

The book is altogether fascinating and is valuable not only for its portrait of Courant but also for the picture which it gives of the mathematical community in Göttingen both in its hey-day and in its time of extreme adversity from 1933 onwards. In this connexion the text is enhanced by a collection placed at the end of more than 30 photographs of Klein, Hilbert, Courant, Weyl, Emmy Noether, Born, Franck and other prominent personalities.

D. MARTIN

SAWYER, W. W., *A First Look at Numerical Functional Analysis* (Oxford Applied Mathematics and Computing Science Series, 1978), £9·00 (boards) and £4·25 (paper).

This book is a very readable introduction to the concepts of functional analysis, with the problems of numerical analysis at the forefront. The topics covered are limits, continuity, convergence, vector space, contraction mappings, Minkowski spaces, linear operators, Fréchet differentiation, integration, Newton–Kantorovich method, polynomial operators and majorants, Hilbert space, functionals and compactness.

The author endeavours to deal with as much non-linear analysis as possible, and deserves much credit for this. In particular the material on the generalisation of the Newton–Raphson Method is excellent, marred only by an error on p. 99, where the use of an incorrect ball invalidates the solution of an example.

The approach used to justify some of the definitions and to analyse some of the theorems is geometric. This works marvellously well, for example, in analysing Hölder's inequality.

This book is successful in the same way as all of Professor Sawyer's texts—he has a wonderful ability to make the subject he writes about come alive. There are a few blemishes. Compactness is given less coverage than it warrants (it does not even appear in the index) and there is no mention of that most useful result for numerical functional analysis, the principle of uniform boundedness. On p. 50 a matrix iteration is seen to converge, but it is stated not to be a contraction, apparently only euclidean norm is to be considered here but that is not so in other sections. Also contraction mappings are only considered in normed vector space although distance functions are presented. On p. 175 it is overlooked that closure is required for compactness in finite-dimensional space. The text is not free from misprints—I noted about 15. There are a fair number of exercises, but no solutions.

Lest these criticisms detract from a very worthwhile text, I would like to underline that it is a welcome addition to the literature, and should prove useful to functional and numerical analysts alike.

D. W. ARTHUR