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ARTICLE

What Mathematics and Metaphysics of Corporeal Nature Offer to Each Other: Kant on the Foundations of Natural Science

Michael Bennett McNulty

University of Minnesota, Minneapolis, MN, USA
Email: mcnu0074@umn.edu

Abstract

Kant famously distinguishes between the methods of mathematics and of metaphysics, holding that metaphysicians err when they avail themselves of the mathematical method. Nonetheless, in the *Metaphysical Foundations of Natural Science*, he insists that mathematics and metaphysics must jointly ground 'proper natural science'. This article examines the distinctive contributions and unity of mathematics and metaphysics to the foundations of the science of body. I argue that the two are distinct insofar as they involve distinctive sorts of grounding relations – mathematics pertains to formal grounding, while metaphysics concerns material grounding – while they are unified insofar as they treat motion, the fundamental determination of the science of body.

Keywords: Kant; natural science; metaphysics; mathematics; grounding

1. Introduction

Although he undeniably esteems mathematics, Kant is well known for seeking to isolate mathematics from the 'queen of the sciences', metaphysics. While his predecessor, Christian Wolff, famously sought to emulate the mathematical method in metaphysics, Kant criticized his predecessor's mathematics-envy. According to Kant, while mathematics may hold insight for metaphysics – particularly, metaphysics ought to mimic the innovation that first placed mathematics (and natural science) on the 'secure path of science' (Bxvi–xviii)[1] – the methods of mathematics and metaphysics are *essentially* different, a thesis that he expounds upon in the Discipline of Pure Reason in its Dogmatic Use of the *Critique of Pure Reason* (A712–38/B740–66). Whereas mathematics achieves synthetic a priori cognitions of things via mathematical construction (through the exhibition of particular mathematical objects in the pure forms of intuition), metaphysics proceeds from concepts (either logically, by analysing concepts, or transcendentally, by seeking the conditions of the possibility of experience). A philosopher, like Wolff, errs when they attempt to apply the mathematical method to the problems of metaphysics, a point Kant states eloquently.

> Now from all this it follows that it is not suited to the nature of philosophy, especially in the field of pure reason, to strut about with a dogmatic gait and to decorate itself with the titles and ribbons of mathematics, to whose ranks philosophy does not belong, although it has every cause to hope for a sisterly union with it. (A735/B763)

Thus, metaphysics, in particular, and philosophy, more generally, ought not ape mathematics, though we may hope for a 'sisterly union' between the two, essentially distinct doctrines.

In the *Metaphysical Foundations of Natural Science*, however, the place and relations of mathematics and metaphysics become even more complicated. One of the main goals of the *Foundations* is to explain the possibility of a proper natural science of corporeal nature. A proper natural science, for Kant, is one that achieves genuine – that is, apodictically certain – knowledge.[2] Such is possible, according to Kant, only through the joint contributions of *both* mathematics and metaphysics. Thus, he writes that '*Properly* so-called natural science presupposes, in the first place, metaphysics of nature' (*MFNS*, 4: 469) and 'I assert, however, that in any special doctrine of nature there can be only as much *proper* science as there is mathematics therein' (*MFNS*, 4: 470).[3] Hence, in particular, a proper natural science *of bodies* is founded on a combination of both the application of mathematics and a metaphysics of corporeal nature.

In this article, I am concerned with the relationship between mathematics and metaphysics in the conceptual framework of Kant's *Metaphysical Foundations of Natural Science*. I focus especially on the most fundamental issues involving this relationship: what sorts of cognitions belong to the mathematics and metaphysics of corporeal nature and how is it possible for them to form a sisterly union, despite their underlying and essential distinctiveness. My approach seeks to address these issues in a systematic context, as informed by Kant's overarching accounts of mathematics, metaphysics, methodology and types of knowledge. While a wide range of scholarship bears on these issues, none adequately illuminates the topics with which I am concerned. First, there are a variety of accounts of the mathematization of nature that make contributions at a different level, not delving into the details of the nature of metaphysics, mathematics and their union. Accounts like those of McNulty (2014), Sutherland (2014), Dunlop (2022), though immensely illuminating of Kant's account of the mathematics of nature, are conceptually downstream from my interests. Second, there are accounts of the framework of the *Foundations* that rest on 'metaphysical construction' as the unique method of the special metaphysics of corporeal nature, like those of Plaass (1965: 74–8) and Stang (2016: 247–8). Given the controversy of the notion of 'metaphysical construction' and my aims for systematicity (an essential role for metaphysical construction in the *Foundations* is difficult to square with Kant's accounts of metaphysics and mathematics in the *Critique of Pure Reason*), I opt to found my account otherwise.[4] Finally, there are interpretations that bear directly on my concerns and spotlight the notion of *real possibility* as central to understanding the roles of mathematics and metaphysics, like Plaass (1965), Washburn (1975) and Friedman (2013). Such accounts are immensely illuminating of the project of the *Foundations*. However, as I will show in section 2, these accounts require supplementation in order to give a comprehensive picture of the respective roles of and sisterly union between mathematics and metaphysics of corporeal nature.

