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THE PRIZE ESSAY

"On the Generalized Simple System for Automatic Stabilisation of a Helicopter in Hovering Flight"

by

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Mr Willmer received the Alan Marsh Award for 1956, enabling him to undertake a short helicopter piloting course at Air Service Training Ltd

SUMMARY

This paper deals with the instability of a hovering helicopter with controls fixed The generalized simple stabilization system, which is composed of rods, springs, dampers and masses and uses the rotor shaft to generate gyroscopic forces, has been analysed Special cases of a Second Order System have been considered and show that good stability can be achieved, thus overcoming the limitations of First Order Systems

1 *Introduction*

One of the helicopter's greatest assets is its ability to hover However, in this condition it is dynamically unstable with the controls fixed In other words, a disturbance from the equilibrium position grows with time This means that the aircraft controls must be operated continuously with resulting pilot fatigue and increased possibility of accident

The alleviation of this instability has been investigated over the past few years Searches have been made for an automatic control device which could be fitted to the helicopter such that, with the controls fixed, a disturbance would decay and the aircraft return to its equilibrium position Of such devices there are those that use as the basis of gyroscopic couples the rotor shaft in contrast to other methods where a separate gyroscope has to be installed At the present time there exist two types which belong to the first category , they are the Bell Stabilizing Bar and the Hiller Servo Blade control Although they do improve the stability they are not completely satisfactory At best they give neutral stability In this paper the principle of their mechanism has been generalized and the possibilities of a more satisfactory device have been investigated

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Attention has been focussed mainly on the Sikorsky R-4B since for this configuration the results can be compared with past investigations We have considered too only the longitudinal motion for it is known that if the criterion for longitudinal stability is satisfied then that for the lateral motion follows

The results are given in terms of the automatic control component in phase with the attitude, that in phase with the rate of change of attitude and their ratio For convenience the dimensionless term $\theta_q\Omega$ is used From these results the stability characteristics are deduced

2 *Stability in Hovering Flight*

Sissingh in reference 1 has shown that by considering a helicopter of arbitrary design and allowing it to have two degrees of freedom, *viz* , attitude and horizontal linear velocity, the equation of longitudinal motion lead to a frequency equation of the form

$$A_3\nu^3 + A_2\nu^2 + A_1\nu + A_0 = 0 \qquad (2\ 1)$$

where the A's are known for a particular aircraft configuration The disturbances in the degrees of freedom are assumed to be of exponential form and the coupling between lateral and longitudinal motion has been neglected Given also are stability charts so that the times to either double or halve the amplitude of the oscillations together with the period can be determined for given values of the constants A_i in equation (2 1)

Attention is focussed mainly on the Sikorsky R-4B and for this type the value of A_1/A_3 is nearly zero From the stability charts it can be seen that the helicopter is unstable and that if A_1/A_3 could be increased stability would be improved

If we introduce a hypothetical autopilot which will impose upon the mean blade setting θ_0 a cyclic pitch $-(\theta_a\alpha + \theta_q q)\sin\psi$ the constants of the frequency equation A_1 and A_2 now become functions of the arbitrary parameters θ_a and θ_q respectively Physically this procedure corresponds to a cyclic pitch variation which is dependent on the attitude and the rate of change of attitude with time

Reference 1 shows that suitable stability can be obtained for $\theta_a = 0\ 12$ and $_q\Omega = 3\ 25$ This converts the unstable helicopter into one where the disturbance is halved in 3 secs with period 11 secs

It is the purpose of the generalized simple stabilization system to use the rotor shaft as the basis of gyroscopic forces in order to obtain the required values of θ_a and $\theta_q\Omega$

3 *The Generalized ' Simple ' Stabilization System*

Unlike fixed wing aircraft the helicopter does not have to be installed with a special gyroscope in order to obtain gyroscopic couples, for it can utilize its own rotor Let us consider therefore a system which is fitted to and rotating with the rotorshaft and which can transmit the required cyclic pitch variation to the rotor blades to give good stability following a disturbance from equilibrium In dealing with ' simple ' systems we are considering ones where masses, springs, rods and dampers are used as the fundamental units of the control mechanism The simple system is attached to the shaft and a rod leads from the control device to a mechanism at the hub which varies the pitch of the blades Following a disturbance of the rotor shaft therefore a change in angle $\Delta\theta$ will be effected The system is represented diagrammatically in Fig 1

When the shaft is in its initial position (t = 0) an arbitrary point on this connecting rod will be at ξ_0 (say) If following a disturbance the mechanism were allowed to follow the rotor without hinderance the arbitrary point would have moved to ξ^1_0 Owing to the gyroscopic forces, the control device does not follow the rotor immediately Let us assume the position of the point be at ξ_t after time t secs

The response therefore to the disturbance is

$$\delta = (\xi_t - \xi^1_0) \tag{3 1}$$

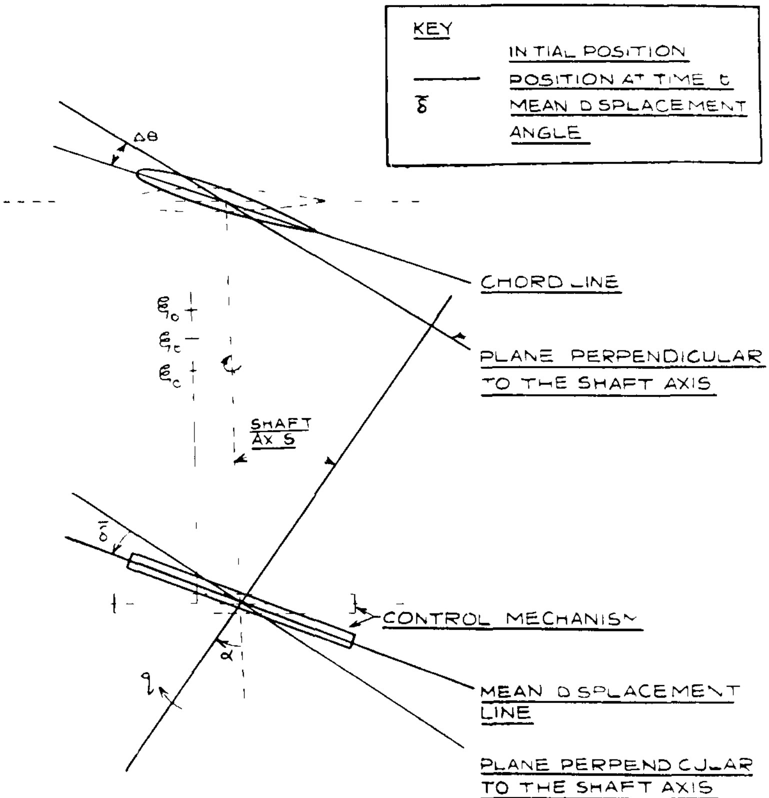
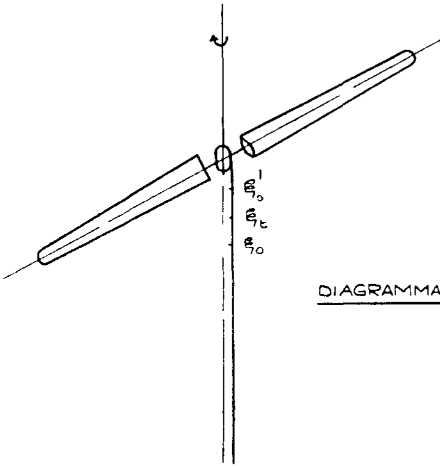


FIG 1 DIAGRAMMATIC REPRESENTATION OF THE GENERALISED SIMPLE SYSTEM

We shall suppose that the mechanism governing the actual pitch variation acts in a manner such that

$$\Delta\theta = G\delta \qquad (3\ 2)$$

where G is a constant and that for a two-bladed rotor the pitch variations are equal but of opposite sign (see Fig 2)



DIAGRAMMATIC REPRESENTATION OF THE HUB MECHANISM

FIG 2

We shall consider the generalized ' simple ' system to be composed of n rods hinged at the axis of rotation and connected to some form of mass The mass may or may not be designed to have any special aerodynamic characteristics The rods are damped at the hinge, between which, and the rotor shaft, springs are attached The rods are also mutually interlinked by springs and dampers and are connected to the main control rod which feeds the response into the mechanism governing the blade pitch angle

The automatic control response is defined by

$$\delta = \sum_{i=1}^{n} n_i \delta_i \qquad (3\ 3)$$

where n_i are constants and the δ_i's are the angular displacements of the respective rods

By considering the equation of motion about the hinge of the i th rod taking into account aerodynamic, mass, hinge damping and interconnecting moments and neglecting feedback due to the other rods through and due to the attachment to the main control, as well as the inertia of the blades about their longitudinal axis we obtain n equations of the form

$$\begin{aligned} \delta_i + 2\Omega\sum_{j=1}^{n} A_{ij}\delta_j + \Omega^2\sum_{j=1}^{n} B_{ij}\delta_j + 2\Omega\ q \sin(\psi + \psi_i) \\ -q\cos(\psi + \psi_i) - 2qK_i\Omega\cos(\psi + \psi_i) = 0 \end{aligned} \qquad (3\ 4)$$

where $i = 1, 2, \quad n$

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The derivation of these equations is given in Appendix I It is a simple extension to prove that the form of the equations is unaltered if we consider additional rods which instead of being attached to the shaft are hinged at points along the present rods

