AN EXTENSION OF NEWTON'S EQUATION OF MOTION

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Classical mechanics failed to solve two problems in own defensive area, namely the motion of Mercury's perihelion, and the high-velcity motion of a charged particle. Today it is generally believed that the concepts of classical mechanics are completely invalid in a treatment of these problems. In this paper, however, we discuss these problems throughly with the concepts of classical mechanics — Euclidean space-time, point of mass and central force. Thereat we introduce a new concept "absolute mass variation", with which we extend the Newton's second law of motion. In following chapters we show that this extended equation explains the motion of Mercury's perihelion, and that it throws new light on the atomic physics. We also make a study of reconstruction of internal structure of mechanics. We discuss the possibility of the revival of the principal frame of Newton's mechanics.

1. BASIC EQUATIONS AND ANGULAR MOMENTUM At first we write "new" equations of motion of a mass-point m.

$$\dot{\bar{P}} \equiv \dot{m}\bar{v} + m\dot{\bar{v}} = \bar{F} \tag{1}$$

$$\dot{m}c^2 = s\bar{F}\cdot\bar{v} \quad , \tag{2}$$

where $s \equiv 3/2$, c=light velocity and

$$\bar{F} = \begin{cases} -GMm\bar{r}/r^3 & \text{(3a)} \\ \mp e^2\bar{r}/r^3 \quad . & \text{(3b)} \end{cases}$$

The introduction of $\dot{m}$-term prescribed by (2) is the fundamental point of our theory. It is evident that the law of conservation of momentum and angular momentum still holds independently of $\dot{m}$-term. Nevertheless areal velocity is no more conservative, but varies in proportion to mass as

$$\dot{m}\bar{h} + m\dot{\bar{h}} = 0 \quad , \quad \text{or} \quad \dot{m}/m = -\dot{h}/h \quad . \tag{4}$$

Generally speaking, the law of conservation exists where dynamical quanti-

R. L. Duncombe (ed.), Dynamics of the Solar System, 53–56.

https://doi.org/10.1017/S007418090001250X Published online by Cambridge University Press

ties are concerned, but corresponding kinematical quantities vary with mass. In view of this conclusion, the "curved space-time" in general relativity can be interpreted as the result of forcing the conservation theorem to the areal velocity.

2. MERCURY'S PERIHELION We apply the new equations of motion to one body problem. Consider motion of a mass-point m around the sun M. From (1), (2) and (3a), the equations of two components of polar coordinates are written as $\dot{m}v_r+ma_r=F_r$, $\dot{m}v_\phi+ma_\phi=F_\phi$. From these, we easily obtain (4) and the orbit equation

$$d^2/d\phi^2(1/r)+1/r=GM/h(r)^2 \simeq GM(1+3GM/c^2r)/h_0{}^2 \quad , \tag{5}$$

where $h(r)=h_0\exp(-3GM/2c^2r) \simeq h_0(1-3GM/2c^2r)$ and $h_0=(\infty)$. This is exactly the Einstein's orbit equation which explains the motion of Mercury's perihelion. In case of hydrogen atom, taking the Coulomb force, R.H.S. of (5) is $e^2/mh^2=e^2m/A^2 \simeq e^2m_0(1+3e^2/2m_0c^2r)/A^2$. This result differs in the perturbation term by numerical factor 3 from the Sommerfeld's (1916) which was deduced from the special relativity.

3. THE ANNUAL CHANGE OF EARTH'S MASS From (2) and (3a) we get a following expression of earth's mass variation

$$m(r)=m_0\exp(3GM/2c^2r) \quad . \tag{6}$$

The difference of earth's mass between perihelion and aphelion is evaluated as

$$\Delta m \simeq 3m_0\varepsilon v^2/c^2 \simeq 5\times10^{-10}m_0 \quad , \tag{7}$$

where ε is orbital eccentricity and v is orbital velocity. The measurement of the annual change of gravity caused by this Δm is expected to prove the existence of $\dot{m}$-effect. The annual variation of earth's rotational velocity is taken for another corroboration of our $\dot{m}$-effect. The moment of inertia I, angular velocity ω and length of one day T also change in proportion to the change of mass. Relation among these quantities are given by $\dot{m}/m=\dot{I}/I=-\dot{\omega}/\omega=\dot{T}/T$. Amplitude of annual variation of length of one day ΔT is evaluated as

$$\Delta T=(\Delta m/m)\times(\text{half a year}) \simeq 8\times10^{-3} \quad (\text{sec}) \quad . \tag{8}$$

The observational value is reported to be 20∿25 ms (Rochester, 1973).

4. INERTIAL SYSTEM For simplicity we consider two mass-points m_1, m_2 whose masses equal to each other. If there is not external force, then the total momentum $(m_1+m_2)\bar{V}_0$ is constant. But in this case, the velocity of center of gravity $\bar{V}_0$ is not always constant, for the total mass m_1+m_2 may vary owing to internal forces. This conclusion can be extended to the general case. Therefore it is concluded that a kinematical definition of inertial system is impossible.

https://doi.org/10.1017/S007418090001250X Published online by Cambridge University Press

5. EXISTENCE OF CRITICAL VELOCITY v_c We consider the direct collision of two mass-points. By eliminating $\bar{F}\cdot\bar{v}$ from $\dot{\bar{P}}\cdot\bar{v}=\bar{F}\cdot\bar{v}$ and (2), and noticing that $\bar{v}$ is parallel to $\dot{\bar{v}}$, we obtain an integral

$$m(v)=m_0/\sqrt{(1-sv^2/c^2)} \quad . \tag{9}$$

Since mass cannot be infinite, velocity of the mass-point cannot exceed the upper bound $v_c=c/\sqrt{s}$. Then it is concluded that the relative velocity of two mass-points cannot exceed $2v_c$.