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In the course of this article, I argue that, although mathematics and metaphysics concern corporeal things, they do so in essentially divergent ways. In particular, the two doctrines concern two distinct sorts of *grounding relations*. Mathematics regards matter as a *formal* ground (as grounding other qualities via its mathematical qualities), whereas metaphysics regards it as a *material* ground (that is, as grounding other things via its causality). The three subsidiary theses that I will defend and elucidate are as follows.

1. Mathematics concerns the *mathematical possibility* of motions and what motions *formally* ground.
2. Metaphysics concerns the analysis of matter, the *real possibility* of matter and what matter (namely, matter's nature) *materially* grounds.
3. Via analysis of matter, metaphysics makes possible mathematization, but, insofar as mathematics and metaphysics concern different sorts of grounding, they remain distinct (and neither is responsible for the other).

I contend that mathematics of corporeal nature has to do with the possibility of matter and its marks conforming to the forms of intuition, space and time, as well as what is grounded on matter's doing so. Conversely, the metaphysics of corporeal nature pertains to the possibility of matter conforming to the categories in general – particularly, the relational categories – as well as what is grounded on matter's doing so – that is, being a substance endowed with causal powers. Thus, metaphysics and mathematics of nature take their own particular tack with respect to one and the same subject (the concept of <matter>) and yet remain essentially distinct. In the following three sections of the article, I clarify and support these theses by discussing, respectively, the mathematics of corporeal nature (§2), the metaphysics of corporeal nature (§3) and their union (§4).

2. Possibility and the fruits of mathematics

My interpretation of the division between and respective goals of mathematics and metaphysics of corporeal nature stems, but diverges, from that of Friedman (2013). Although Friedman complicates this picture in important ways (see below), he initially describes mathematics' distinctive contribution to proper natural science and its contrast with metaphysics as follows.

> The difference between a metaphysical and mathematical foundation for a proper natural science therefore depends on the difference between actuality (existence) and possibility. A metaphysical foundation provides a priori principles governing the existence or actuality of things, while a mathematical foundation provides a priori principles governing their real (as opposed to merely logical) possibility. Whereas the real possibility (objective reality) of things standing under the pure concepts of the understanding is secured by the transcendental deduction of the categories independently of mathematical construction, the real possibility of more specific or determinate kinds of things falling under an *empirical* concept – such as the empirical concept of matter – cannot be a priori established in this way. The only remaining

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> alternative, therefore, is mathematical construction, which exhibits intuitions corresponding to the concept a priori. (Friedman 2013: 27–8)

The observation that metaphysics pertains to existence while mathematics pertains to possibility is a keen one and is further elaborated throughout this article. Friedman specifies that in natural science mathematics is meant to demonstrate the *real* possibility of things falling under relevant empirical concepts. Real possibility can be understood in distinction to logical possibility, a contrast that Kant draws as follows.

> *Logical* possibility, actuality, and necessity are cognized according to the principle of contradiction. Logical necessity does not prove the existence of a thing. But logical possibility is, as shown, not real possibility. Real possibility is agreement with the conditions of a possible experience. (Met-L_2, 28: 557)

On the one hand, something is logically possible when its concept is non-contradictory (see A220–1/B267–8), whereas, on the other hand, something is really possible when it conforms to the conditions of possible experience, or, what Kant elsewhere titles the 'formal conditions of experience (in accordance with intuition and concepts)' (B265) – that is, space, time and the categories of the understanding. There are a variety of logically possible things that are really impossible. For instance, Kant gives the example of a figure enclosed between two straight lines: this concept holds no contradiction – and is thus logically possible – but conflicts with the nature of space (a formal condition of experience) – and is thus really impossible (A220/B268).[5] Real possibility is, for Kant, equivalent to objective reality: a concept is objectively real just in case that it is really possible (see, for example, A596n./B624n.).[6]

The key role of mathematics vis-à-vis proper natural science is thus apparently to demonstrate the real possibility of things falling under relevant empirical concepts like <motion> and <matter>. Metaphysics, alone, cannot demonstrate such real possibilities, as it is concerned rather with the conditions and implications of *existence*, or actuality. Instead, it is left up to mathematics to demonstrate a priori real possibility in proper natural science.[7]

Some passages from the preface to the *Foundations* lend credence to the tight interconnection between mathematics and real possibility. For instance, in the critical passage providing the warrant for the claim that the application of mathematics is necessary for proper natural science, Kant describes that which mathematics uniquely provides as follows.

> Now to cognize something a priori means to cognize it from its mere possibility. But the possibility of determinate natural things cannot be cognized from their mere concepts; for from these the possibility of the thought (that it does not contradict itself) can certainly be cognized, but not the possibility of the object, as a natural thing that can be given outside the thought (as existing). (*MFNS*, 4: 470)

In this passage, Kant declares that mathematics is necessary to achieve a priori knowledge from the *possibility* of determinate natural things. He also explicitly specifies that mathematics does not concern mere logical possibility ('the possibility of the thought (that it does not contradict itself)'), which suggests understanding the reference as to *real* possibility.