It is assumed too that the moments due to the linking of the rods by springs and dampers act in the same plane as the other moments If therefore two rods, i and j, are at a large angle the interlinking moment would act in a plane nearly orthogonal to the planes of the other moments This

FIG 3

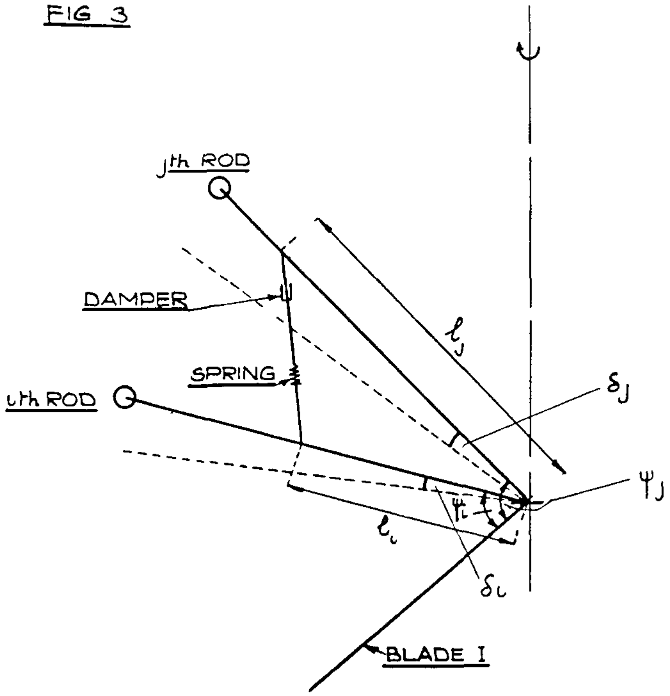


ILLUSTRATION OF THE CONNECTION BETWEEN THE ith AND jth RODS

has no advantage from the response point of view and it may provide structural worries If therefore $|\psi_i - \psi_j|$ is large the appropriate coefficients in equation (3 4) will be small (see Fig 3)

Equations (3 2-4) define the response of the control system to a disturbance in pitch of the helicopter in the hovering condition In general $\Delta\theta$ will be of the form

$$\Delta\theta = \theta_s \sin\psi + \theta_c \cos\psi \qquad (3\ 5)$$

with θ_s, θ_c non-zero This corresponds to displacements in the longitudinal and lateral control The lateral component θ_c will be neglected, i e, we shall neglect coupling between the two motions This lateral component can in fact be used to compensate for the already existing coupling

The component in the longitudinal plane θ is itself composed of two parts Firstly a component in phase with the change of attitude and secondly a part in phase with the rate of change of attitude with time

Adopting the frequency response technique we consider the response to a sinusoidal variation in a

$$a = a_o \sin \nu t \tag{3 6}$$

We can then obtain on solving equations 3 2—6

$$\theta_a = \theta_a \left(\nu, A_{1j}, B_{1j}, K_1, \psi_1, n_1, G, \Omega\right) \tag{3 7}$$

and

$$\theta_q = \theta_q \left(\nu, A_{1j}, B_{1j}, K_1, \psi_1, n_1, G, \Omega\right) \tag{3 8}$$

The method is itself a first approximation We should strictly consider a decreasing or increasing oscillation for a but the above assumption (3 6) simplifies the mathematics involved and investigations have shown (Ref 1) that the introduction of a varying amplitude has only a small effect on θ_a and θ_q

Equations 3 7—8 show that θ_a, θ_q are functions of the parameters of the system By their appropriate choice we can obtain the desired values of θ_a, θ_q to give the type of stability required In this paper we are looking at the problem from the theoretical viewpoint and it will be shown later that the desired values of θ_a, θ_q can be achieved In practice, however, the constants of the system will not be independent but will functionally be related by either engineering limitations or factors intrinsic to a proposed design Here we are content to show that good stability is possible using simple systems as defined

No attempt has been made to solve the equation 3 2—6 generally but concentration has been fixed on particular values of n, i e, for systems with given degrees of freedom

Let us first consider the simplest case

4 *First Order Systems* ($n = 1$)

For a system with one degree of freedom the equation of motion becomes

$$\begin{aligned} \delta + 2A_{11}\Omega\delta + \Omega^2 B_{11}\delta + 2\Omega q \sin(\psi + \psi_1) - q \cos(\psi + \psi_1) \\ -2K_1 q\Omega \cos(\psi + \psi_1) = 0 \end{aligned} \tag{4 1}$$

and the change in the pitch angle of the blade is given by

$$\Delta\theta = G\delta = -\left(\theta_a a + \theta_q q\right) \sin \psi \tag{4 2}$$

The systems in current use can be divided into two classes Firstly there are those that incorporate only mechanical damping at the hinge and little aerodynamic damping and secondly there are those where the aerodynamic moment forms the major contribution

The first case is the well-known Bell Stabilizing Bar and in effect is governed by the equation

$$\delta + 2K\delta\Omega + \Omega^2\delta + 2\Omega q \cos\psi + q \sin\psi = 0 \tag{4 3}$$

$$i\,e \quad K_1 = 0\,,\ B_{11} = 1\,,\ A_{11} = K\,,\ \psi_1 = \pi/2$$

The second class is typified by the Hiller Servo Blade Control and the equation of motion here becomes

$$\delta + 2K\delta\Omega + \Omega^2\delta + 2\Omega q \cos\psi + q \sin\psi + 2Kq\Omega \sin\psi = 0 \tag{4 4}$$

$$i\,e \quad A_{11} = K_1 = K\,,\ B_{11} = 1\,,\ \psi_1 = \pi/2$$

In each case it has been shown (Refs 1 and 2) that the values of θ_a, $\theta_q\Omega$ are given approximately by

$$\theta_a = \bar{v}^2 / (K^2 + \bar{v}^2) \tag{4 5}$$

$$\theta_q\Omega = K / (K^2 + \bar{v}^2) \tag{4 6}$$

Figs 4 and 5 show the variation of θ_a and $\theta_q\Omega$ with the damping constant K for a frequency ratio of $\bar{v} = 0\ 01$ This value is of the same order of magnitude as that experienced by a helicopter when slightly disturbed from the equilibrium position It can be seen that θ_a decreases rapidly with increasing K and that $\theta_q\Omega$ reaches a maximum and then decays

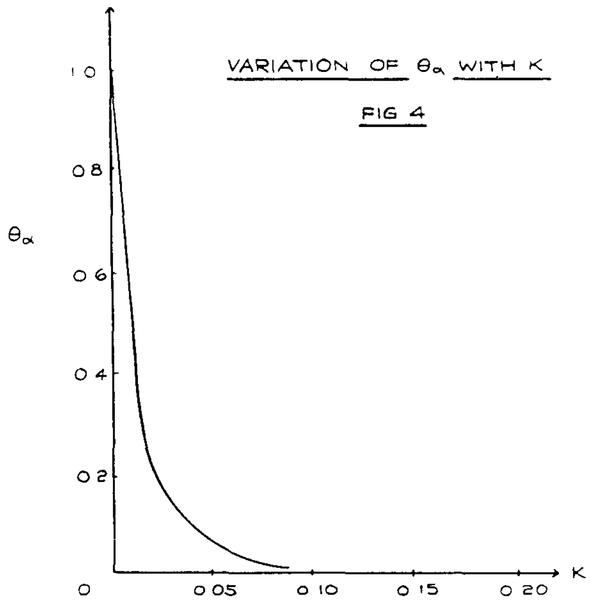
From the stability charts of Ref 1 we see that A_1/A_3 and hence θ_a must be greater than zero Therefore since the decrease of θ_a with K is fast we must have a small value of K This leads to a large $\theta_q\Omega$ The minimum practical value for K is of the order 0 03

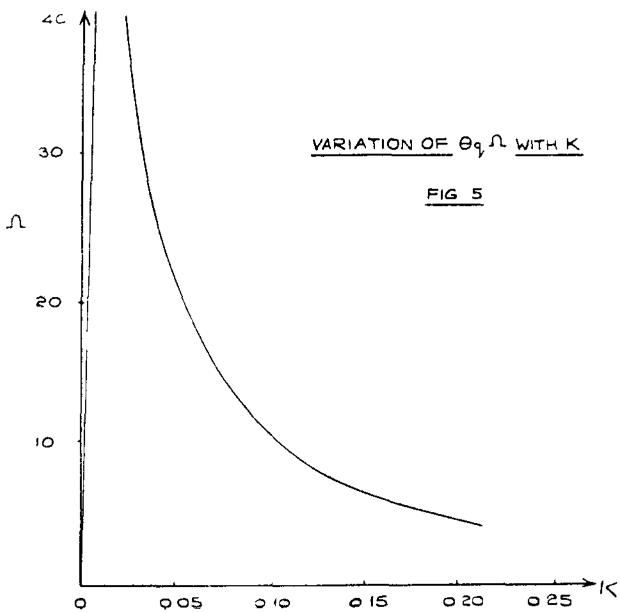
In the first of the two cases the excitement due to the gyroscopic forces associated with the mass of the bar has a frequency which is practically equal to its natural frequency Thus the motion must contain damping

