B. COSMIC RAY METHODS OF EXPLORING INTERPLANETARY SPACE

PAPER 36

THE 27-DAY VARIATION IN COSMIC RAY INTENSITY AND IN GEOMAGNETIC ACTIVITY

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ABSTRACT

The amplitude of the average 27-day wave in cosmic ray intensity, at Huancayo, Peru, and its phase relative to that for the 27-day wave in international magnetic character figure (ICF) is determined from results of harmonic analysis of data for each of 246 intervals (or solar rotations) of 27 days. From these data, the variability of which is essential for tests of statistical significance, the amplitude of the average 27-day wave in cosmic ray intensity and its phase relative to that in geomagnetic activity is determined for each of three groups of solar rotations selected according to the average of the amplitudes of the 27-day waves in magnetic activity. A fourth group contained only 27-day intervals in which large cosmic ray decreases occurred. Relative to that in magnetic activity, the phase of the 27-day wave in cosmic ray intensity is found for the averages, to be the same for the four groups.

The maxima of the average cosmic ray waves occur about 1·5 days after the minima of the corresponding waves in ICF. In general, the amplitude of the average 27-day wave in cosmic ray intensity, in the co-ordinate system in which its phase is relative to that of the 27-day wave in ICF tends to be greater for the selected groups of rotations with larger average ICF amplitudes. For most years near sunspot minimum the amplitude of the 27-day cosmic ray wave does not differ significantly from zero.

Bartels found for 27-day waves in ICF the effective number of statistically independent 27-day waves for N successive solar rotations to be $N/3$; the number found for cosmic ray intensity is $N/2$. Thus, on the average the 27-day recurrence tendency is less for cosmic ray intensity than for magnetic activity.

https://doi.org/10.1017/S0074180900237923 Published online by Cambridge University Press

1. INTRODUCTION

Probably all of the established variations in cosmic ray intensity are in some way connected with solar activity. The large sudden decreases of cosmic ray intensity occur during magnetic storms although storms without decreases in cosmic ray intensity often occur [1]. The cosmic ray intensity averaged for the five magnetically disturbed days of each month is nearly always less than that for the five magnetically quiet days of the month. These facts imply a close connexion between the mechanism responsible for magnetic storms and geomagnetic activity and that for many of the variations in cosmic ray intensity. Morrison [2] has proposed one ingenious mechanism to explain many of the variations of cosmic ray intensity based on the hypothesis that the solar streams responsible for magnetic storms and magnetic activity generally contain a low density of cosmic rays, so that a decrease of cosmic ray intensity may be observed when the earth is inside the stream. The relation between the 27-day waves in magnetic activity and those in cosmic ray intensity should provide information of value to theories for the time variations.

Since our first [3] investigation of the relation between the 27-day variations in magnetic activity and cosmic ray intensity, the amount of data has greatly increased. As a measure of geomagnetic activity the international character figure [4] is used. This measure is chosen mainly because Bartels, in a famous paper [5] made a classic investigation of the waves in magnetic activity based on daily international magnetic character (ICF) figures for 378 solar rotations of 27 days each, beginning 11 January 1906.

2. DATA AND METHOD OF ANALYSIS

The basic ICF data were the daily mean ICF figures for each Greenwich day for the period 13 June 1936 through December 1955. The 27-day waves for the ICF data were determined by a six ordinate scheme for harmonic analysis using differences [6] to eliminate non-cyclic change. The six ordinates used were means for alternate sequences of 4 and 5 days for each 27-day interval.

The cosmic ray data from Compton–Bennett meters [1] at Huancayo (starting 13 June 1936) and at Cheltenham (starting 6 April 1937) were similarly analysed to provide, for each station, a set of harmonic coefficients: A and B for the 27-day waves; similarly, the analysis of the ICF data provided another set a and b, for corresponding 27-day intervals. In the 27-day harmonic dial for ICF each pair of coefficients a and b defines

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the end-point of a vector **c**, the length, $c=\sqrt{(a^2+b^2)}$, of which is the amplitude of the 27-day wave. The time of maximum of the wave is indicated by the number on the scale on the periphery of the harmonic dial [5] to which the vector points.