Additionally, at the outset of the *Foundations*, Kant contrasts the nature of a thing, 'the first inner principle of all that belongs to the existence of [the] thing' (*MFNS*, 4: 467), from its essence, 'the first inner principle that belongs to the possibility of [the] thing'. He then proceeds to explain that 'one can attribute only an essence to geometrical figures, but not a nature (since in their concept nothing is thought that would express an existence)' (*MFNS*, 4: 467n.). Kant's opposition of natures and essences and association of mathematics with essence indicate both that mathematics pertains to the possibility of natural things and that existence is the concern of the other part of the doctrine of nature: metaphysics.[8]

However, the interpretation that mathematics of nature demonstrates the real possibility of determinate natural things faces various serious obstacles, as Friedman (2013: 28–33) also recognizes. First, upon reflection, mathematics does not appear capable of securing the real possibility of things falling under the relevant concepts of natural science. Mathematics deals specifically with *mathematical* concepts, that is, those that are a priori, made and refer to delimitations of the pure intuitions of space and time (A723–4/B751–2, A729/B757). Yet, the concepts of the natural science of body, like <motion> and <matter>, are given, empirical concepts, whose content goes beyond spatial or temporal determination and whose objective reality is proven via *experience* (A84/B117).[9]

Second, there are counterexamples to the thesis that mathematics secures the objective reality of natural-scientific concepts. In the General Remark to the Dynamics of the *Foundations*, Kant opposes two approaches to natural philosophical explanation (*MFNS*, 4: 523–5). To wit, he distinguishes the metaphysical-dynamical mode of explanation – according to which inherent forces of matter are the basis of explanation – from the mathematical-mechanical approach – according to which explanations of natural occurrences come down to motions of absolutely impenetrable bits in otherwise empty space. By way of illustration, whereas the metaphysical-dynamist would explain the resistance of a body to penetration by another by appeal to a resisting force, the mathematical-mechanist would appeal to the absolute impenetrability of the constituent material parts of the body. Although he prefers the metaphysical-dynamical approach, Kant reports that 'the mathematical-mechanical mode of explanation has an advantage over the metaphysical-dynamical [mode], which cannot be wrested from it' (*MFNS*, 4: 524–5), namely that its explanans – shapes of impenetrable bodies and the empty spaces among them – are mathematically constructible. Conversely, 'if the material itself is transformed into fundamental forces', as the metaphysical-dynamical mode of explanation has it, 'we lack all means for *constructing* this concept of matter, and presenting what we thought universally as possible in intuition' (*MFNS*, 4: 525). Yet the mechanical approach's mathematical adequacy is offset by its metaphysical incoherence: namely, that 'it must take an empty concept (of absolute impenetrability) as basis' (*MFNS*, 4: 525). An empty concept, for Kant, is one that lacks objective reality: as Kant remarks in the Amphiboly, 'For every concept there is requisite, first, the logical form of a concept (of thinking) in general, and then, second, the possibility of giving it an object to which it is to be related. Without this latter it has no sense, and is entirely empty of content' (A239/B298).[10] Hence, when Kant claims that the mathematical-mechanical mode of explanation takes an empty concept at its basis, he means one that lacks objective reality, or one that is not really possible. So, the explanations of the mathematical-mechanist are mathematically constructible, as Kant reports, though they are not really possible, thereby driving a wedge between these notions.

Third, systematic considerations expose a gap between a concept being mathematically constructible and its being really possible. Recall that (real) possibility is defined by Kant as conformity with 'formal conditions of experience (in accordance with intuition and concepts)'. That a concept is mathematically constructible does not show that it conforms with the full arsenal of formal conditions of experience; being constructible simply shows that it agrees with the formal conditions of intuition and the category of <quantity>. In particular, that something is mathematically constructible has nothing to do with its synthesis according to the categories of *relation*, chief among them being <causality>. Kant himself observes as much when he notes that mathematics does not concern existence (*MFNS*, 4: 467n.; A714–B747). By this, he means mathematics is not relevant to necessary relations of existence, like substance-accident, cause-effect and community, which are captured by the categories of relation (A723–4/B751–2). Thus, since real possibility regards the agreement between a concept and all of the formal conditions of experience, including the categories of relation, mathematical construction, alone, cannot demonstrate any thing's real possibility. Being the product of mathematical construction is a necessary condition for real possibility, no doubt, but it is insufficient.[11]

The deficiency of mathematics with respect to accounting for the real possibility of the concepts of natural science is well recognized, even among those that underline the relation between mathematics and real possibility. For Washburn, mathematics can only demonstrate real possibility in concert with metaphysics: 'A successful metaphysical construction brings the basic concept of a science into conformity with the understanding while a successful mathematical construction brings it into conformity with the special conditions of sensibility, thus demonstrating *a priori* the objective reality of that concept and the real possibility of its object' (1975: 289).[12] Likewise, for Pollok, '[The mathematical construction of material objects] does not demonstrate the existence of matters, but rather their spatio-temporal conditions of existence' (2001: 88). According to Plaass (1965), mathematics ultimately proves the real possibility of the *general* concept of matter, but only subsequent to its *metaphysical* construction as an a priori predicable. Finally, on similar grounds, Friedman (2013: 30) contends that the mathematics of corporeal nature instead concerns those 'partial concepts' whose real possibility *is* demonstrated by mathematical construction, like those of composite motions. So <matter>, itself, and the causal concepts belonging to it, like <force>, are not mathematically constructed, and, therefore, their real possibility is not demonstrated mathematically (see Friedman 2013: 119).

