ABSTRACT OF THESIS

Roy Westwick, Linear transformations of a Grassmann product space, presented at the University of British Columbia, March 1960 (Supervisor, M.D. Marcus).

The objective of this thesis is to determine the linear transformations of a Grassmann product space which send the set of non-zero Grassmann product vectors (also called pure vectors) into itself.

Let U be an n-dimensional vector space over a field F and let r be an integer such that $0 \leq r \leq n$. The r-th Grassmann product space will be denoted by $A_r(U)$. Subspaces of $A_r(U)$ consisting entirely of pure vectors are called pure subspaces.

With each non-zero pure vector of $A_r(U)$ we associate an r-dimensional subspace of U. By studying the set of subspaces of U corresponding to a basis set of a pure subspace of $A_r(U)$ we are able to deduce the form of this pure subspace. In this way we are able to classify the pure subspaces of $A_r(U)$, arriving at only two essentially distinct types.

We next study the maximal pure subspaces, i.e. the pure subspaces which are not contained in larger pure subspaces. They are of importance because the assumptions on the linear transformations under consideration imply that a maximal pure subspace is mapped into another maximal pure subspace. The form of the transformation is now almost completely determined by examining the incidence relations between pairs of maximal pure subspaces before and after the transformation is applied. Some algebraic manipulations are then needed in order to display the form of the transformation completely.

With the suitable assumptions, our results state that the transformations under consideration are induced by linear transformations of the vector space U, except possibly when $2r = n$. When $2r = n$ two types of transformations are possible. This arises from the fact that the two types of maximal pure subspaces have the same dimensions, (unlike the situation when $2r \neq n$). One type of transformation (those induced by linear maps of U) does not alter the type of pure subspaces, while the other interchanges the two types.

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