To determine the phase of 27-day waves in cosmic ray intensity relative to that for ICF each pair of coefficients A and B for cosmic ray intensity, CRI, is transformed (by rotation of the axes) to the pair A_R and B_R referred to a set of axes in which the vector **c** for ICF for the same 27-day interval has its time of maximum at zero. This co-ordinate system is designated the rotated co-ordinate system (RCS), or by a subscript R, to distinguish if from the original co-ordinate system (OCS).

3. STATISTICAL PARAMETERS FOR THE VARIABILITY OF THE 27-DAY WAVES

Fig. 1 shows the harmonic dials for the 27-day waves in cosmic ray intensity in the rotated co-ordinate system (RCS) for three different ranges in amplitude for the 27-day waves in ICF. The starred point on the vertical axis, in each of the three dials, indicates by its distance from the origin the average of the ICF amplitudes for that group. Table 1 indicates the number of rotations, n, for each of the three dials, together with the average of the ICF amplitudes, $\overline{A}_R$, $\overline{B}_R$, and the amplitude $\overline{C}_R$ of the average CRI waves in the RCS. The average $\overline{C}_R$ vector is shown for each dial.

The means, standard deviations, and other parameters, determined for the distributions, in the rotated co-ordinate system, for Fig. 1 (as well as for Figs. 2 and 3) are indicated in Table 1. In Table 1 the standard deviations, s_{A_R} and s_{B_R} of A_R and B_R are about their means (in Figs. 1, 2 and 3, A_R is plotted on the vertical axis and B_R on the horizontal axis). It will be noted from the first four rows of Table 1 that the standard deviation, s_{B_R}, of B_R is in every case somewhat greater than that of A_R which would suggest that the two-dimensional frequency distributions for the points in Figs. 1, 2 and 3 may be slightly elliptical. However, using the results from line four of Table 1 derived from 230 rotations, the ratio of the variance of B_R to that for A_R is 1·31. If samples of 230 statistically independent pairs A_R, B_R are drawn from populations with the same variance, then in about 5% of such samples the ratio of the variance would equal or exceed 1·31. Thus, the difference in the variances is barely statistically significant at the 5% level and the distributions may for practical purposes be regarded as circularly symmetric, with