Nevertheless, in the following, I take another tack, one which better clarifies the contribution that mathematics makes to natural science as well as the relations between mathematical and metaphysical knowledge. Even if mathematical construction does not demonstrate real possibility, that a concept is mathematically constructible *does* show something about the concept *beyond* its mere logical possibility. Namely, it demonstrates that the concept conforms with the pure

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intuitions of space and time as formal conditions of experience. On this ground, I submit that there is an intermediate level of possibility between logical and real possibility, which I dub 'mathematical possibility'. Things falling under a concept are mathematically possible when the concept as such is mathematically constructible.[13] Mathematical construction of a concept shows that the things falling under the concept conform with space and time as conditions of the possibility of intuition, but does not necessarily show that they are really possible in the sense of conforming with the full slate of conditions of experience. When Kant writes that mathematics has to do with possibility in the Preface to the *Foundations*, I understand him to mean *mathematical possibility*. Thus, the claim that mathematics is necessary to provide us with knowledge of natural things 'from [their] mere possibility' signifies that it is to do so from their mere *mathematical* possibility.[14]

But mathematics is not only meant to demonstrate (mathematical) possibility of natural-scientific concepts. It is also supposed to be an engine for the production of further a priori cognitions about natural things, providing us knowledge of them *from* their mathematical possibility. In a letter to Karl Leonard Reinhold, Kant clarifies the way in which mathematics reveals further a priori truths about mathematical concepts in terms of a unique sort of grounding relation.

> [T]he real ground is again twofold: either the *formal* ground (of the *intuition* of the object) – as, for example, the sides of a triangle contain the ground of the angles – or the *material* ground (of the *existence* of the thing). The latter determines that whatever it contains will be called *cause*. (Letter to Reinhold, 12 May 1789; *Corr*, 11: 36).[15]

In general, according to Herder's notes from Kant's lectures on metaphysics, 'a ground is . . . something which, if it is posited, something else is posited' (Met-Herder, 28: 11).[16] Grounds are therefore by definition connected with consequences, those things that are posited in virtue of the grounds. Grounding relations may be divided into those that hold in virtue of the law of non-contradiction, in which case they involve *logical grounding*, and those that do not hold in virtue of the law of non-contradiction, in which case they involve *real grounding* (Met-Herder, 28: 11). With these preliminaries covered, we can turn to the distinction between two types of real grounds – formal grounds and material grounds[17] – mentioned in Kant's letter to Reinhold. On the one hand, the material ground for a thing grounds its existence. The paradigmatic example of such grounding involves the causal relation: a cause is the material ground of the effect insofar as it grounds the existence of the effect. (I return to material grounding in §4.) On the other hand, mathematics is concerned with formal grounding. Although Kant does not elaborate much on formal grounding, specifying only that it is the ground of the '*intuition* of the object', his example is revealing. Kant explains that the sides of a triangle are the formal ground of the angles of the triangle. This is a reference to one of the triangle congruence theorems, colloquially known as the 'Side-Side-Side' theorem, proven as proposition 8 in book I of Euclid's *Elements*: '*If two triangles have the two sides equal to two sides respectively, and also have the base equal to the base, then they also have the angles equal which are contained by the equal straight lines*' (Euclid 2002: 8). So, given the specification of the lengths of the sides of a triangle, the measures of the angles are fixed and determinable.

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The sides of the triangle are neither a *logical ground* of the angles – one cannot simply logically analyse the concept of the sides and discover the angles – nor a material ground – the sides of the triangle do not *cause* the angles.[18] Formal grounding relations, those disclosed by mathematical construction, are of a distinctive sort, irreducible to logical or material grounding.[19]

One and the same concept can be considered from the perspective of logical grounding or from that of formal grounding. Certain of the properties of the objects falling under the concept will belong to it logically, others formally. So, <triangle> can be considered as a logical ground by analysing it. Such analysis shows that that all triangles are figures, insofar as <figure> is contained in the concept of <triangle>. But <triangle> may also be considered as a formal ground. Thus, by means of a series of mathematical proofs, one can demonstrate that the triangularity of a figure – its falling under the concept <triangle> – is the ground for the interior angles the figure summing to two right angles. This feature cannot be discovered via logical analysis of <triangle> and can only be shown via mathematical construction (A716–17/B744–5). For this reason, the mathematical construction of <triangle> *formally* grounds triangles' possession of interior angles summing to two right angles.

The same holds in proper natural science: we can study natural-scientific concepts – chief among them, <motion> – either as logical or as formal grounds. The concept <change> is contained in the concept <motion>, so something's being a motion is the logical ground of it being a change. However, <motion> can also be considered as a formal ground insofar as it is mathematically constructible. In the Proposition of the Phoronomy in his *Foundations*, Kant shows that motions can be composed according to (his idiosyncratic version of) the parallelogram law (*MFNS*, 4: 490–3). The composite motion of two given motions, represented by line segments, is represented by the diagonal of the parallelogram they define. This entails that the composed motions formally ground the velocity and direction of the composite motion. This formal grounding relation, which is disclosed by the mathematical construction of <motion>, is the mathematical centrepiece both of the *Metaphysical Foundations of Natural Science* as well as Kant's account of proper natural science.