6. INTEGRALS From (2) and (3b) we have

$$m(r)=m_0+3e^2/2c^2r \quad , \tag{10}$$

where $m_0=m(\infty)$. In case of the Coulomb force, by taking scalar product of $\bar{P}$ and the expression (1), and using (10) we obtain the integral

$$D_1=m^2v^2-(m+m_0)e^2/r \quad . \tag{11}$$

Similarly from $\bar{P}\cdot\dot{\bar{P}}=\bar{F}\cdot\bar{P}$, (2) and (3a) or (3b), we have another integral

$$D_2=m^2v^2-m^2c^2/\sqrt{s} \quad . \tag{12}$$

7. SUNDMAN'S RESULT Let us apply our $\dot{m}$-effect to hydrogen atom. From (10) we get the increment of an electron mass at Bohr radius a_0 as $\Delta m \equiv m(a_0)-m_0=(3/2)m_0\alpha^2$, where α is Sommerfeld's fine structure constant. In case of free fall, D_1 in (11) is zero. Therefore, considering the process of free fall at which m is nearly equal to m_0, we get $r(dr/dt)^2 =2e^2/m_0$ from (11). By integrating this equation we finally have

$$r(t)=(9e^2/2m_0)^{1/3}(t-t_c)^{2/3} \quad . \tag{13}$$

It is surprising that this Sundman's result holds not at $r\simeq 0$, but at $r\simeq a_0$.

8. ELECTRON RADIUS r_0 We consider the direct collision of an electron and a positron with the coordinates of center of gravity. From (9) and (10), replacing r by 4r, we obtain an integral

$$4mrv^2=e^2(1+m_0/m) \quad . \tag{14}$$

In this integral, let us notice the moment when r goes to zero. At this moment v goes to critical velocity $c/\sqrt{s}$ and m tends to infinity, but R.H.S. stays at the finite value e^2, so L.H.S. also must be finite. On the other hand we know the experimental fact that two photons radiate after pair annihilation of an electron and a positron. On the view point of the conservation of total energy we may assume that at the moment of the pair annihilation m(t) becomes equal to m_0. From this assumption and from our discussion about (14), we can define the electron radius r_0 uniquely:

$$r_0=3e^2/8m_0c^2 \quad . \tag{15}$$

Velocity and mass of electron at r_0 are given as $m(r_0)=2m_0$, $v(r_0)=c/\sqrt{2}$.

9. A DYNAMICAL INTERPRETATION OF MICROSCOPIC MAGNETIC FIELD Our non-relativistic modification of Newton's mechanics naturally suggest the critique of electromagnetic field theory. Depending on the Bohr's semi-classical model of hydrogen atom we show that the magnetic field has its origin in our $\dot{m}$-effect. From (1), (2) and (3b) we get the equation of motion of an electron

$$m d\bar{v}/dt=\bar{F}-s(\bar{F}\cdot\bar{v})\bar{v}/c^2 \quad . \tag{16}$$

On the other hand, that of special relativity is

$$m d\bar{v}/dt=\bar{f}-(\bar{f}\cdot\bar{v})\bar{v}/c^2 \quad , \tag{17}$$

where $m=m_0/\sqrt{(1-v^2/c^2)}$ and $\bar{f}$ is the Lorentz's force. Both equations consist of corresponding three terms, but substances of each term are different. Let us compare our equation (16) to relativistic equation (17) in v^2/c^2 approximation. At first, it is noticed that the absolute mass variation (10) can be rewritten as $m=m_0/(1-v^2/c^2)^{3/2}+O(v^4/c^4)$, where $m_0=m(r=\infty)$ has the same physical meaning as proper mass in special relativity. This expression corresponds to longitudinal mass in electrodynamics. We artificially devide the increment of mass Δm given by (10) into two parts $\Delta m/3$ and $2\Delta m/3$. Then $m_0+\Delta m/3=m_0/\sqrt{(1-v^2/c^2)}+O(v^4/c^4)$. This part is substantially equal to the relativistic mass in (17). To analyze the second part $2\Delta m/3$, we use a kinematical relation of relative motion $d\bar{v}/dt=\bar{\omega}\times\bar{v}$, where $\bar{\omega}=\bar{r}\times\bar{v}/r^2$ is the angular velocity of the electron. Then $(2/3)\Delta m d\bar{v}/dt =e^2\bar{\omega}\times\bar{v}/c^2r=(e/c)\bar{v}\times\bar{H}$, where $\bar{H}(\bar{r})=-(e/c)\bar{r}\times\bar{v}/r^3$. Usually, this $\bar{H}$ is regarded as the microscopic magnetic field at the position of the electron. Consequently, we can express our equation (16) as

$$m_0/\sqrt{(1-v^2/c^2)}\, d\bar{v}/dt=-e(\bar{E}+(1/c)\bar{v}\times\bar{H})-s(\bar{F}\cdot\bar{v})\bar{v}/c^2 \quad . \tag{18}$$

Thus we traced the dynamical origin of the magnetic field. The difference between the results of two theories, namely factor $(\bar{F}\cdot\bar{v})\bar{v}/2c^2 \sim O(v^2/c^2)$ gives us the hint for the experimental veritification of our $\dot{m}$-effect.

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