$$M=\{S^2_{A_R}+S^2_{B_R}\}^{1/2}$$

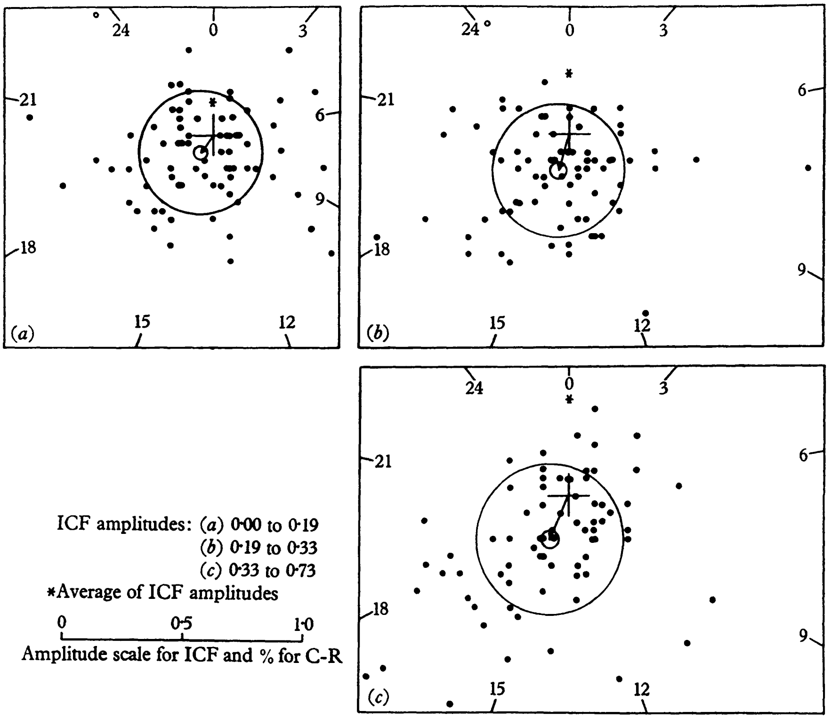


Fig. 1. Harmonic dials 27-day waves cosmic ray intensity (C–R), Huancayo (1936–54) phases relative to those for waves in international character figure (ICF) rotations with large C–R storm effects excluded.

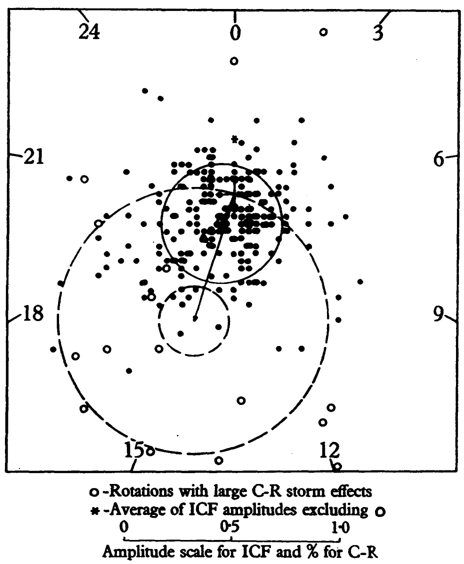


Fig. 2. 27-Day harmonic dial using data pooled from Fig. 1 (a), (b) and (c) and including 16 rotations with large C–R storm effects.

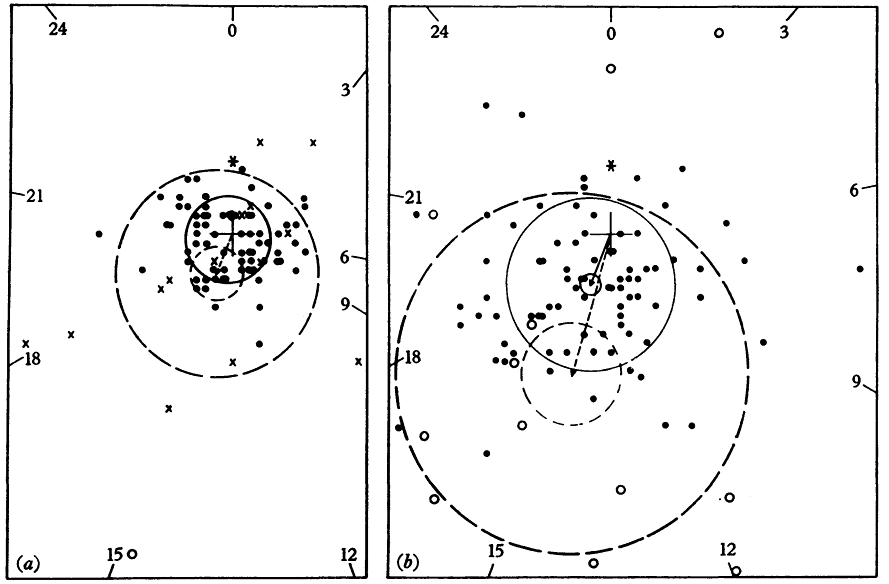
Table 1. *Data for 27-day harmonic dials in Figs. 1, 2 and 3*

