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Subnormal structure in infinite soluble groups

D.J. McCaughan

The basic concepts of this thesis - those of subnormal subgroup and subnormal index - are due initially to Wielandt ([2]). Our aim is to investigate the properties of groups in some classes defined by subnormality conditions, for example the class of groups in which the subnormal indices are bounded. Associated with this are two larger and distinct classes, the first consisting of those groups which have the subnormal intersection property, that is, in which the intersection of any family of subnormal subgroups is again subnormal, the second consisting of those groups which have the subnormal join property, defined analogously.

We attempt to give some answers to two general questions.

(i) Under what restrictions will a soluble group with some subnormality condition be nilpotent?

(ii) Under what restrictions will a soluble group with the subnormal intersection property have a bound on its subnormal indices?

After an introductory chapter, the definitions and properties of the various classes are treated in Chapter 2. In Chapter 3 we present some technical results, mainly on π-radicability. Chapter 4 concerns "rank" in soluble groups and leads up to the result that an extension of a group with the minimal condition on subnormal subgroups by a group with bounded subnormal indices again has bounded subnormal indices.

In Chapter 5 we consider metanilpotent groups with the subnormal intersection property, in the context of (i). Various conditions are given

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under which such groups are nilpotent, and in the simpler cases the general structure is elucidated. As a first application of these results we prove that an abelian-by-finite group with the subnormal intersection property has a bound on its subnormal indices. The same is true of a nilpotent-by-(finite nilpotent) group, but it seems difficult to extend these two results to show that a restriction to nilpotent-by-finite groups will suffice in (ii).

In Chapter 6, using an apparently new restriction, two further theorems of type (i) are proved. From the first of these we deduce that a soluble minimax group with the subnormal intersection property has a bound on its subnormal indices. An example is constructed to show that this fails for the larger class of soluble groups of finite reduced rank.

Also included in the thesis is the joint paper [1] which applies some of the previous results to investigate and characterise nilpotent-by-(periodic nilpotent) groups with bounded subnormal indices. We show (for example) that a reduced periodic group is of this type if and only if its Sylow subgroups are all nilpotent of bounded class. Similar but weaker results hold for a group which has both the subnormal join and the subnormal intersection property. An example is given to show the incompleteness of the results in the second case.

References

[1] D.J. McCaughan and D. McDougall, "The subnormal structure of metanilpotent groups", *Bull. Austral. Math. Soc.* 6 (1972), 287-306.

[2] Helmut Wielandt, "Eine Verallgemeinerung der invarianten Untergruppen", *Math. Z.* 45 (1939), 209-244.