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CORRIGENDUM

Ship generated mini-tsunamis – CORRIGENDUM

John Grue

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Equation (4.2) in Grue (2017) employs the approximation $\partial\phi_F/\partial t \simeq -p/\rho$ (ϕ_F the potential at the surface, p the given surface pressure, ρ the density). This is a valid approximation in the supercritical case with $Fr^2 = U^2/gh \gg 1$ (U the speed of the moving pressure, g the acceleration due to gravity, h the water depth, Fr the depth Froude number). In the subcritical case with $Fr^2 = U^2/gh < 1$ the hydrostatic term in the dynamic boundary condition at the free surface gives a significant contribution, however. This results in a multiplicative factor of $-Fr^2/(1-Fr^2)$ of the asymptotic upstream wave elevations derived in § 4. The revised result is obtained from the linear free surface boundary condition: $\partial^2\phi/\partial t^2 + g\partial\phi/\partial y = -\partial(p/\rho)/\partial t$, at $y=0$, where ϕ is the velocity potential. In a frame of reference moving with the speed U of the surface pressure, along the x_1-direction, where $\partial/\partial t = -U\partial/\partial x_1$, the free surface boundary condition becomes $U^2\partial^2\phi_F/\partial x_1^2 + g\partial\phi/\partial y = (U/\rho)\partial p/\partial x_1$. Fourier transformation gives

$$-U^2k_1^2\hat{\phi}_F + g\widehat{\frac{\partial\phi}{\partial y}} = \mathrm{i}k_1 U\frac{\hat{p}}{\rho}, \tag{0.1}$$

where a hat denotes a Fourier transform and $\boldsymbol{k} = (k_1, k_2)$ is the wavenumber in Fourier space. The normal velocity $\partial\phi/\partial y$ is connected to the velocity potential at the free surface by the solution of the Laplace equation, expressed in terms of the Fourier transform, for a fluid layer of constant depth h by $\widehat{\partial\phi/\partial y} = k\tanh kh\,\hat{\phi}_F = (\omega^2/g)\hat{\phi}_F$, giving

$$(\omega^2 - U^2k_1^2)\hat{\phi}_F = \mathrm{i}k_1 U\frac{\hat{p}}{\rho}, \tag{0.2}$$

and yielding

$$-\mathrm{i}k_1 U\hat{\phi} = Fr^2\mathcal{C}\frac{\hat{p}}{\rho}, \quad \mathcal{C} = \frac{k_1^2/k^2}{1-(k_1^2/k^2)Fr^2}, \tag{0.3a,b}$$

where $k = |\boldsymbol{k}|$ and the spectral wave speed $c = \omega/k \to \sqrt{gh}$ for $k \to 0$. The constant $\mathcal{C}$ is positive in the subcritical case ($Fr^2 < 1$) and negative for $(k_1^2/k^2)Fr^2 > 1$. The product $Fr^2\mathcal{C} \to -1$ for $Fr^2 \gg 1$. The two-dimensional case gives the coefficient as $\mathcal{C}_0 = 1/(1-Fr^2)$, yielding $\partial\phi_F/\partial t = Fr^2\mathcal{C}_0\,p/\rho$.

In three dimensions, in the subcritical case, we obtain

$$-\mathrm{i}k_1 U\hat{\phi}_F = \frac{\hat{p}}{\rho}\left(Fr^2\frac{k_1^2}{k^2} + Fr^4\frac{k_1^4}{k^4} + Fr^6\frac{k_1^6}{k^6} + \cdots +\right). \tag{0.4}$$

Assuming that the pressure distribution is a delta function in the two horizontal directions with $p(x_1, x_2) = \rho g V_0\delta(x_1)\delta(x_2)$ where V_0 is the volume of the pressure