In the second case the damping is provided by the air forces The specific damping of a typical rotor blade considering only air forces is approximately $K = \gamma_o/16$ The damping is still too large even if we use very heavy blades, *i e*, for small values of γ_o In order therefore to obtain the necessary small amount of damping the servo-blade, located at the outer part of the radius, is relatively short, *i e*, of small aspect ratio

For the value of $K = 0\ 03$ we see that $\theta_a = 0\ 10$ and $\theta_q\Omega = 30$ Unfortunately these values only serve to give neutral stability This can be seen from the stability charts of Ref 1 where for a given value of A_1/A_3 and A_0/A_3 an increase in A_2/A_3 leads to an increase in the time for a disturbance oscillation to decay to half its initial amplitude

The large value of $\theta_q\Omega$ has also a detrimental effect upon the control sensitivity In his lecture to the Helicopter Association of Great Britain in 1948 (Ref 3) Sissingh gives a plot of control effectiveness against θ_a for various values of $\theta_q\Omega$ It is shown that control displacements in phase with the rate of change of attitude θ_q play the major part in determining the effectiveness of the pilot's controls An increase in θ_q causes a decrease in the effectiveness Although the definition of control sensitivity used, *viz*, the ratio of the amplitude of the forced oscillation of the helicopter to





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the amplitude of the manual control displacement when the pilot applies a manual periodic control of period 4 secs, is not a completely satisfactory criterion for judging the response of an automatically stabilized helicopter, it does show in a simple way that loss in control sensitivity is mainly caused by the component θ_q becoming too large

The limitations of the system in current use can be summarized thus

i θ_a is too small

ii $\theta_q\Omega$ is too large

For these systems the ratio $\frac{\theta_q\Omega}{\theta_a}$ is of the order of 300, good stability can be obtained if this ratio is about 30

These limitations are caused through the minimum practical value of K being too great, so let us consider the properties of a system with an additional degree of freedom

5 *Second Order Systems* (n = 2)

The equations of motion for a second order system are

$$\delta_1 + 2\Omega A_{11}\delta_1 + 2\Omega A_{12}\delta_2 + \Omega^2 B_{11}\delta_1 + \Omega^2 B_{12}\delta_2$$
$$+ 2\Omega\, q \sin(\psi + \psi_1) - q\cos(\psi + \psi_1) - 2qK_1\Omega\cos(\psi_1 + \psi) = 0$$

and (5 1)

$$\delta_2 + 2\Omega A_{21}\delta_1 + 2\Omega A_{22}\delta_2 + \Omega^2 B_{21}\delta_1 + \Omega^2 B_{22}\delta_2$$
$$+ 2\Omega\, q \sin(\psi + \psi_2) - q\cos(\psi + \psi_2) - 2qK_2\Omega\cos(\psi + \psi_2) = 0$$

and the variations of the blade pitch angle are given by

$$\Delta\theta = G(\delta_1 + \bar{n}\delta_2) - G(\theta_{1s} + \bar{n}\theta_{2s})\sin\psi \qquad (5\ 2)$$

where $\bar{n} = n_2/n_1$

From these equations we obtain

$$\theta_a/G = \frac{L\bar{v}^2}{M + N\bar{v}^2} + 0(\bar{v}^4) \qquad (5\ 3)$$

and

$$\theta_q\Omega/G = \frac{P + Q\bar{v}^2}{M + N\bar{v}^2} + 0(\bar{v}^4) \qquad (5\ 4)$$

where the constants P, M, Q and N are functions of the parameters of the system defined by (5 1) They are to be chosen such that terms in powers of $\bar{v}$ greater than the third can be neglected The detailed account of the derivation of equations 5 3 and 4 is given in Appendix II

From these equations it is seen that in order to reduce the magnitude of $\theta_q\Omega$ in comparison with θ_a we have to make P small since it is generally impossible to make L sufficiently large We shall put P = 0 and consider

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the position where we have only mechanical damping at the hinge with no interconnecting moments but with varying azimuth angles Afterwards we shall introduce springs and aerodynamic damping and consider their effect on the results

Equations (5 1) become for a system with only mechanical damping

$$\ddot{\delta}_1 + 2\Omega A_{11}\dot{\delta}_1 + \Omega^2\delta_1 + 2\Omega \dot{q} \sin(\psi + \psi_1) - \dot{q}\cos(\psi + \psi_1) = 0$$

and (5 5)

$$\ddot{\delta}_2 + 2\Omega A_{22}\dot{\delta}_2 + \Omega^2\delta_2 + 2\Omega \dot{q} \sin(\psi + \psi_2) - \dot{q}\cos(\psi + \psi_2) = 0$$

In Appendix III it is shown that

$$\theta_a/G\bar{\nu}^2 = \frac{(A_{22} - A_{11})\sin\psi_1 \sin\psi_2 + \frac{1}{2}A_{11}A_{22}\sin(\psi_1 - \psi_2)}{A^2_{11}A_{22}\sin\psi_2} \tag{5 6}$$

and

$$\theta_q\Omega/G\bar{\nu}^2 = \frac{A_{22}\sin\psi_2\cos\psi_1 - A_{11}\sin\psi_1\cos\psi_2 + \left(\frac{A_{11}}{A_{22}} - \frac{A_{22}}{A_{11}}\right)\sin\psi_1\sin\psi_2}{A^2_{11}\,A_{22}\,\sin\,\psi_2} \tag{5 7}$$

where we also neglect $N\bar{\nu}^2$ when compared with M and the value of $\bar{n}$ required to give $P = 0$ is

$$\bar{n} = -\frac{A_{22}\,\sin\,\psi_1}{A_{11}\,\sin\,\psi_2} \tag{5 8}$$

The constant G in these equations is a gearing ratio and increases the value of θ_a and $\theta_q\Omega$ whilst leaving the ratio $\theta_q\Omega/\theta_a$ constant It is considered to be positive

The neglected terms in the series expansions used to determine θ_a and $\theta_q\Omega$ have no significant effect This is because of the low value of $\bar{\nu}$ which is approximately 0 01 for a typical helicopter

From the form of equations (5 6) and (5 7) we see that θ_a and $\theta_q\Omega$ are not necessarily positive It can be seen from the stability charts of Ref 1 that for the values of the coefficients of the stability equation (2 1) used, we must have both θ_a and $\theta_q\Omega$ positive in order to improve the dynamic stability

Let us therefore consider the boundaries where θ_a and $\theta_q\Omega$ are zero in both the $(\psi_1 - \psi_2)$ plane and the $(A_{11} - A_{22})$ plane The areas in these planes where θ_a and $\theta_q\Omega$ are both greater than zero will be called ‘ available regions ’

Figs 6 and 7 show the boundaries and available regions for the range $-\pi/2 < \psi_1, \psi_2 < \pi/2$ We have plotted for convenience tan ψ_1 and tan ψ_2 It can be seen that for $A_{22} > A_{11}$ there are regions for $\psi_1, \psi_2 < 0$, $\psi_1, \psi_2 > 0$ and $\psi_1 > 0$, $\psi_2 < 0$ For $A_{22} < A_{11}$ the only region occurs when $\psi_1 < 0$ and $\psi_2 > 0$

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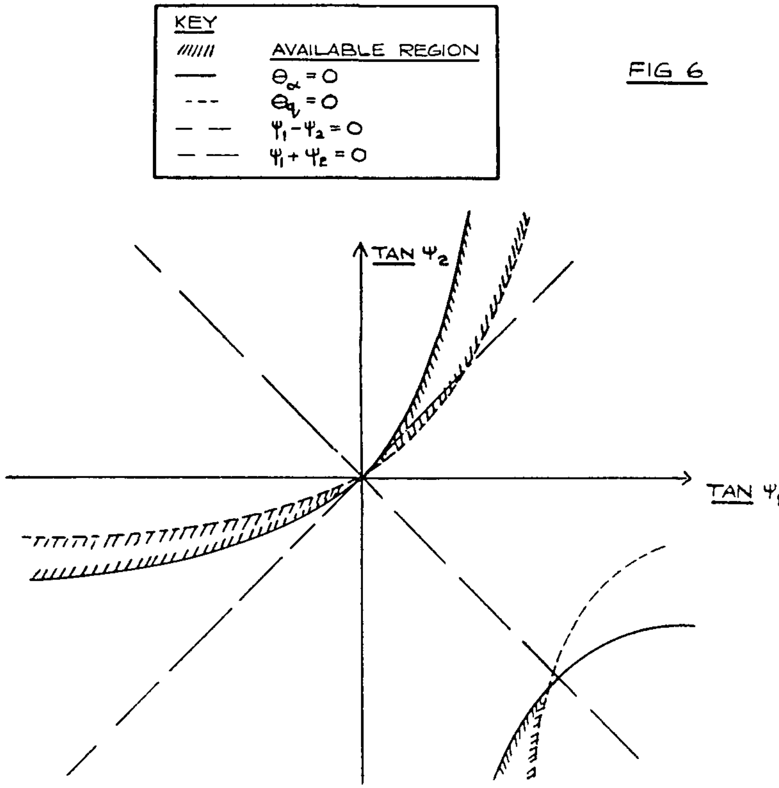
FIG 6