Fig. no.	No. of roations n	ICF amplitude	For cosmic ray intensity A_R %	B_R %	C_R %	s_{A_R} %	s_{B_R} %	M %	ρ %	Points inside ρ	Points outside ρ	κ	$P=e^{-\kappa^2}$	T days
1 (*a*)*	76	0·126	−0·076	−0·053	0·093	0·195	0·233	0·304	0·253	41	35	2·64	1×10^{-3}	2·6
1 (*b*)*	79	0·248	−0·155	−0·043	0·161	0·207	0·258	0·329	0·274	45	34	4·36	8×10^{-9}	1·2
1 (*c*)*	75	0·419	−0·173	−0·077	0·189	0·248	0·264	0·362	0·301	43	32	4·54	1×10^{-9}	1·8
2*	230	0·262	−0·135	−0·057	0·147	0·219	0·252	0·234	0·278	129	101	6·70	3×10^{-20}	1·7
2†	16	0·290	−0·600	−0·190	0·629	0·642	0·447	0·782	0·651	7	9	3·27	3×10^{-5}	1·3
3 (*a*)‡	14	0·300	−0·154	−0·066	0·167	0·312	0·347	0·467	0·389	8	6	1·61	8×10^{-2}	1·8
3 (*a*)§	75	0·254	−0·024	−0·010	0·026	0·127	0·155	0·200	0·167	40	35	0·12	$9{\cdot}8\times10^{-1}$	1·8
3 (*b*)*	82	0·258	−0·199	−0·080	0·214	0·250	0·304	0·394	0·328	46	36	4·93	4×10^{-11}	1·6
3 (*b*)†	12	0·270	−0·534	−0·154	0·556	0·693	0·449	0·827	0·689	5	7	2·33	5×10^{-3}	1·2

* Excluding intervals with large C–R storm effects. † Only intervals with large C–R storm effects. ‡ 1952 only. § All except 1952.

as the parameter governing the distribution. The radius of the so-called probable error (p.e.) circle is given [5] by 0·833 M. From Table 1 it will be seen that in most cases a few more points lie inside the circles than outside, but the difference in no case is statistically significant, based on the χ-square test.

A more important point is whether in the rotated dial the points for successive rotations are statistically independent. To test this point the



x From 1952 only   o Rotations with large C–R storm effects   * Average of ICF amplitudes excluding x and o

0   0·5   1·0

Amplitude scale for ICF and % for C–R

Fig. 3. 27-Day harmonic dials C–R intensity Huancayo, Peru, phases relative to those for ICF: (*a*) near SS minimum, (*b*) near SS maximum.

sample of 230 rotations (plotted in Fig. 2 omitting the open circles) was used. The standard deviation, $s(1)$, of individual values of B_R from the mean was computed as well as the standard deviation, $s(h)$, for means of h chronologically successive values of B_R with $h=5$, 10, and 15. The characteristic [5] $c(h)=s(h)\sqrt{(h)}/s(1)$ was determined for the above values of h. The resulting values of $c(h)$ were essentially constant and independent of h. Thus [5] these successive values of B_R are statistically independent, and their standard deviation $s(n)$ for means of n is reliably given by $s(n)=s(1)/\sqrt{(n)}$.

A similar test for successive values of A_R (plotted as the vertical co-ordinate in Fig. 2) showed successive values of A_R were definitely not statistically independent, since for these, the characteristic $c(h)$ increased with $\sqrt{(h)}$ with no indication of reaching an asymptotic value (such as is indicated for example in Fig. 6). A plot of means of A_R for 5 successive, non-overlapping rotations, showed these means tended definitely to cluster in groups I and II, around two quite different average values which were: $-0{\cdot}193\%$ and $-0{\cdot}025\%$ respectively for 150 and 75 rotations. The smaller values of A tended to occur roughly near the two sunspot minima and the others near the maxima. This fact is also indicated by the data in rows two and three from the bottom of Table 1, from which it is evident that the average cosmic ray vector for the 75 rotations near sunspot minimum does not differ significantly from zero; whereas the average for the 82 rotations near sunspot maximum is eight times larger.

