BOOK REVIEWS

KREYSZIG, ERWIN, *Differential Geometry* (2nd ed., Mathematical Expositions No. 11, Toronto and Oxford University Presses, 1964), pp. 377, 68s.

The first edition of this valuable book appeared in 1959 and a review by Professor A. G. Walker is to be found in Vol. 12 (series II) Part 3 of these Proceedings. This second edition differs little from the first; minor corrections have been made to the text and further problems for solution have been added.

The author describes his book as an introduction to differential geometry of curves and surfaces in three-dimensional Euclidean space, and presents his material in eight chapters whose headings are as follows: I. Preliminaries; II. Theory of curves; III. Concept of a surface. First fundamental form. Foundations of tensor calculus; IV. Second fundamental form. Gaussian and mean curvature of a surface; V. Geodesic curvature and geodesics; VI. Mappings; VII. Absolute differentiation and parallel displacement; VIII. Special surfaces.

The reader will appreciate the clarity of the writing, the abundance of excellent illustrative figures and the supply of interesting problems scattered throughout the text—especially as thirty pages towards the end of the book are devoted to the solution of the latter. The author makes full use of tensor calculus, introducing the basic concepts and rules early in his initial chapter on surfaces and presenting all subsequent results in its compact notation. For the reader previously unfamiliar with tensors this is not the easiest approach to a study of surfaces, but for the reader already acquainted with tensor calculus or already familiar with the properties of surfaces the presentation could not be improved.

Chapter V contains the Gauss-Bonnet theorem and some of its corollaries. In chapter VI discussion of the properties of isometric and conformal mappings is followed by investigation of the mappings of Mercator, Lambert, Sanson and Bonne, names perhaps more familiar to the cartographer than to the mathematician. Chapters VII and VIII seem to the reviewer to be the best in the book—chapter VII for its extreme clarity of exposition and chapter VIII partly for its fascinating figures.

The printing and layout of the volume are excellent.

ELIZABETH A. MCHARG

JANS, J. P., *Rings and Homology* (Holt, Rinehart and Winston, London, 1964), 88 pp., 28s.

This monograph comprises an introduction to the structure theory of rings with minimum condition combined with an introduction to the homological algebra of modules. Briefly, the idea behind the monograph is to use the concepts of homological algebra to motivate and develop the structure theory of such rings and then to consider the homological algebra in more detail. After an introductory chapter, semi-simple rings with minimum condition are introduced as those rings R with the property that every R-module is projective (or, alternatively, injective). From this definition, it is deduced in fifteen pages that any such ring is the direct sum of matrix rings over division rings and this decomposition is unique. This chapter is a masterpiece of compression and may, consequently, be found difficult by a reader new to the subject. In chapter III the reader is taken from the definition of a complex and homology groups up to the definition of the extension functors as the derived functors of Hom, and the proof of the basic exact sequences for the extension functors. In the last two chapters this is applied to a discussion of the various homological dimensions and an introduction to duality theorems and quasi-Frobenius rings.

This book covers a remarkable amount of ground in 88 pages. Although, as a consequence, the style is occasionally a little compressed, it is not a difficult book to read. In the opinion of the reviewer, the book would form an excellent basis for a lecture course aiming at introducing research students to the tools of homological algebra, and the excellent sets of exercises at the end of each chapter enhance its value in this respect.

D. REES

HADWIGER, H., DEBRUNNER, H. AND KLEE, V., *Combinatorial geometry in the plane* (Holt, Rinehart & Winston, London, 1964), vii+113 pp., 30s.

This most interesting book is a translation by Klee, with additional material, of the monograph *Kombinatorische Geometrie in der Ebene* (Geneva, 1960) by the first two authors; this in turn was based on an article by Hadwiger in *L'Enseignement Mathématique*, (2) 1 (1955), 56-89, subsequently translated into French by J. Chatelet. So the exposition is not entirely new, but it is convenient to have this compact presentation of the material in English. In their Introduction the authors comment on the basically elementary nature of the methods of plane combinatorial geometry. It is perhaps surprising that the elementary properties of convexity are often not included in the undergraduate syllabus, though as far as (lack of) difficulty is concerned they are well within the scope of the course. Apart from the intrinsic interest of the subject, so apparent in this book, and its application to the now fashionable topics of game theory and linear programming, convexity is of importance later in functional analysis. It is also interesting to note that a recently published survey of " new school mathematics " (*Some Lessons in Mathematics*, C.U.P., 1964) includes a short chapter on convexity in which Helly's theorem (see below) is mentioned. Lest, however, it be thought that mathematical silk purses can be made too easily from sows' ears, it should be added that the simple nature of the tools has to be matched with considerable ingenuity in their use!

The first 40 pages of Part I of the book contain the statements of 98 propositions, set out as problems, with connecting explanatory material and references. Many of the problems derive from the fundamental theorem of E. Helly that if in a family of bounded closed convex sets in real n-dimensional space, every $(n+1)$ of the sets have a common point, then there is a point common to all the sets of the family. The proofs are given or sketched in Part II (pp. 57-98). Whether this mode of presentation, used so successfully in Polya and Szegö's famous collection of problems on analysis, is ideal here is open to some doubt. Certainly it gives a good bird's-eye view of the subject, but there will probably be few readers among the uninitiated who progress far without rather frequent glances at Part II! As an (admittedly extreme) example it may be mentioned that problem 76 is van der Waerden's celebrated theorem on arithmetic progressions, appearing here in a slightly unfamiliar geometrical guise. This is in Section 9, whose subject matter overlaps most closely with " combinatorics " as the word is understood today. It also includes F. P. Ramsey's combinatorial theorem and the " marriage problem " theorem of P. Hall (though the standard necessary and sufficient condition for that problem does not appear to be stated explicitly). The relevance of this section to the kind of problem treated elsewhere in the book becomes clearer on reading the corresponding part (pp. 52-54) of the translator's addendum. The latter, entitled " Further development of combinatorial