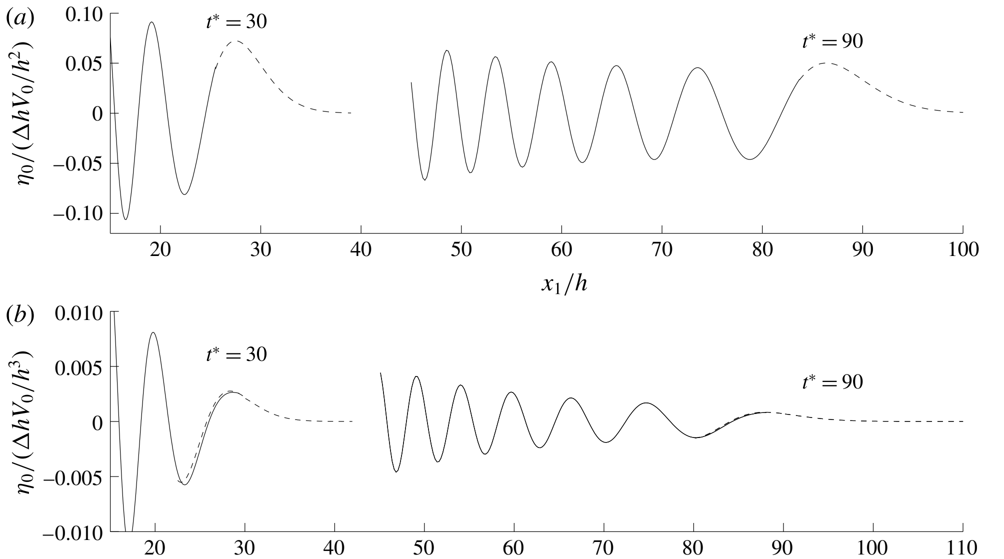

FIGURE 1. Asymptotic upstream waves. Delta function moving over a bottom step, $\Delta h > 0$, $Fr = 0.5$, $t^* = t\sqrt{g/h} = 30$, 90. Stationary phase, ——; wave front expressions, – – –. (*a*) Two dimensions. (*b*) Three dimensions.

distribution, its Fourier transform becomes $\hat{p} = \rho g V_0$. Evaluating the inverse Fourier transform of the first term on the right-hand side of (0.4) gives

$$\phi_F \simeq \frac{V_0 U}{(2\pi)^2 h} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\mathrm{i}k_1 \mathrm{e}^{\mathrm{i}(k_1 x_1 + k_2 x_2)}}{k_1^2 + k_2^2} \, \mathrm{d}k_1 \, \mathrm{d}k_2 = -\frac{V_0 U}{2\pi h} \frac{x_1}{x_1^2 + x_2^2}. \tag{0.5}$$

The contribution to the right-hand side of (4.1) becomes

$$\hat{h}_1 \simeq -\mathrm{i}k_1 \Delta h \int_{-\infty}^{\infty} \phi_F|_{x_1=-Ut} \, \mathrm{d}x_2 = \frac{\mathrm{i}k_1 \Delta h V_0 U}{2\pi h} \int_{-\infty}^{\infty} \frac{-Ut}{(Ut)^2 + x_2^2} \, \mathrm{d}x_2 = -\frac{\mathrm{i}k_1 \Delta h V_0 U}{2h} \frac{t}{|t|}. \tag{0.6}$$

The contribution to the Fourier-transformed wave elevation becomes

$$\hat{\eta}_0 = -\frac{\mathrm{i}k_1 \Delta h V_0 U}{2h} \int_{t_0}^{t} \cos \omega(s-t) \frac{s}{|s|} \, \mathrm{d}s = -\frac{\mathrm{i}k_1 \Delta h V_0 \, Fr^2}{U\omega/g} \sin \omega t. \tag{0.7}$$

The successive contributions from the expansion give

$$\hat{\eta}_0 = -\frac{\mathrm{i}k_1 \Delta h V_0 Fr^2 (1 + Fr^2 + Fr^4 \ldots)}{U\omega/g} \sin \omega t = -\frac{\mathrm{i}k_1 \Delta h V_0 Fr^2 \mathcal{C}_0}{U\omega/g} \sin \omega t, \tag{0.8}$$

which replaces (4.4) in § 4, where $\mathcal{C}_0 = 1 + Fr^2 + Fr^4 + \cdots = 1/(1 - Fr^2)$ $(Fr^2 < 1)$. The subsequent results for the elevations in § 4 are multiplied by the factor $-Fr^2\mathcal{C}_0$. The asymptotics show a leading wave of elevation, for a positive step with $\Delta h > 0$; see the corrected figure 1. This correction does not affect the results of the fully dispersive calculations.

REFERENCES

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