The value of the variance of the A_R in group I about the mean for group I, pooled with the variance of the A_R in group II about the mean for group II resulted in a standard deviation (s.d.) of $0{\cdot}206\%$ for A_R. It will be noted that this is only slightly less than the s.d. of $0{\cdot}219\%$ for A_R in line four of Table 1 for which the deviations were from a single mean $(-0{\cdot}135)$ for all 230 rotations.

Next, the standard deviation for deviations of A_R (for single rotations) was computed with the deviation of each A_R in group I measured from the mean $(-0{\cdot}193\%)$ for group I, and with the deviation of each A_R in group II measured from the mean $(-0{\cdot}025\%)$ for group II. The successive individuals in this set of deviations were found to be statistically independent.

Since successive values of B_R (from one mean) and of A_R (using two means) are statistically independent, then the expectancy, M, for successive rotations will also be statistically independent. In Table 1 it will be noted that s_{A_R} and s_{B_R} are smaller for years near sunspot minimum than for years near sunspot maximum (see second and third rows from bottom of Table 1). This indicates that M is also less near sunspot minimum. Thus the waves for the 230 rotations (for example) summarized in line four of Table 1 comprise a sample composed of two sub-samples, one from each of two populations having different means and variances. For this reason the distribution derived from the single parameter, M, provides only an approximation to the actual distribution. This accounts for the tendency for slightly more than half the points falling inside the p.e. circles. Nevertheless, the distribution of means from such composite samples, one part of which is from one population with a certain mean and variance and the remainder of which is from another population with different mean and variance, will be governed by a single M, provided the individuals from the two populations are statistically independent. Consequently, the expectancy for means of n vectors in Table 1, is obtained from the expectancy M for single vectors on division by $\sqrt{(n)}$.

4. TESTS FOR STATISTICAL SIGNIFICANCE OF AVERAGE WAVES AND RESULTS

From the results of the preceding discussion the expectancy for the averages of n waves is obtained from M, the expectancy for single waves or vectors by dividing M by $\sqrt{(n)}$. Then the ratio, κ, of the amplitude of the average vector (for example $\bar{C}_R$ in Table 1) to the expectancy [5] for the average is $\kappa = \bar{C}_R \sqrt{n}/M$. The probability, P, of obtaining an average vector as large or larger than $\bar{C}_R$, from a population with $\bar{C}_R = 0$, is given [5] by $P = e^{-\kappa^2}$.

These values of P are given in Table 1 for different samples, the harmonic dials for which are plotted in the indicated figures. The tabulated values of P indicate only two average vectors $\bar{C}_R$, both in Fig. 3 (*a*), one from 14 rotations for 1952 only, and the other from 75 rotations for the years 1942, 1943, 1944, 1945, 1953 and 1954, which are too small to be regarded as differing significantly from zero. Incidentally, the sample for Fig. 3 (*b*) with 82 rotations is derived from the years 1937, 1938, 1939, 1946, 1947, 1948 and 1949. For the remaining samples the average vector $\bar{C}_R$ is large enough to be quite definitely statistically significant. The small circles centered on the end-points of the average vectors in Figs. 1, 2 and 3 are the probable error circles for these averages. Finally, the last column of Table 1 gives the time of maximum of the 27-day CRI wave in days after the minimum of the ICF wave. To these values of T, 0·2 day should be added since daily means of ICF are for G.M.T. days and the daily mean CRI values are for 75° W.M.T. days.

Thus, on the average, the maxima and minima of the 27-day waves in CRI occur respectively about 1·9 days after the minima and maxima of the ICF waves. Earlier results [4] from a smaller sample of only 34 rotations indicated the average CRI wave to be essentially opposite in phase to the ICF wave.

Using Chree's method Simpson [7] finds that the peaks or selected *maxima* from curves of daily mean neutron intensity (during 19 months, 1 May 1951 to 30 November 1952) tend to occur about 1 *day after* a minimum in magnetic activity. To this extent his results are in approximate agreement with those obtained herein. He also found, however, that the same neutron peaks were followed after about 1 day by a maximum in magnetic activity. This differs from the result for the 27-day waves.