This overview thus clarifies the basic contributions of mathematics to proper natural science. Mathematics concerns the mathematical possibility of the empirical concepts of natural science as well as what can be proven a priori on the basis of this bare mathematical possibility. The a priori cognitions that are grounded by natural-scientific concepts' being mathematically constructible are formally grounded, not logically or materially. So the mathematics of corporeal nature has to do with the mathematical possibility of concepts of bodies (especially motion and matter) and what they formally ground. This, however, secures neither the full real possibility of material concepts nor codifies what they materially ground; such are rather the contributions of the metaphysics of corporeal nature.

3. The nature of metaphysics and the metaphysics of nature

3.1 Existence, nature and material grounding

Whereas mathematics is concerned with the mere (mathematical) possibility of objects and what may be proven based thereupon, metaphysics of nature pertains to the

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existence of things and relations of existence. Thus, in the Preface to the *Foundations*, Kant describes the necessity of metaphysics for proper natural science as follows.

> Properly so-called natural science presupposes, in the first place, metaphysics of nature. For laws, that is, principles of the necessity of that which belongs to the *existence* of a thing, are concerned with a concept that cannot be constructed, since existence cannot be presented a priori in any intuition. (*MFNS*, 4: 469, my emphasis)

Understanding the role of metaphysics vis-à-vis proper natural science requires first elucidating Kant's conception of *existence*. Existence, or actuality, is a category of modality. As Kant writes, 'That which is connected with the material conditions of experience (of sensation) is actual', or exists (A218/B266). Kant is clear that the material of experience is perception, or sensation with consciousness, but hastens to add that this does not entail that the only actual things are those that are directly perceived. Hence, he writes, actuality requires 'not immediate perception of the object itself the existence of which is to be cognized, but still its connection with some actual perception in accordance with the analogies of experience, which exhibit all real connection in an experience in general' (A225/B272).[20] So to exist is to be connected with other perceptions in accordance with the Analogies of Experience, or by means of the categories of relation: it is to stand in relations of substance and accident, cause and effect, or community with other things. Something is thus actual, or exists, insofar as it belongs to the causal nexus of nature. Stang helpfully distinguishes the Kantian notion of existence from that of bare being, rightly noting that, for Kant, existence is '[b]eing causally efficacious, either as a substance or the accidents inhering in a substance due to its interactions with other substances' (Stang 2016: 238). Consequently, whereas mathematics considers something just as conforming with the forms of intuition and the category of <quantity>, metaphysics especially deals with 'principles of that which belongs to the *existence* of a thing', meaning the laws that govern the robust existence of the thing under the categories of relation.[21]

This overview accords with Kant's earlier definition of 'nature', which opposed that of 'essence', discussed above. A nature of a thing, for Kant, is 'the first inner principle of all that belongs to the existence of [the] thing' (*MFNS*, 4: 467). With existence understood as described above, this means that the nature of a thing is the first, most fundamental principle governing its causality. Furthermore, this entails that natures of things are a subject of the metaphysics of nature: because metaphysics deals with 'principles of the necessity of that which belongs to the *existence* of a thing' it must be concerned specifically with the 'first' such principle. Finally, for these reasons, the metaphysics of nature also regards relations of material grounding, insofar as such grounds are 'of the *existence* of [a] thing'. Thus, the metaphysics of nature concerns the existence of things (their being under the categories of relation), what such existence grounds (material grounding) and the first principle of this grounding (a nature).

3.2 Analysis and the projects of the **Foundations**

Yet Kant is clear that there is also an analytic dimension of this metaphysics. To wit, in the preface of the *Foundations*, he declares that an analysis of the concept of

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<matter> is requisite for the application of mathematics to matter and a part of the metaphysics of corporeal nature.

> But in order to make possible the application of mathematics to the doctrine of body, which only through this can become natural science, principles for the *construction* of the concepts that belong to the possibility of matter in general must be introduced first. Therefore, a complete analysis of the concept of matter in general will have to be taken as the basis, and this is a task for pure philosophy – which, for this purpose, makes use of no particular experiences, but only that which it finds in the isolated (although intrinsically empirical) concept itself, in relation to the pure intuitions in space and time, and in accordance with laws that already essentially attach to the concept of nature in general, and is therefore a *genuine metaphysics of corporeal nature*. (*MFNS*, 4: 472)

What does an analysis of matter have to do with the construction of the concepts belonging to the possibility of matter? Fortunately, Kant subsequently describes the relevant dimension of the analytic project of the *Foundations*, writing that 'The understanding traces back all other predicates of matter belonging to its nature to [motion], and so natural science, therefore, is either a pure or applied *doctrine of motion*' (*MFNS*, 4: 476–7). The crucial analytic project of the metaphysics of corporeal nature is relating the relevant concepts to motion, for, as mentioned above, <motion> is the mathematically constructible concept of the proper natural science of body. It is through the logical analysis of concepts relating to matter, by means of which their connection with motion is discovered, that they become mathematically constructible. This is part of how the metaphysics of corporeal nature makes possible the application of mathematics to body.[22]

So, putting together the various considerations of the article up to this point, there are three relevant ways of considering things – in particular, corporeal things – that correspond to the three main projects of the *Foundations*.