AVAILABLE REGIONS FOR $-\frac{\pi}{2} < \psi_1, \psi_2 < \frac{\pi}{2}$ AND $m > 1$

In Figs 8 and 9 we have moved the origin of the azimuth angle through a right angle by substituting

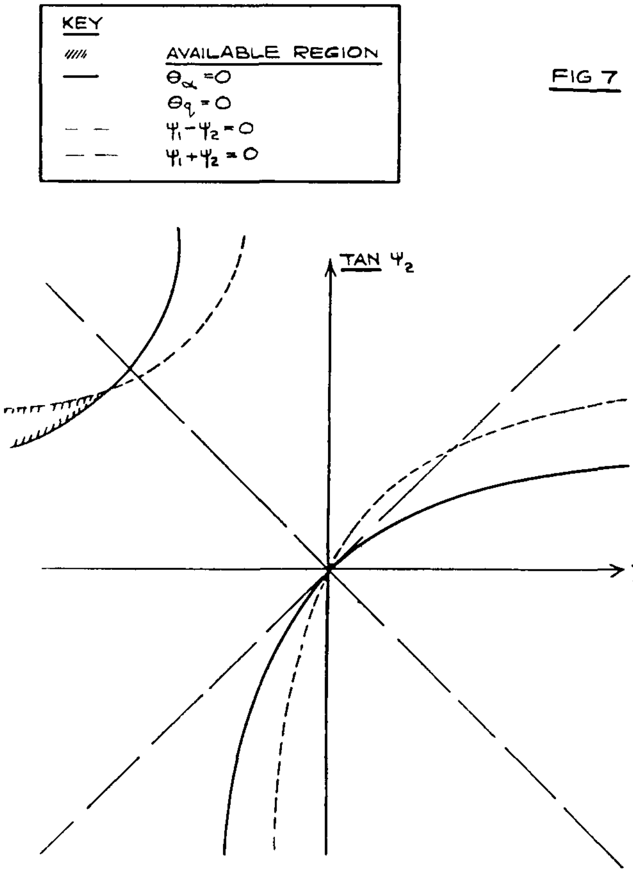
$$\psi_1 = \phi_1 + {}^{\pi}/_{2} \qquad (5\ 9)$$

$$\text{and } \psi_2 = \phi_2 + {}^{\pi}/_{2} \qquad (5\ 10)$$

From these diagrams we see that for $A_{22} > A_{11}$, regions occur for $\psi_1 < {}^{\pi}/_{2}$, $\psi_2 < {}^{\pi}/_{2}$ and $\psi_1 < {}^{\pi}/_{2}$, $\psi_2 > {}^{\pi}/_{2}$ For $A_{22} < A_{11}$ we have regions for $\psi_1 > {}^{\pi}/_{2}$, $\psi_2 > {}^{\pi}/_{2}$ and $\psi_1 > {}^{\pi}/_{2}$, $\psi_2 < {}^{\pi}/_{2}$

The diagrams are only sketches of the shape of the boundaries which are expected The position and gradients are functions of both A_{11} and A_{22} The theory behind the curves is given in Appendix IV If we now plot the boundaries in the damping plane for given values of ψ_1 and ψ_2 we have only to consider positive values of A_{11} and A_{22} Figs (10—16) show the available regions when A_{22}/A_{11} is plotted against A_{11} Fig 10 is for the case where $-\pi/2 < \psi_1 < \psi_2 < 0$, Fig 11 shows the regions when $-\pi/2 < \psi_1 < 0 < \psi_2 < \pi/2$ and $\psi_1 + \psi_2 < 0$ The region for both $-\pi/2 < \psi_2 < 0 < \psi_1 < \pi/2$, $\psi_1 + \psi_2 < 0$ and $0 < \psi_1 < \pi/2 < \psi_2 < \pi$, $\psi_1 + \psi_2 < \pi$ is shown in Fig 12 In Fig 13 we show the region for $0 < \psi_2 < \psi_1 < \pi/2$ and in Fig 14 the region for $0 < \psi_1 < \psi_2 < \pi/2$ When $0 < \psi_2 < \pi/2 < \psi_1 < \pi$ and $\psi_1 + \psi_2 > \pi$ the available region is given in Fig 15 Finally in this section, Fig 16 shows the region for $\pi/2 < \psi_2 < \psi_1 < \pi$ These results follow from the data in Appendix V and it can be shown that there are no available regions for the following

FIG 7



AVAILABLE REGIONS FOR $-\frac{\pi}{2} < \psi_1, \psi_2 < \frac{\pi}{2}$ AND $m < 1$

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$$-\pi/2 < \psi_2 < \psi_1 < 0$$
$$-\pi/2 < \psi_1 < 0 < \psi_2 < \pi/2 \text{ with } \psi_1 + \psi_2 > 0$$
$$-\pi/2 < \psi_2 < 0 < \psi_1 < \pi/2 \text{ with } \psi_1 + \psi_2 > 0$$
$$0 < \psi_1 < \pi/2 < \psi_2 < \pi \text{ with } \psi_1 + \psi_2 > \pi$$
$$0 < \psi_2 < \pi/2 < \psi_1 < \pi \text{ with } \psi_1 + \psi_2 < \pi$$

and $$\pi/2 < \psi_1 < \psi_2 < \pi$$

On comparing Figs 6—9 with Figs 10—16 we see that they correspond very closely The regions for $A_{22} > A_{11}$ given in Fig 6 correspond to

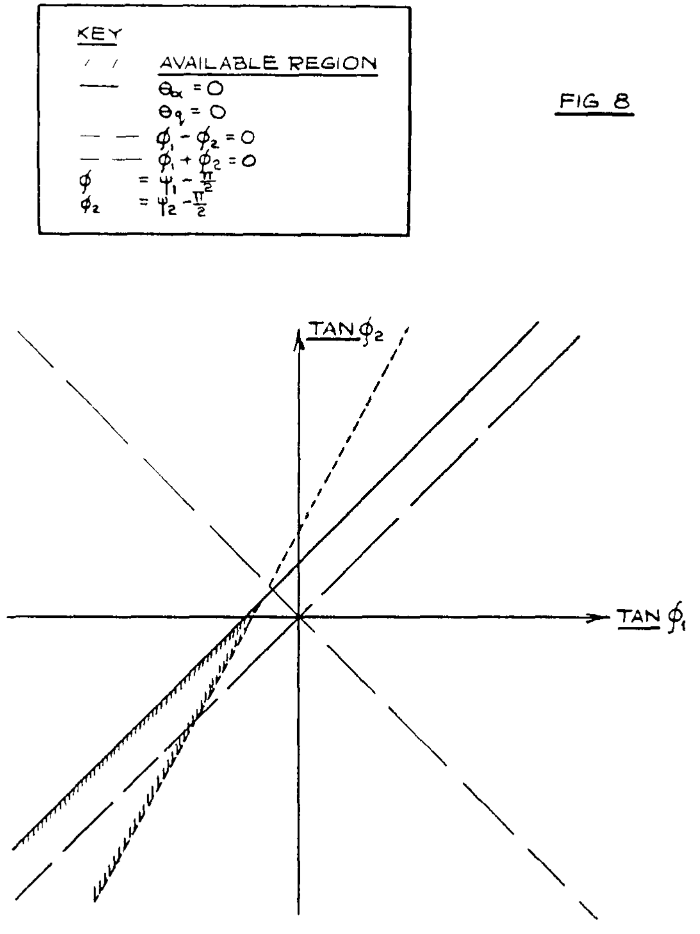

FIG 8

AVAILABLE REGIONS FOR $0 < \psi_1, \psi_2 < \pi$ AND $m > 1$

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Figs 10, 12, 13 and 14 The region for $A_{11} > A_{22}$ given in Fig 7 corresponds to Fig 11 The regions of Fig 8 correspond to Figs 12, 13 and 14 whilst those of Fig 9 are equivalent to Figs 15 and 16 It can be seen also that the regions given above where no available regions exist correspond to similar regions in Figs 6—9