During those magnetic storms with large decreases in cosmic ray intensity, the beginning of the cosmic ray decrease generally occurs [1, 8] near the start of the main phase of the magnetic storm. The minimum cosmic ray intensity during the storm usually coincides with the minimum of *H*, the northward geomagnetic component at the equator. It is most likely that the minimum of *H* coincides closely with the maximum of geomagnetic activity as measured by *K* indices [8] or by ICF. Thus, for the

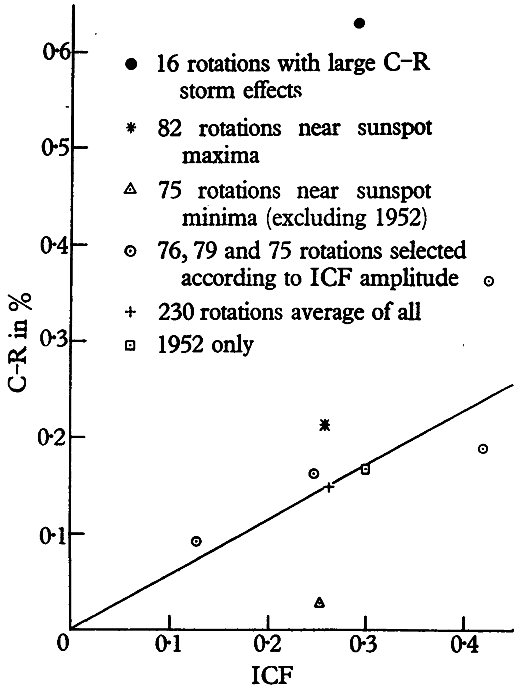


Fig. 4. Amplitude of 27-day waves in cosmic ray intensity (C–R) as function of average character-figure amplitudes (ICF)

large C–R decreases associated with some geomagnetic storms, the minimum CR intensity probably coincides with the maximum geomagnetic activity, whereas, on the average, the maximum of the 27-day waves in CRI occurs about 2 days after the minimum for ICF. This difference may possibly indicate that in geomagnetic storms the solar stream strikes the earth head on, giving rise to a maximum magnetic activity and a minimum of cosmic ray intensity within hours after the sudden commencement of the storm.

Fig. 4 indicates a plot of the amplitude, $\bar{C}_R$, of the average vector for CRI in the RCS, as a function of the ICF amplitudes (from data in Table 1).

https://doi.org/10.1017/S0074180900237923 Published online by Cambridge University Press

In a rough way the former increases with the latter excepting for rotations with large C–R storm effects for which $\bar{C}_R$, relative to the ICF amplitude, is several times greater than for any other group. Also $\bar{C}_R$ for years near sunspot minimum is relatively much smaller than for any other group.

5. THE DEGREE OF QUASIPERSISTENCE OF 27-DAY WAVES IN COSMIC RAY INTENSITY AND IN INTERNATIONAL MAGNETIC CHARACTER FIGURE

Fig. 5 shows a summation harmonic dial for 27-day waves in CRI for Huancayo from 264 rotations (vectors in the original non-rotated co-