Consideration	First principle	Mode of grounding
Principles of all that belongs to the (logical) possibility of a thing	Essence	Logical
Principles of all that belongs to the (mathematical) possibility of a thing	Essence	Formal
Principles of all that belongs to the existence of a thing	Real Essence/ Nature	Material

The first column describes the different ways of considering a thing as a basis for further cognition: that is, how the thing, as possible or as actual, grounds knowledge about it or other things. In a logical context, one may consider what else must be the case given the logical possibility of a thing; in mathematics, one considers

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mathematical possibility; and, in metaphysics, one considers the existence of the thing, understood robustly as its being in the causal nexus of the world. For each of these approaches to considering a thing, there are corresponding first principles, which are referenced in the second column. Such first principles, being fundamental, ground the other knowledge under each mode of consideration. So, the logical essence of a thing – the marks that make up its concept – grounds all that belongs to the logical possibility of the thing. Mathematical consideration of a thing does not involve a different first principle – Kant is clear that mathematics considers the essences of things – yet, mathematics deals differently with essences. Whereas a merely logical, or philosophical, consideration of an essence simply discloses what is analytically contained in the concept – recall the philosopher's attempt to prove Euclid's proposition 32 in book 1 of the *Elements* via analysis, discussed in the first *Critique* (A716–17/B744–5) – mathematics involves exhibiting the essence in intuition and inferring synthetic a priori truths therefrom. So logic and mathematics do not involve different first principles, but only different approaches to them. Metaphysics, for its part, concerns not the mere possibility of a thing but rather its actuality and what is the case based on this actuality. As such, the metaphysical consideration of a thing takes a real essence, or nature, the 'first principle of all that belongs to the existence of a thing' (*MFNS*, 4: 467), as its basis. Finally, to each manner of regarding a thing there also corresponds a particular mode of grounding, each of which is described in the third column. When considering all that belongs to the logical possibility of a thing, one regards that thing as a logical ground. Likewise, when one considers it as mathematically possible, one regards it as a formal ground. A thing is regarded as a material ground when one examines the principles of its existence. In this last case, one is especially concerned with the thing *as a cause*.

To clarify the metaphysical consideration of matter – that is, matter *as existing* – it is incumbent to understand what it means for matter to be causally efficacious. What is the *nature* of matter, the first principle for that which belongs to its existence? Insofar as existence is being a member of the causal nexus, the first principle for matter's existence must be its fundamental causal power. In the Dynamics chapter of the *Foundations*, Kant argues that two fundamental, moving forces are essential to matter: the fundamental repulsive force, through which matter diminishes the motion of matter entering its space, and the fundamental attractive force, through which it accelerates other bodies toward it.[23] These forces serve as the first principle of the existence of matter, the basis for its causal influence on the world: 'Therefore, only these two kinds of forces can be thought, as forces to which all moving forces in material nature must be reduced' (*MFNS*, 4: 499; see also 4: 523, 532). The moving forces comprise the nature of matter; the metaphysical project of studying all that belongs to the existence of material things comes down to reducing or explaining natural phenomena by means of these fundamental forces.[24]

To sum up, there are two main metaphysical projects in the *Foundations*. First, there is an analytic project that seeks to reduce the moments of matter to motion, which makes possible their mathematical construction. Second, there is a project of clarifying what matter really grounds. Specifically, this project aims at deriving laws regarding matter's nature (causality) from application of the categories. The determination of matter by the categories generates propositions governing matter's moving force that necessarily connect certain perceptions of matter, particularly of bodies accelerating

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and decelerating.[25] For example, the perception of my desk is necessarily connected to the perception of the motion of the bouncy ball after striking the table, insofar as the nature of the desk – in particular, its possession of the fundamental force of repulsion – causes the ball to change its motion. In service of this metaphysical project, Kant discusses the different manifestations of moving force in the Dynamics chapter, providing accounts of the necessity of the fundamental attractive and repulsive forces for the existence of matter, and proceeds to examine the implications of these forces for the communication of motion in the Mechanics chapter.

4. Mathematics and metaphysics together (and apart)

As referenced above, Kant writes that mathematics and metaphysics, despite their fundamental differences, must form a 'sisterly union'. At this point, it may be difficult to conceive such a union, so fundamental are the divisions between metaphysics and mathematics. The metaphysics of corporeal nature pertains to the logical analysis of concepts belonging to matter's existence, matter's real essence (nature) and its expression of causal powers. In contrast, mathematics produces synthetic a priori cognition about motions via construction, and such construction concerns relations of formal grounding. Mathematics particularly tells us that, given composite motions of a particular sort, the compound motion that they produce has other particular qualities (by means of the parallelogram law). What, then, are the implications of the preceding account of the mathematics and metaphysics of corporeal nature for the relation and union between the two?

