## Book reviews

over B, and the inverse images  $p^{-1}(b)$  for  $b \in B$  are called fibres. The basic idea is to carry out constructions on fibres individually; for example, if X and Y are fibrewise spaces with maps  $p: X \to B$  and  $q: Y \to B$ , then their fibrewise product is a space  $X \times_B Y$  with a map  $r: X \times_B Y \to B$  such that

$$r^{-1}(b) = p^{-1}(b) \times q^{-1}(b)$$

for each b in B. Some things can be done quite mechanically, but not many; for example, in the case of the fibrewise product, one has to choose a suitable topology for  $X \times_B Y$ .

The subject originated with the study of fibre bundles; these are spaces X with maps  $p: X \rightarrow B$  such that p is locally like the projection of a cartesian product onto a factor. This makes nearby fibres homeomorphic in a coherent way. For example, if X is a space on which a topological group G acts freely, then X is often a fibre bundle over the orbit space X/G. Fibrewise homotopy theory works much more generally, but for many significant results the spaces must still be something like fibre bundles. Unfortunately, this excludes some important spaces with singular fibres which come from algebraic geometry; for this reason, the authors hope that the theory can be developed further.

The book is written for readers who are familiar with ordinary homotopy theory. It is in two parts; the first part gives a general introduction, while the second part covers fibrewise stable homotopy theory. There is a lot of information, although the authors concentrate on applications rather than foundations. In particular, in the second part, they omit most of the elaborate machinery of spectra and infinite loop spaces, and they deal only with suspension spectra. The applications in the second part include fixed point indices, duality, transfer, manifolds, and homology. The highlights are a previously unpublished proof of the Adams conjecture and a fibrewise proof of Miller's stable splittings of the unitary groups.

One possible approach to fibrewise topology would be to start with ordinary topology, generalize it systematically, and see what happens. This was the approach taken by James in an earlier work [1]. The present book is more interesting but also more difficult. The details of arguments are often omitted; this makes it easy to see what is going on but requires a lot of work for a conscientious reader. The subject can be rather confusing, because there are many similar concepts with similar names. Sometimes the same name is used for different concepts: there are two different definitions for a fibrewise manifold. The authors are generally very helpful; they give lots of definitions and make careful distinctions. The book is interesting to read through, and it should also be useful for reference.

R. J. STEINER

## References

1. JAMES, I. M., *Fibrewise topology* (Cambridge Tracts in Mathematics 91, Cambridge University Press, 1989).

FERES, R. Dynamical systems and semisimple groups: an introduction (Cambridge Tracts in Mathematics 126, Cambridge, 1998), xvi + 245 pp., 0 521 59162 7 (hardback), £35 (US\$54.95).

The term *dynamical systems* has come to cover a wide range of topics and approaches. But in most of these approaches Lie groups play a prominent part, whether as symmetries in the case of integrable, Hamiltonian systems or, more generally, as the transformations parametrized by the (time) evolution parameter itself. In the present text we are dealing with the interplay between group representations and ergodic theory, the actions of semisimple Lie groups on manifolds with finite, group invariant measure. In accordance with the general philosophy of modern nonlinear dynamics one has first to understand the linearization of the action along its orbits and then to ask how the linearization helps one to understand the global structure. The book deals essentially with the first part of this programme, culminating in the rigidity theorems of Margulis and Zimmer.

After a basic introduction to topological dynamics and ergodic theory, which assumes knowledge of measure theory, the text launches into an introduction to Lie theory and group actions on smooth manifolds. This is couched in the language of differential geometry and culminates in some representation theory for semisimple Lie groups. I would have said this is a bit brisk for a graduate student (at whom the book is aimed) without an acquaintance with Lie theory and would need supplementing with a more thorough treatment. A knowledge of differential geometry up to principal bundles, for which a brief introduction is provided, is assumed.

After this the book moves on to its true goal with further discussion of ergodic theory, the Moore and Birkhoff theorems, Anosov systems and a proof of the Oseledec theorem. In the final chapter the rigidity theorems of Margulis and Zimmer appear.

An important feature of the text is the inclusion of many exercises, always a helpful thing. But the author might have provided some hints or, occasionally references, for their solution.

## C. ATHORNE

## GOLDMAN, W. M. Complex hyperbolic geometry (Oxford Mathematical Monographs, Clarendon Press, 1999), xx + 316 pp., 0 19 853793 X, £65.

The unit ball in  $\mathbb{C}^n$ , and complex manifolds for which this is the universal cover, have been studied from many points of view ranging from complex analysis to algebraic geometry. Until recently the literature contained comparatively little about these subjects from a purely geometrical viewpoint. After an auspicious beginning with major work of Picard, Giraud and Élie Cartan, the geometrical side of the subject fell into decline. A revival of interest began about 25 years ago with major contributions by Chen, Greenberg, Mostow and others. This resurgence has been intensified over the last 10 years, largely inspired by Goldman's interest. This makes the publication of the book under review very timely as well as an invaluable guide to the recent developments.

The book is a monumental result of an investigation of complex hyperbolic geometry conducted over more than a decade. It contains a wealth of useful, beautiful and intriguing facts that the author has discovered during this study. Many of these results are of fundamental importance for those studying complex hyperbolic geometry. As well as pioneering new areas of the subject, the book is anchored into the existing literature (for example, it contains a commentary on Giraud's seminal paper). There is a long bibliography and references for further reading are provided throughout the text. There are many different conventions and systems of notation in the literature. This potential source of confusion is minimized by Goldman, who fixes conventions and notation throughout. It is to be hoped that (unless there are clear reasons for not doing so) writers of future papers and books in this subject will either adopt Goldman's notation or else provide a clear means of translating between their conventions and his.

The unit ball in  $\mathbb{C}^n$  has a natural metric of constant negative holomorphic sectional curvature, called the Bergman metric. As such it forms a model for *complex hyperbolic n-space*  $\mathbf{H}^n_{\mathbb{C}}$  analogous to the ball model of (real) hyperbolic space  $\mathbf{H}^n_{\mathbb{R}}$ . The main difference is that the (real) sectional curvature is no longer constant, but is pinched between two negative numbers whose ratio is 4. Goldman normalizes so that the holomorphic sectional curvature is -1 which means that the sectional curvatures lie in the interval [-1, -1/4]. (There seems to be no consensus about which interval to take. Different choices lead to awkward factors of 2 or 4 in various key places.) The geometry of  $\mathbf{H}^n_{\mathbb{C}}$  is not a completely straightforward generalization of  $\mathbf{H}^n_{\mathbb{R}}$ . Aspects such