6 *Three Special Cases*

Let us now consider some special cases Firstly let $A_{11} = 0\ 30$ and $A_{22} = 0\ 35$ Fig 17 shows the available regions near the origin and it can be seen that it is of the form predicted by Fig 6 The assumption that the second term in the denominator of the expressions (5 3) and (5 4) for θ_a/G and $\theta_q\Omega/G$ is justified for, with the given values of A_{11} and A_{22} and a

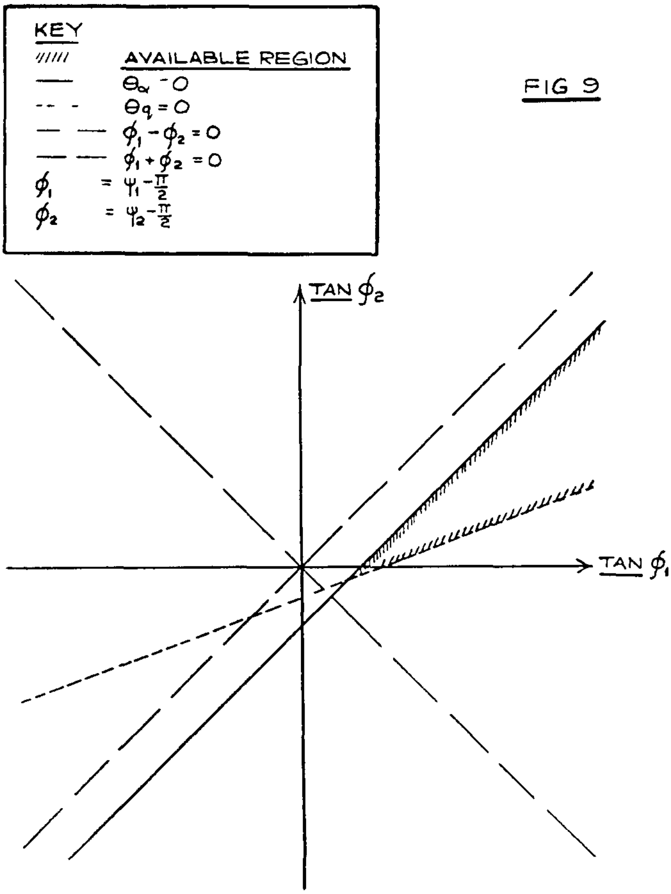


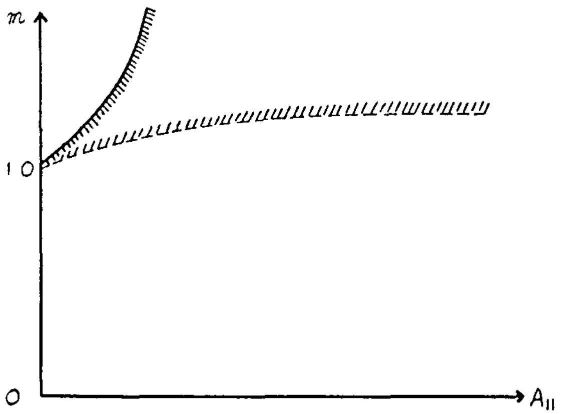
FIG 9

AVAILABLE REGION FOR $0 < \psi_1, \psi_2 < \pi$ AND $m < 1$

frequency ratio of about 0 01, it can easily be proved that $N\bar{v}^2/M$ is of the order of 0 005 Effects of this order of magnitude can be neglected

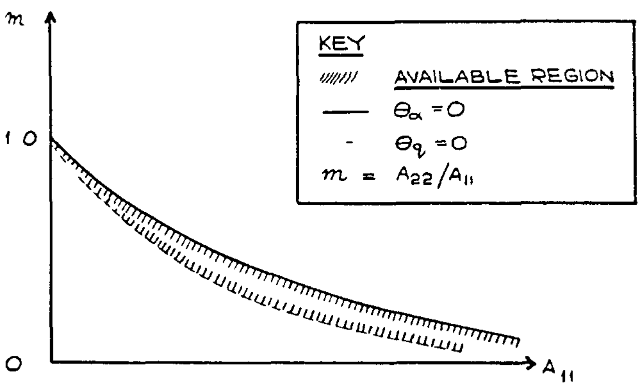
For the second example we have chosen $\psi_1 = -\pi/3$ and $\psi_2 = -\pi/6$ In Fig 18 $m = A_{22}/A_{11}$ is plotted against A_{11} and the available region corresponds to the expected area given by Fig 10

Thirdly, we have chosen $\psi_1 = \pi/6$ and $\psi_2 = \pi/3$ The available region Fig 19 agrees with the predicted region Fig 14 If we now choose $A_{22} = 2A_{11}$ and plot $\theta_\alpha/G\bar{v}^2$, $\theta_q\Omega/G\bar{v}^2$ and $\theta_q\Omega/\theta_\alpha$ against A_{11} we obtain Fig 20 From this figure it can be seen that small values of the ratio $\theta_q\Omega/\theta_\alpha$ are possible and hence good stability characteristics can be obtained We now introduce springs and aerodynamic damping into the system and investigate their effect Let us first consider the spring effect



AVAILABLE REGION FOR $-\frac{\pi}{2} < \psi_1 < \psi_2 < 0$

FIG 10



AVAILABLE REGION FOR $-\frac{\pi}{2} < \psi_1 < 0 < \psi_2 < \frac{\pi}{2}$, $\psi_1 + \psi_2 < 0$

FIG 11

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7 *Effect of Spring Restraint*

The equation of motion can now be written in the form

$$\ddot{\delta}_i + 2\Omega A_{ii}\dot{\delta}_i + \Omega^2(1+\Delta B_{ii})\delta_i + 2\Omega q \sin(\psi + \psi_i) - \dot{q}\cos(\psi + \psi_i) = 0 \qquad (i = 1, 2) \qquad (7\ 1)$$

The original B_{ii} of equations (5 1) has been replaced by $(1 + \Delta B_{ii})$ for convenience in the algebra and because $\Delta B_{ii} = 0$ reduces (7 1) to the equations (5 5) where the spring effect is absent By conducting a similar analysis as performed in Appendix III it can be shown that both θ_a and $\theta_q\Omega$ are of the form

$$\frac{(\alpha_1 \tan\psi_1 \tan\psi_2 + \alpha_2 \tan\psi_1 + \alpha_3 \tan\psi_2 + \alpha_4)\cos\psi_1}{\beta_1 + \beta_2 \tan\psi_2} \qquad (7\ 2)$$

where α_j and β_k are functions of both A_{ii} and ΔB_{ii} ($j = 1 \ldots 4$, $k = 1, 2$, $i = 1, 2$)

Thus the major changes in the picture for the available regions in the (ψ_1, ψ_2) plane are that the curve $\theta_a = \theta_q\Omega = 0$ suffer changes in their asymptotes Let us consider a numerical example Taking

$$A_{11} = 0\ 30\ ,\ A_{22} = 0\ 35\ ,\ \Delta B_{11} = 0\ ,\ \Delta B_{22} = 0\ 10$$

and considering that part of the ψ plane where $0 < \psi_1, \psi_2 < \pi/2$ we obtain Fig 21 From this figure we see that larger values of ψ_1 may now be used and that the previous values when the spring effect is absent are no longer ' available '

In Fig 22 we have plotted $\theta_a/G\nu^2$, $\theta_q\Omega/G\nu^2$ and $\theta_q\Omega/\theta_a$ against $\tan\psi_1$ for $\psi_2 = \pi/4$ The figure shows that in the new available region the desired value of $\theta_q\Omega/\theta_a$ necessary for good dynamic stability can be achieved Thus we see that the introduction of the spring effect does not harm the system's ability to produce the desired ratio but only alters the position of the available areas

8 *Effect of Aerodynamic Damping*

On the introduction of damping due to air forces and neglecting spring effects the equation of motion becomes

$$\ddot{\delta}_i + 2\Omega(A_{ii} + K_i)\dot{\delta}_i + \Omega^2\delta_i + 2\Omega q \sin(\psi + \psi_i) \qquad (8\ 1)$$

$$- \dot{q}\cos(\psi + \psi_i) - 2qK_i\Omega\cos(\psi + \psi_i) = 0 \qquad (i = 1, 2)$$

The aerodynamic effect is given by the K_i terms and it can be seen that when $K_1 = K_2 = 0$, equation (8 1) reduces to equations (5 5) where only mechanical damping at the hinge is considered

The effect on the boundaries $\theta_a = \theta_q\Omega = 0$ is the same as in the case of the introduction of springs into the system