First, mathematics of corporeal nature is an appendage, of sorts, of its metaphysical counterpart. Mathematics concerns something distinct from metaphysics: namely, relations of formal grounding, but its application to natural science in general depends on the analytic project of metaphysics. Motion is the constructible concept of physics; to apply more broadly to physics, other properties of bodies need to be reduced. Metaphysics and mathematics form a union insofar as they both concern motion, and, more importantly, the boundaries of mathematization of body are just those of the metaphysics of body, insofar as motion is matter's fundamental determination. Second, mathematics and metaphysics make fundamentally distinct contributions to proper natural science. Mathematics can provide a priori knowledge of formal grounding, whereas metaphysics has to do with material grounding. Strictly speaking, casual relations – considered in metaphysics and described by laws of nature – are *not* mathematically constructible: 'principles of the necessity of that which belongs to the *existence* of a thing, are concerned with a concept that cannot be constructed, since existence cannot be presented *a priori* in any intuition' (*MFNS*, 4: 469). Causal and mathematical relations are essentially distinct. Indeed, it is something of a category error to think that causal relations, those of material grounding, *can* be mathematized.[26]

Finally, this account makes sense of the relative disregard for mathematics in the body of the *Foundations*. Although Kant underscores the importance of mathematical construction for a priori knowledge of bodies in the Preface, mathematical content comes up rather sparingly throughout the rest of the book. In fact, Kant offers only two mathematical constructions in the book: of composite motion in the Phoronomy (*MFNS*, 4: 490–3) and of the communication of motion in the Mechanics (*MFNS*, 4:

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544–7). Furthermore, he stops short in the Dynamics chapter of the book, declining to endorse a technique for mathematical construction of the filling of space definitively. After arguing that the fundamental forces of repulsion and attraction are essential to matter's filling of space,[27] he deems the presentation of a mathematical construction inessential to his goals.

> And thus the dynamical concept of matter, as that of the movable filling its space (to a determinate degree), would be constructed. But for this one needs a law of the ratio of both original attraction and repulsion at various distances of matter and its parts from one another, which … is a purely mathematical task, which no longer belongs to metaphysics – nor is metaphysics responsible if the attempt to construct the concept of matter in this way should perhaps not succeed. (*MFNS*, 4: 517–18)

Hence, in the Dynamics chapter, specifically, and in the *Foundations*, as a whole, Kant apparently sidelines mathematics from his considerations. This stands in tension with his strong rhetoric about the importance of mathematics for proper natural science, according to which 'a pure doctrine of nature concerning *determinate* natural things (doctrine of body or doctrine of soul) is *only* possible by means of mathematics' (*MFNS*, 4: 470).[28]

Fortunately, my account of metaphysics, mathematics and the relation between the two straightforwardly clarifies why for Kant metaphysics is not 'responsible' for the task of constructing the dynamical concept of matter. Since mathematics and metaphysics fundamentally concern different sorts of grounding, mathematical construction and its relations of formal grounding are not topics that belong to or are relevant to metaphysics. In its logical moment, metaphysics makes possible the application of mathematics to the filling of space by explicating it in terms of motion – as diminishing penetrating motions. But the details of the formal grounding, the way in which the quantity of the diminution of penetrating motion is related to other quantities, such as the volume of a body, are not concerns of the metaphysics of corporeal nature.

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Notes

1 I use the following abbreviations for Kant's writings: *Corr* = *Correspondence*, *CPrR* = *Critique of Practical Reason*, *MFNS* = *Metaphysical Foundations of Natural Science*, Met-Herder = Herder Metaphysics, Met-L_2 = Metaphysics L_2, Ph-Berliner = Berlin Physics, PM = 'What Real Progress Has Metaphysics Made in Germany since the Time of Leibniz and Wolff?'. Citations of the *Critique of Pure Reason* refer to the standard A/B pagination. All other citations of Kant's works refer to the Akademie edition volume number and page number(s) (Kant 1900–). Where available, English translations from the Cambridge Edition of the Works of Immanuel Kant are used.

2 For varying accounts of proper natural science, see Watkins (1998), van den Berg (2014: 15–51), McNulty (2014), Zammito (2017) and Breitenbach (2022).