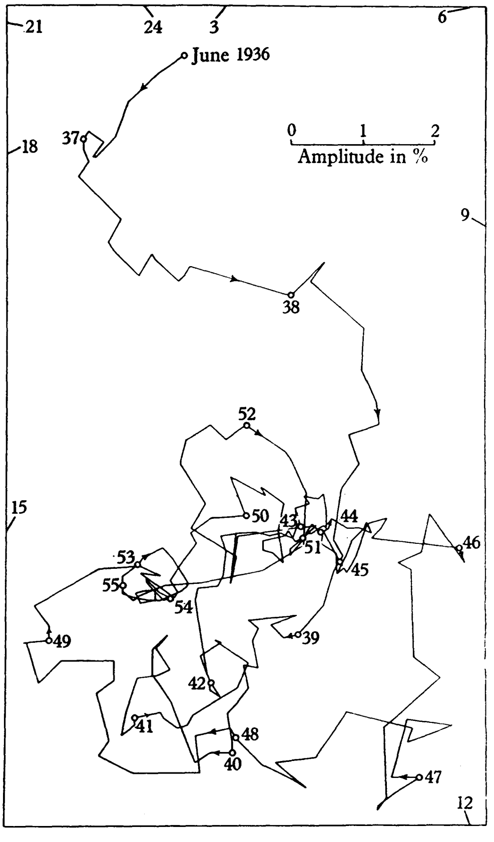


Fig. 5. Summation dial 27-day cosmic ray waves, Huancayo 1936–55.

ordinate system). Several stretches in the same direction indicating quasi-persistence are evident in the intervals: 1937–8, 1938–9, 1946–7 and 1949–50. Bartels [5] determined the degree of quasipersistence for 27-day

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waves in ICF. His results are shown by the upper curve (*a*) of Fig. 6. With $m(h)$ equal to the expectancy (or two dimensional standard deviation) for means of h successive vectors $c(h) = m(h)\sqrt{(h)}$ and $c(1) = m(1)$. $m(1)$ is simply the r.m.s. amplitude for single vectors. For statistically independent vectors $c(h)/c(1) = 1$ for all h. With increasing values of $\sqrt{(h)}$ for ICF the characteristic $c(h)/c(1) = 1$ approaches the asymptotic value of 1·74 indicating that the effective number of statistically independent vectors [5] in a sample of N is $N/1{\cdot}74^2 = 3{\cdot}0$, or that the equivalent length of sequences is 3·0 rotations. For the 27-day waves in cosmic ray intensity

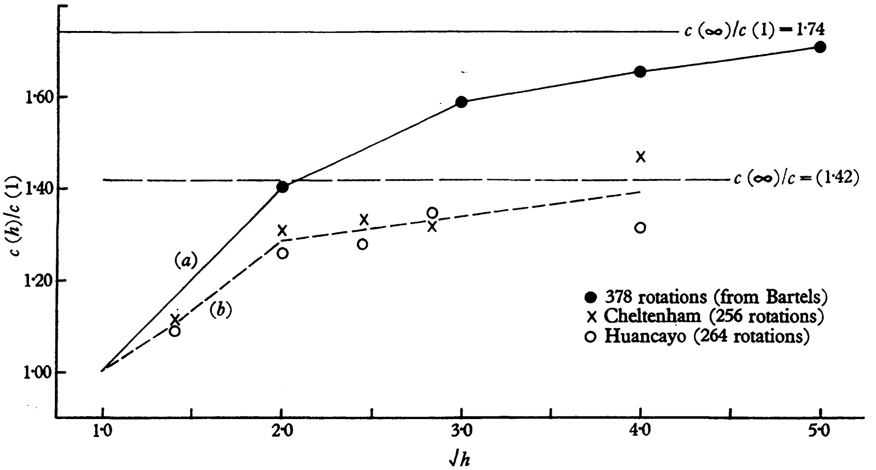

Fig. 6. Quasipersistence in 27-day waves for international character figure (*a*), and for cosmic ray intensity (*b*).

at Huancayo and at Cheltenham the characteristic is shown in Fig. 6 (*b*) with asymptote 1·41 so that 2·0 is the equivalent length of sequences for 27-day waves in cosmic ray intensity. It may be noted that Bartels characteristic for ICF (Fig. 6 (*a*)) was derived [6] from 378 rotations starting 11 June 1906. He gives the value 0·262 for the expectancy M of single vectors and shows that the arithmetic mean is $0{\cdot}886 \times M$ or 0·232 in the units of ICF. From Table 1 the arithmetic mean of the ICF amplitudes for 246 rotations is 0·264 or only about 14% greater.