Let us consider the case where

$$A_{11} = 0\ 30\ ,\ A_{22} = 0\ ,\ K_1 = 0\ ,\ K_2 = 0\ 35$$

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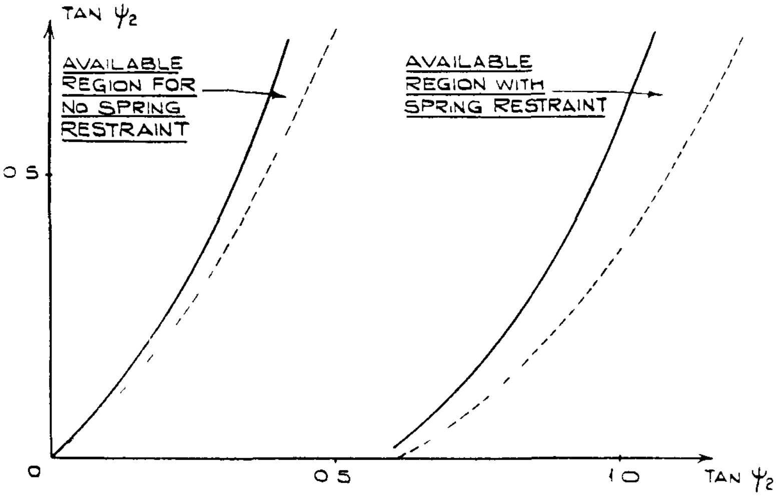
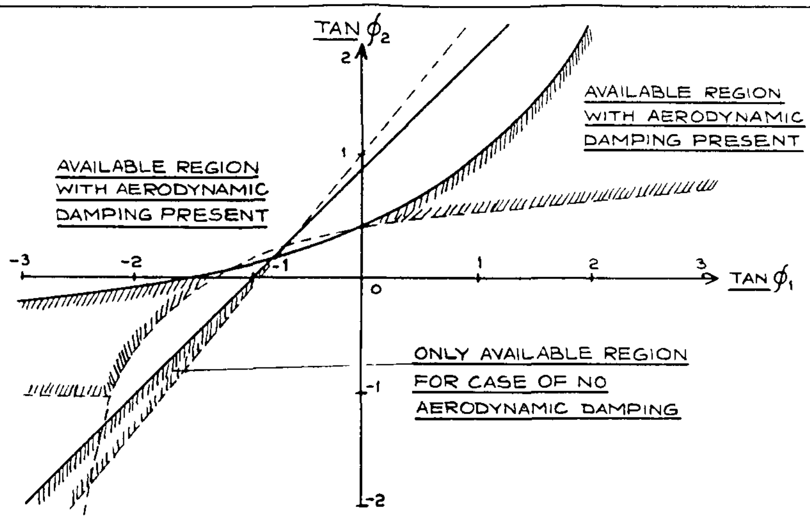
KEY —— $\theta_{\alpha} = 0$ ---- $\theta_{q} = 0$

FIG 21



THE EFFECT OF SPRING RESTRAINT ON AVAILABLE REGIONS FOR $0 < \psi_1, \psi_2 < \pi/2$

$A_{11} = 0\ 30$, $A_{22} = 0\ 35$, $\Delta B_{11} = 0$ $\Delta B_{22} = 0\ 10$



THE EFFECT OF AERODYNAMIC DAMPING ON AVAILABLE REGIONS

FOR $0 < \psi_1, \psi_2 < \pi$, $A_{11} = 0\ 30$, $A_{22} = 0$, $K_1 = 0$, $K_2 = 0\ 35$

FIG 23

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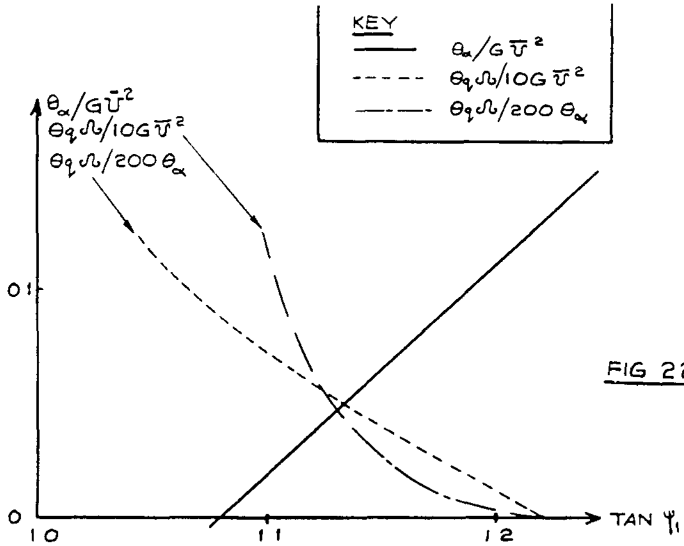

FIG 22

VARIATIONS OF $\theta_\alpha/G\bar{v}_1^2$, $\theta_q\Omega/G\bar{v}^2$ AND $\theta_q\Omega/\theta_\alpha$ WITH TAN ψ_1

FOR $A_{11} = 0{\cdot}30$, $A_{22} = 0{\cdot}35$, $\Delta B_{11} = 0$, $\Delta B_{22} = 0{\cdot}10$, TAN $\psi_2 = 1{\cdot}0$

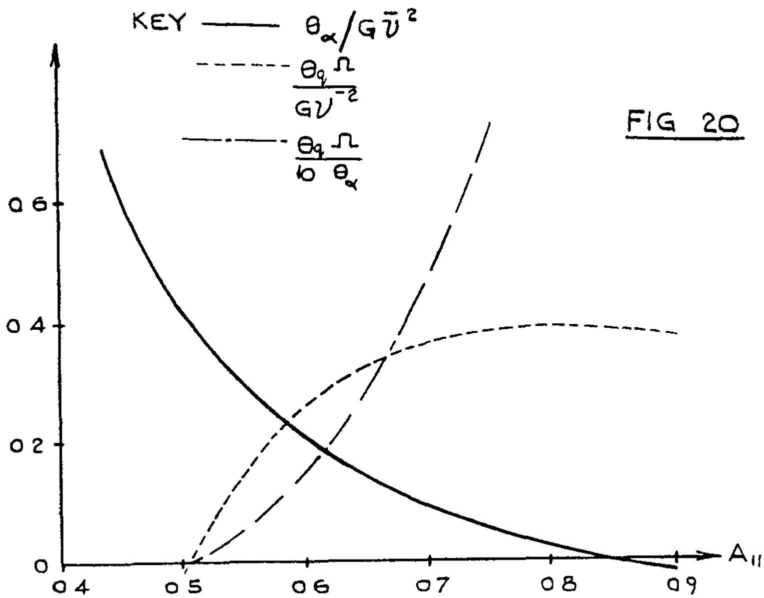

FIG 20

VARIATIONS OF $\theta_\alpha/G\bar{v}^2$, $\dfrac{\theta_q\Omega}{G\bar{v}^2}$, $\dfrac{\theta_q\Omega}{10\,\theta_\alpha}$ WITH A_{11}

FOR $\psi_1 = \pi/6$, $\psi_2 = \pi/3$, $m = 2$

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This corresponds to a system where one rod is mechanically damped at the hinge and the other is damped aerodynamically Fig 23 shows the changes produced in the ψ plane where $0 < \psi_1, \psi_2 < \pi$ It can be seen that the available region before the introduction of aerodynamic damping has disappeared and has been replaced by two larger regions The reason why the new curves fail to degenerate into straight lines as in the case of pure mechanical damping, is because of the introduction of the α_4 term in equation (7 2) In Fig 24 we have plotted $\theta_\alpha/G\bar{v}^2$, $\theta_q\Omega/G\bar{v}^2$ and $\theta_q\Omega/\theta_\alpha$ against $\tan \phi_1$ for $\phi_2 = \pi/4$, *i e*, $\psi_2 = \frac{3}{4}\pi$ We see again that the required value of $\theta_q\Omega/\theta_\alpha$ for good stability can be obtained

9 *Conclusions*

1 Theoretically a helicopter can be fitted with an automatic control device utilizing the rotor shaft as the origin of gyroscopic couples to give any required stability characteristic with controls fixed A second order system will give the small values of the ratio $\theta_q\Omega/\theta_\alpha$ required for good stability

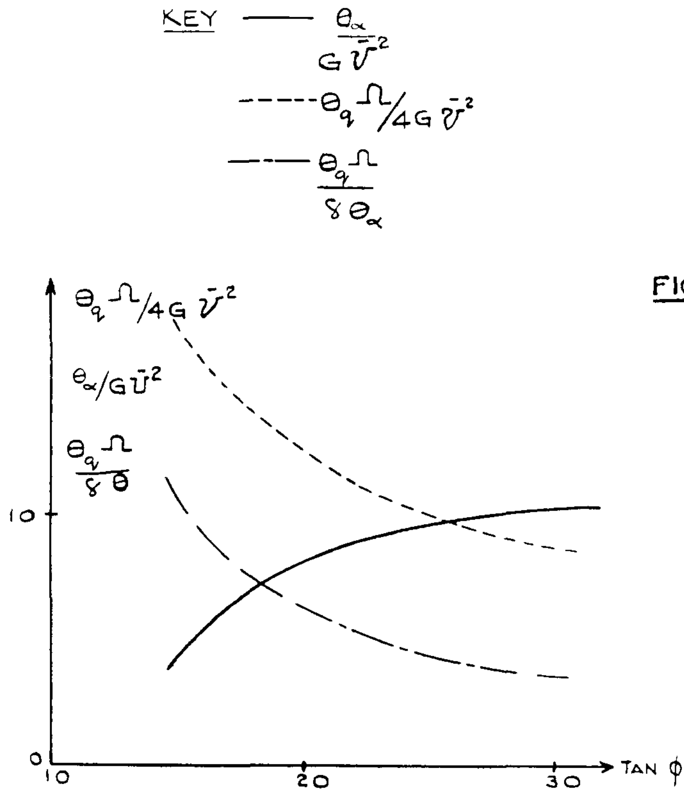

FIG 24

VARIATIONS OF $\frac{\theta_\alpha}{G\bar{v}^2}$, $\frac{\theta_q\Omega}{4G\bar{v}^2}$, $\frac{\theta_q\Omega}{8\theta_\alpha}$