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3 See also his denials that chemistry and psychology are proper natural sciences (*MFNS*, 4: 470–1), which likewise appeal to the application of mathematics as the decisive consideration.
4 To flesh out the issues with attributing a pivotal role to metaphysical construction, consider first that the use of construction is precisely that which is supposed to *distinguish* metaphysics from mathematics; see the Discipline of Pure Reason in the first *Critique* (A714/B742). The systematic deficiencies I have in mind are the following. In the Discipline, Kant also divides up rational (a priori) cognition into three sorts. At one level, there are mathematical cognitions (from mathematical construction) and philosophical cognitions. Philosophical cognitions may be differentiated into those that are logical and those that are transcendental. This *exhaustive* division of the sorts of rational cognition puts pressure on interpretations ascribing a central role to metaphysical construction. If there is such a thing as metaphysical construction, then it must be assimilated into the framework of philosophical/metaphysical cognitions described in the Discipline. Either metaphysical construction cannot be assimilated into this framework – in which case there is lacking textual evidence and systematic justification for its existence – or it can – in which case its distinctiveness is vitiated. For more on metaphysical construction, see also Pollok (2001: 88–9) and Dunlop (2022: 70–4).
5 For more on the distinction between logical and real possibility, see Plaass (1965: 52–8) and Stang (2016: 197–9). Additional connections between these notions and the categories of modality are well outlined by Leech (2017).
6 See also the *Critique of Pure Reason* (Bxxvi, B301–2), PM (20: 325) and Plaass (1965: 56). Chignell (2010: 178–81) explains that Kant assumes their equivalence in *CPrR* (5: 134).
7 Presumably, empirical proofs of real possibility are more commonplace in *improper* natural sciences. That proper natural science aspires to a priori knowledge of natural things – i.e. knowledge from their mere (real) possibility – indicates that a priori proofs of real possibility are necessary.
8 Friedman (2013: 27) also references the essence/nature distinction in support of his interpretation.
9 For more on the prima facie impossibility of directly mathematically constructing empirical concepts, see McNulty (2014: 398–9).
10 Recall the first half of Kant's famous slogan: 'Thoughts without content are empty' (A51/B75; see also A90–1/B122–3).
11 The very same synthesis by means of which mathematical concepts are constructed is that by means of which the space and time of an object are synthesized (see the Axioms of Intuition, A162–6/B202–7; Sutherland 2005), whereby it falls under the category of <quantity>. For that reason, mathematical construction is a necessary condition of real possibility. Furthermore, it is in virtue of the unity of this synthesis that mathematics applies to objects of possible experience. But this argument only guarantees that the *space and time* of objects of experience is mathematically constructible. Guaranteeing that other, empirically determinate aspects of objects can be mathematically constructed depends upon special metaphysics, like the metaphysics of corporeal nature, as Kant outlines in the Preface of the *Foundations* (*MFNS*, 4: 472) (see n. 13).
12 The language of metaphysical construction and lack of detail notwithstanding, Washburn's interpretation resembles my own insofar as I too recognize these different components in proving real possibility (see below).
13 The proviso 'as such' is critical: objects falling under the concept <acid> are mathematically constructible *as* outer objects. In synthesizing the space of an object, like a particular acid, one brings mathematical construction procedures to bear, as Kant describes in the Axioms of Intuition (see n. 11). However, this does not mean that the concept of <acid> can itself be exhibited a priori in intuition. Rather, its *space*, say the shape of the acid in a cylindrical flask, is constructible.
14 When we speak of mathematical objects (triangles, numbers, icosahedra, Archimedean spirals and so forth), we mean to refer to them only as pure representations of space and time, in abstraction from empirical objects that bear relations of existence. So, although real possibility, in general, refers to the *existence* of empirical objects, in the context of mathematics, a mathematical object is really possible in the case that it conforms to the forms of pure intuition. Thanks to an anonymous referee for pressing me on this.
15 A mistake in the Cambridge edition version of this passage, corrected in the above translation, obscures Kant's otherwise obvious reference to the triangle congruence theorem (Kant 1999: 299). The phrase 'den Grund der Winkel' should be translated as 'the ground of the angles' instead of as 'the ground of the angle', the latter of which Zweig uses.

16 For more detailed treatments the concept of <ground>, which delve into many knotty issues, see Stratmann (2018) and Stang (2019). For analyses of the elusive notion of 'positing', see Stang (2019) and Sala (2020).
17 For legibility and flow, in the following, I mostly refer to such grounds as 'formal grounds' and 'material grounds', omitting the reference to their being real.
18 Indeed, mere mathematical figures can neither possess nor be material grounds, as such are grounds of existence, and 'in [geometrical figures'] concept nothing is thought that would express an existence' (*MFNS*, 4: 467n.).
19 For another reading of the Reinhold letter and the distinction between formal and material grounds, see Stratmann (2018: 5–7). Although his interests primarily lie with material grounds, I agree with Stratmann that formal grounds are *partial* grounds of (real) possibility of appearances.
20 Leech (2017) discusses Kant's conception of material conditions in his broader theory of modality and examines the idea of the sum total of empirical reality as, itself, a material condition of experience. Leech's careful and illuminating analysis aside, in the context of natural science and laws understood as principles relating to the existence of things, the global perspective of the sum total of empirical reality can be passed over.
21 That said, in the following, I especially focus on causality as a relation of existence.
22 For more on Kant's argument for the fundamentality of motion to the concept <matter>, see McLear (2018).
23 In a few places, Kant claims that the moving forces are 'essential' to matter or belong to its 'possibility' (see *MFNS*, 4: 511; Ph-Berliner, 29: 75). That the moving forces belong to the logical essence or possibility of matter does not preclude that they also belong to the real essence or existence of matter.
24 For more on reduction of the specific variety of matter to the fundamental forces, see McNulty (2022).
25 For this reason, such a proposition is 'transcendental', that is, a 'rule[] of the synthesis of perceptions' (A722n./B750n.). Expounding on the transcendental aspect of these metaphysical propositions would take this article far afield, but a related account is Watkins' (1998) 'Transcendental Argument Interpretation' of the *Foundations*.
26 Dunlop's (2022) recent account of the relation between metaphysics and mathematics of corporeal nature is broadly consonant with that on offer in this article, although she does not draw such a bright line between the doctrines. For Dunlop, Kant intends for his metaphysics of corporeal nature to rule out those alternative metaphysical pictures that create obstacles for the mathematization of nature. So the role of metaphysics is primarily to clear the way for mathematization. The principled division of tasks and responsibilities of metaphysics and mathematics accords with my interpretation.
27 Kant argues for the essentiality of these forces by means of the so–called 'balancing argument'. See Warren (2010), Smith (2013) and Friedman (2013: 180–94).
28 The warrant for Kant's excusing himself of constructing the fundamental forces is a matter of much scholarly disagreement. For two recent interventions, see Dunlop (2022) and Warren (2022).

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