6. VARIABILITY OF DAILY MEANS BEFORE AND AFTER REMOVING 27-DAY WAVES IN COSMIC RAY INTENSITY

Table 2 summarizes for Huancayo the pooled values for each year of the standard deviation s_d of CRI daily means from monthly means. The third column of Table 2 gives the r.m.s. value, n, of the amplitudes of all the

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27-day CRI waves in each year. Since the c.d. of daily means from the monthly means will differ little when pooled for the year from the yearly pooled value of c.d. of daily means from the means for 27-day intervals, then the standard deviation s_k for the residuals with the 27-day waves deducted [5] is closely approximated by: $s_k = (s_d^2 - n^2/2)^{1/2}$. A plot of s_d as function of s_k from the values in Table 2 shows that $s_k = 0{\cdot}78\, s_d$.

Table 2. *Standard deviation of daily mean CRI at Huancayo before and after removing 27-day waves; harmonic coefficients for yearly mean 27-day wave*

Year	s.d. of daily means from monthly means pooled for each year (%)	r.m.s. C-R amplitude (%)	s.d. of residuals after removing C-R wave (%)	Harmonic coefficients for yearly mean 27-day C-R wave		
				$\bar{A}_R$ (%)	$\bar{B}_R$ (%)	$\bar{C}_R$ (%)
1937	0·43	0·35	0·35	−0·09	−0·05	0·10
1938	0·67	0·61	0·51	−0·32	−0·07	0·33
1939	0·49	0·37	0·42	−0·20	−0·02	0·20
1940	0·38	0·39	0·26	−0·15	−0·06	0·16
1941	0·37	0·31	0·30	−0·21	−0·09	0·23
1942	0·39	0·41	0·26	−0·09	−0·06	0·11
1943	0·25	0·17	0·22	+0·03	−0·03	0·04
1944	0·24	0·16	0·21	−0·08	−0·01	0·08
1945	0·29	0·24	0·24	+0·04	−0·10	0·11
1946	0·83	0·69	0·68	−0·17	+0·03	0·17
1947	0·61	0·57	0·45	−0·30	−0·23	0·38
1948	0·52	0·50	0·38	−0·20	−0·15	0·25
1949	0·50	0·44	0·39	−0·10	−0·07	0·12
1950	0·38	0·36	0·28	−0·13	−0·08	0·15
1951	0·49	0·52	0·33	−0·27	−0·01	0·27
1952	0·52	0·48	0·39	−0·15	−0·07	0·17
1953	0·28	0·21	0·24	+0·01	+0·06	0·06
1954	0·21	0·15	0·18	−0·06	+0·08	0·10
1955	0·31	0·26	0·25	—	—	—

Finally, in Table 2 the harmonic coefficients A_R and B_R for CRI at Huancayo are tabulated together with the amplitude $\bar{C}_R$ of the average CRI wave.

7. ACKNOWLEDGMENT

It is a pleasure to acknowledge the invaluable contribution of Miss Isabelle Lange whose expert assistance made possible this investigation.

REFERENCES

[1] Forbush, Scott E. *J. Geophys. Res.* **59**, no. 4, 525–42, 1954.
[2] Morrison, P. *Phys. Rev.* **101**, no. 4, 1397–404, 1956.
[3] Forbush, Scott E. Transactions Washington Meeting, 1939; Internat. Union Geod. Geophys. Assoc. Terr. Mag. Electr., Bull. no. 11, 438–52, 1940.
[4] Chapman, S. and Bartels, J. *Geomagnetism* (Oxford, Clarendon Press, 1940).
[5] Bartels, J. *J. Geophys. Res.* **40**, no. 1, 1–60, 1935.
[6] Bartels, J. *Beitr. Geophys.* **28**, 1–10, 1930.
[7] Simpson, J. *Phys. Rev.* **94**, 426–40, 1954.
[8] Bartels, J. Presented at the Symposium on Geophysics, April 1956. (To be published in *Proceedings National Academy of Sciences.*)