FOR $A_{11} = 0\,30$, $A_{22} = 0$, $K_1 = 0$, $K_2 = 0\,35$, $\tan\phi_2 = 1\,0$

2 The position of the bars must be in ‘ available ’ regions The location of these regions can be changed by the introduction of aerodynamic damping and elastic restraint

3 The results are sufficient to indicate how mechanical apparatus should be designed for practical application of the principle

References

1 ‘ Investigation on automatic stabilization of the helicopter,’ by G J Sissingh (R A E Rep Aero 2277)

2 ‘ The Frequency Response of the Ordinary Blade, the Hiller Servo Blade and the Young-Bell Stabilizer,’ by G J Sissingh (R and M 2860)

3 ‘ Automatic Stabilization of Helicopters,’ by G J Sissingh (Journal of the Helicopter Ass of Gt Britain, 1948)

Appendix I

The Generalized ‘ Simple ’ Stabilization System

We shall consider the system to be composed of n rods hinged at the axis of rotation and connected to some form of mass If we neglect the moment of inertia of the blades about their axes, the forces in the blade setting device which oppose the motion δ, *i e* , friction and feedback to the separate rods due to their connection with the main control bar, the moments about the hinge give for the i th rod,

$$M_{ai} + M_{mi} + \sum_{j=1}^{n} M_{ij} = 0 \qquad \text{(A 1)}$$

and M_{ai} = Moment due to Aerodynamic forces

M_{mi} = Moment due to Mass forces

M_{ij} = Moment due to the connection of j th rod with i th rod

M_{ii} = Moment due to the connection of i th rod to the shaft

It is to be noted that in general $M_{ij} \neq M_{ji}$

These separate moments are given by

$$M_{ai} = 2\Omega^2 K_i I_{si}[\delta_i/\Omega + C'\delta_i - (q/\Omega)\cos(\psi + \psi_i)] \qquad \text{(A 2)}$$

where

I_{si} = Moment of Inertia of i th mass about hinge

K_i = Specific damping and

C' = coupling between the incidence of the i th mass and its angular displacement

$$M_{mi} = I_{si}[\delta_i + \Omega^2\delta_i + 2\Omega q \sin(\psi + \psi_i) - q\cos(\psi + \psi_i)] \qquad \text{(A 3)}$$

$$M_{ii} = 2K'_i\Omega I_{si}\delta_i + \epsilon_i\delta_i I_{si}\Omega^2 \qquad \text{(A 4)}$$

where

K'_i = specific damping at hinge of i th rod

ϵ_i = specific elastic constant

$$M_{ij} = S_{ij}(l_i\delta_i - l_j\delta_j) + D_{ij}(l_i\delta_i - l_j\delta_j) \quad \text{(A 5)}$$

$$= \Omega^2 I_{si} \{ S_{ij}(l_i\delta_i - l_j\delta_j) + \frac{2}{\Omega} d_{ij}(l_i\delta_i - l_j\delta_j) \quad \text{(A 6)}$$

assuming $$l^2_i + l^2_j - 2l_il_j \cos(\psi_i - \psi_j) < (l_i\delta_i - l_j\delta_j)^2 \quad \text{(A 7)}$$

where S_{ij} and d_{ij} are specific spring and damping constants The condition A 7 is imposed in order that the moment due to the interlinking forces acts in the same plane as the other moments Fig 3 illustrates the situation

Substituting these expressions into (A 1) gives equations of the form

$$\delta_i + 2\Omega\sum_{j=1}^{n} A_{ij}\delta_j + \Omega^2\sum_{j=1}^{n} B_{ij}\delta_j + 2\Omega q \sin(\psi + \psi_i)$$

$$- q\cos(\psi + \psi_i) - 2qK_i\Omega\cos(\psi + \psi_i) = 0 \quad \text{(A 8)}$$

$$(i = 1, 2, \quad n)$$

Appendix II

The equations of motion for a Second Order System with no inter-connecting moments are

$$\delta_1 + 2\Omega\delta_1 A_{11} + \Omega^2(1 + \Delta B_{11})\delta_1 + 2\Omega q \sin(\psi + \psi_1) - q\cos(\psi + \psi_1)$$
$$- 2K_1\Omega q\cos(\psi + \psi_1) = 0 \quad \text{(A 9)}$$

$$\delta_2 + 2\Omega\delta_2 A_{22} + \Omega^2(1 + \Delta B_{22})\delta_2 + 2\Omega q \sin(\psi + \psi_2) - q\cos(\psi + \psi_2)$$
$$- 2K_2\Omega q\cos(\psi + \psi_2) = 0 \quad \text{(A 10)}$$

Substituting $\delta_i = \Theta_{is}\sin\psi + \Theta_{ic}\cos\psi \ (i = 1, 2)$ (A 11)

and
$$a = a_0^{\,i\upsilon t}$$
$$\Theta_{is} = \theta_{is}\, e^{i\upsilon t} \quad \text{(A 12)}$$
$$\Theta_{ic} = \theta_{ic}\, e^{i\upsilon t}$$

we obtain

$$\frac{-\theta_{is}}{a_0} = \frac{\bar{\upsilon}^2(Z'_0X'_2 + Z'_1X'_1) + i\bar{\upsilon}Z'_0X'_1 + i\bar{\upsilon}^3(Z'_0X'_3 + X'_1Z'_2 - X'_2Z'_1)}{Z'^2_0 + \bar{\upsilon}^2(Z'^2_1 + 2Z'_0Z'_2)}$$

(A 13)

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neglecting the terms in higher powers of $\bar{v}$,

where $$Z'_0 = \Delta B^2_{11} + 4A^2_{11} \quad \text{(A 14)}$$

$$Z'_2 = -2\Delta B_{11} - 4A^2_{11} - 4 \quad \text{(A 15)}$$

$$Z'_1 = 4A_{11}\ \Delta B_{11} + 8A_{11} \quad \text{(A 16)}$$

$$X'_2 = (-4 - \Delta B_{11} - 4A_{11}K_1) \sin\psi_1$$

$$+ (-2A_{11} + 4K_1) \cos\psi_1 \quad \text{(A 17)}$$

$$X'_1 = (4A_{11} + 2\Delta B_{11}K_1) \sin\psi_1 + (-4A_{11}K_1 + 2\Delta B_{11}) \cos\psi_1 \quad \text{(A 18)}$$

and $X'_3 = (-2A_{11} - 2K_1) \sin\psi_1$ (A 19)

Similarly we obtain θ_{2s}/a_0

Now $$\theta_{1a} = -\text{Re}\,(\theta_{1s}/a_0) \quad \text{(A 20)}$$

$$\theta_{1q}\Omega\bar{v} = -\text{Im}\,(\theta_{1s}/a_0) \quad \text{(A 21)}$$

and $$\Delta\theta = G\,(\delta_1 + \bar{n}\delta_2) \quad \text{(A 22)}$$

so that $$\theta a = G\,(\theta_{1a} + \bar{n}\theta_{2a}) \quad \text{(A 23)}$$

$$\theta_q\Omega = G\,(\theta_{1q} + \bar{n}\theta_{2q})\,\Omega \quad \text{(A 24)}$$

Hence we can write $$\theta_a/G = L\bar{v}^2/(M + N\bar{v}^2) \quad \text{(A 25)}$$

and $$\theta_q\Omega/G = \frac{P + Q\bar{v}^2}{M + N\bar{v}^2} \quad \text{(A 26)}$$

where $$L = [Z_0''^2(Z'_0X'_2 + Z'_1X'_1) + \bar{n}Z'^2_0\,(Z''_0X''_2 + Z''_1X''_1] \quad \text{(A 27)}$$

$$P = Z'_0Z''_0[X'_1Z''_0 + \bar{n}Z'_0X''_1] \quad \text{(A 28)}$$

$$Q = [Z''_0{}^2(Z'_0X'_3 + Z'_2X'_1 - Z'_1X'_2) + Z'_0X'_1\,(Z_1''^2 + 2Z''_0Z''_2)$$
$$+ \bar{n}\,Z''_0X''_1(Z'^2_1 + 2Z'_0Z'_2) + \bar{n}\,Z'^2_0\,(Z''_0X''_3 + X''_1Z''_2 - X''_2Z''_1)] \quad \text{(A 29)}$$

$$M = Z'^2_0Z_0''^2 \quad \text{(A 30)}$$

and $$N = Z'^2_0\,(Z_1''^2 + 2Z''_0Z''_2) + Z_0''^2\,(Z'^2_1 + 2Z'_0Z'_2) \quad \text{(A 31)}$$

The double primed terms are the equivalent expressions of A (14—19) when the second bar is considered

Since, from A 14, $Z' \neq 0$ except when both ΔB_{11} and A_{11} are zero, we must have, for $P = 0$,

$$X'_1Z''_0 + \bar{n}Z'_0X''_1 = 0 \quad \text{(A 32)}$$

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Appendix III

When we consider only damping at the hinge A 14—19 become

$$Z'_0 = 4A^2_{11} \tag{A 33}$$

$$Z'_2 = -4\,(1 + A^2_{11}) \tag{A 34}$$

$$Z'_3 = 8A_{11} \tag{A 35}$$

$$X'_2 = -4 \sin \psi_1 - 2A_{11} \cos \psi_1 \tag{A 36}$$

$$X'_1 = 4A_{11} \sin \psi_1 \tag{A 37}$$

and

$$X'_3 = -2A_{11} \sin \psi_1 \tag{A 38}$$

and the condition that P should be zero is that

$$\bar{n} = -\frac{A_{22} \sin \psi_1}{A_{11} \sin \psi_2} \tag{A 39}$$

When these expressions are substituted into equations A 27—30,

$$\frac{\theta_a}{G\bar{v}^2} = \frac{(A_{22} - A_{11}) \sin \psi_1 \sin \psi_2 + \frac{1}{2}A_{11}A_{22} \sin (\psi_1 - \psi_2)}{A^2_{11}\, A_{22} \sin \psi_2} \tag{A 40}$$

and

$$\frac{\theta_q\Omega}{G\bar{v}^2} = \frac{A_{22} \sin \psi_2 \cos \psi_1 - A_{11} \sin \psi_1 \cos \psi_2 + \left(\frac{A_{11}}{A_{22}} - \frac{A_{22}}{A_{11}}\right) \sin \psi_1 \sin \psi_2}{A^2_{11}\, A_{22} \sin \psi_2} \tag{A 41}$$

It is assumed that the ratio $N\bar{v}^2/M$ is small and can be neglected

---

Appendix IV

The Regions of Possible Azimuth Angles ψ_1, ψ_2

Let us consider the regions in the ψ_1, ψ_2 plane where both θ_a and $\theta_q\Omega$ are positive

Let us write

$$F = (m - 1) \tan \psi_1 \tan \psi_2 + \tfrac{1}{2}\, m\, A_{11}\, (\tan \psi_1 - \tan \psi_2) \tag{A 42}$$

and

$$f = -\tan \psi_1 + m \tan \psi_2 + \left(\frac{1 - m^2}{mA_{11}}\right) \tan \psi_1 \tan \psi_2 \tag{A 43}$$

where

$$m = A_{22}/A_{11} \tag{A 44}$$

Then

$$\frac{\theta_a}{G} = \frac{F\, \bar{v}^2 \cos \psi_1 \cos \psi_2}{A_{11}A_{22} \sin \psi_2} \tag{A 45}$$

and

$$\frac{\theta_q\Omega}{G} = \frac{f\, \bar{v}^2 \cos \psi_1 \cos \psi_2}{A_{11}A_{22} \sin \psi_2} \tag{A 46}$$

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Range I $\quad -\frac{\pi}{2} < \psi_1 , \psi_2 < \frac{\pi}{2}$

Here
$$\cos \psi_1 \cos \psi_2 > 0 \tag{A 47}$$

Let
$$e = \tfrac{1}{2}mA_{11}/(m-1) \text{ and } h = \frac{mA_{11}}{m^2-1} \tag{A 48}$$

The boundaries are given by

$$F = 0 \text{ i e } \tan \psi_2 = \frac{e \tan \psi_1}{e - \tan \psi_1} \tag{A 49}$$

and
$$f = 0 \text{ i e } \tan \psi_2 = \frac{h \tan \psi_1}{mh - \tan \psi_1} \tag{A 50}$$

Considering $G > 0$ and using (A 47) we can write

$$\theta_a = \gamma^2 F/\sin \psi_2 \tag{A 51}$$

and
$$\theta_q = \gamma'^2 f/\sin \psi_2 \tag{A 52}$$

where γ and γ' are real quantities

The condition $G > 0$ corresponds to $\Delta\theta$ being in the same sense as δ The available areas are found by considering the various ψ_1, ψ_2

(*a*) $m > 1$ i e $A_{22} > A_{11}$ The results are shown in fig 6

(*b*) $m < 1$ i e $A_{11} > A_{22}$ The results are shown in fig 7

Range II

$$0 < \psi_1, \psi_2 < \pi$$

Let
$$\psi_1 = \phi_1 + \pi/2 \tag{A 53}$$

and
$$\psi_2 = \phi_2 + \pi/2 \tag{A 54}$$

Then
$$\theta_a/G\bar{v}^2 = \frac{F' \cos \phi_1}{A_{11}A_{22}} \tag{A 55}$$

and
$$\theta_q \Omega/G\bar{v}^2 = \frac{f' \cos \phi_1}{A_{11}A_{22}} \tag{A 56}$$

where
$$F' = (m-1) + \tfrac{1}{2}mA_{11} (\tan \phi_1 - \tan \phi_2) \tag{A 57}$$

$$f' = \tan \phi_2 - m \tan \phi_1 + \left(\frac{1}{m} - m\right)/A_{11} \tag{A 58}$$

and the boundaries are given by

$$F' = 0 \text{ i e } \tan \phi_2 = \phi_1 + {}^1/_e \qquad \text{(A 59)}$$

and $$f' = 0 \text{ i e } \tan \phi_2 = m \tan \phi_1 + {}^1/_h \qquad \text{(A 60)}$$

The results are illustrated in Fig 8 and 9 The regions are found by inspection

---

Appendix V

By writing $$a = \sin \psi_1 \qquad \text{(A 61)}$$

$$b = \tfrac{1}{2} \sin (\psi_1 - \psi_2)/\sin \psi_2 \qquad \text{(A 62)}$$

$$c = \sin \psi_1 \cos \psi_2/\sin \psi_2 \qquad \text{(A 63)}$$

$$d = \cos \psi_1 \qquad \text{(A 64)}$$

$$H = -a + ma + \tfrac{1}{2} mA_{11} b \qquad \text{(A 65)}$$

and $$H' = -c + md + \left(\frac{1}{m} - m\right)a/A_{11} \qquad \text{(A 66)}$$

we see that $$\frac{\theta_a}{G} = \frac{Hv^2}{A_{11}A_{22}} \qquad \text{(A 67)}$$

and $$\frac{\theta_q \Omega}{G} = \frac{H'v^2}{A_{11}A_{22}} \qquad \text{(A 68)}$$

By considering the groups—

1 $-\pi/2 < \psi_1 < \psi_2 < 0$

2 $-\pi/2 < \psi_2 < \psi_1 < 0$

3 $-\pi/2 < \psi_1 < 0 < \psi_2 < \pi/2$

4 $-\pi/2 < \psi_2 < 0 < \psi_1 < \pi/2$

5 $0 < \psi_1 < \psi_2 < \pi/2$

6 $0 < \psi_2 < \psi_1 < \pi/2$

7 $0 < \psi_1 < \pi/2 < \psi_2 < \pi$

8 $0 < \psi_2 < \pi/2 < \psi_1 < \pi$

9 $\pi/2 < \psi_1 < \psi_2 < \pi$

10 $\pi/2 < \psi_2 < \psi_1 < \pi$

we obtain Figs 10—16

List of Symbols

A_i	Coefficients of frequency equation ($i = 0, 1, 2, 3$)
θ_a	automatic control component in phase with attitude
θ_q	automatic control component in phase with rate of change of attitude
α	angle in pitch (in radians)
q	rate of change of attitude with respect to time (in rads/sec)
ψ	azimuth angle measured from rear position in direction of rotation (in radians)
Ω	angular velocity of rotor (in rads/sec)
δ	automatic control device response
δ_i	angular displacement of i th rod of the device (in radians)
$\Delta\theta$	change of pitch setting (in radians)
θ_o	mean pitch setting of rotor blade (in radians)
G	gearing ratio
ψ_i	azimuth angle of i th rod measured from blade I (in radians)
A_{ij}	specific damping coefficient in generalized equations
B_{ij}	specific spring constant in generalized equations
K_i	specific aerodynamic damping in generalized equations
θ_s	cyclic pitch component (in radians)
θ_c	ditto
υ	frequency of the oscillation (in sec $^{-1}$)
$\bar{\upsilon}$	$= \upsilon/\Omega$ frequency ratio
$\bar{n}$, i i	linkage ratios
γ_o	inertia number

Other symbols are defined in the text as required