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CORRIGENDUM

SYMPLECTIC FILLINGS OF LINKS OF QUOTIENT SURFACE SINGULARITIES – CORRIGENDUM

MOHAN BHUPAL AND KAORU ONO

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D. Shin informed us of his joint work [3] and that one case is missing in our list in [2]. We claimed that the link of the quotient surface singularity $I_{30(5-2)+23}$ has four minimal symplectic fillings; however, there is a fifth one. Recall that in [2] we showed that the compactifying divisor of a tetrahedral, octahedral or icosahedral singularity of Type $(3, 1)$ can be chosen to have the form indicated in Figure 1, where D is a rational cuspidal curve, A and B are rational curves meeting D at the cusp point and $C_1, \ldots, C_k$ are rational curves. The missing filling is a Case II filling. In [2], we represented these using the notation $(m; D \cdot D, -c_1, \ldots, -c_k; i, j; a_1 \times i_1, \ldots, a_l \times i_l)$, where m denotes the index of the singularity and $-c_1, \ldots, -c_k$ denote the self-intersection numbers of the curves $C_1, \ldots, C_k$. This notation is meant to indicate that if we glue the corresponding symplectic filling X to a regular neighborhood of the compactifying divisor, then there will be a pair of (-1)-curves, one intersecting B and C_i and one intersecting D and C_j. In addition, there will be a further a_i (-1)-curves intersecting C_i for each i. In this notation, the missing filling can be represented by

$$(30(5-2)+23; 5, -2, -2, -3, -3; 4, 4; 2).$$

We would like to point out that this omission in our original list was not due to a theoretical error. Rather, it was due to our overlooking of a case—the original list was prepared by hand using the restrictions given in Propositions 4.5, 4.8 and 4.10. The omission arose by our not considering all possibilities arising from Proposition 4.10 in the case of the link of the singularity $I_{30(5-2)+23}$. It has been verified by the first author using

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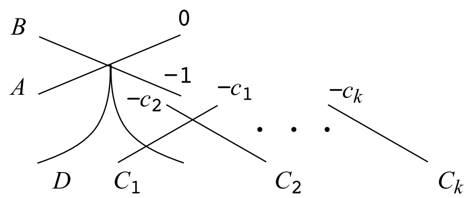

Figure 1.
Compactifying divisor for a tetrahedral, octahedral, or icosahedral singularity of Type $(3, 1)$.

a program in C++ [1] that this is the only case missing in our original list. Details of the algorithm found in the program in the case of Type $(3, 1)$ singularities follow. The algorithm for the case of Type $(3, 2)$ singularities is similar and the details are omitted.

Algorithm for finding symplectic fillings of links of quotient surface singularities of Type $(3, 1)$

As mentioned above, corresponding to each link of a quotient surface singularity of Type $(3, 1)$ there is a standard compactifying divisor K consisting of a cuspidal rational curve D, a string of rational curves $C_1, \ldots, C_k$ and two more rational curves A and B. For our purposes it is sufficient to record the self-intersection numbers $d = D \cdot D$, $-c_1 = C_1 \cdot C_1, \ldots, -c_k = C_k \cdot C_k$. We will write $Q = Q_K = (d, -c_1, \ldots, -c_k)$. We handle Case I and Case II symplectic fillings of the singularity link associated with the data Q separately. For Case I fillings, by Proposition 4.8, it is sufficient to list the numerical data associated to all preadmissible configurations $A' \cup B' \cup D' \cup C'_1 \cup \cdots \cup C'_k$ such that $D' \cdot D' = D \cdot D$ and $C'_i \cdot C'_i \geqslant C_i \cdot C_i$ for all i. For Case II fillings, by assumption and Proposition 4.10, first we have to identify the curves C_i and C_j such that there are exceptional curves E and F intersecting B and C_i and D and C_j, respectively, in some rational surface R containing the compactifying divisor K. Blowing down the curves E and F and denoting the resulting configuration $A' \cup B' \cup D' \cup C'_1 \cup \cdots \cup C'_k$, by Proposition 4.10, it is then sufficient to find all preadmissible configurations $A'' \cup B'' \cup D'' \cup C''_1 \cup \cdots \cup C''_k$ such that $D'' \cdot D'' = D' \cdot D'$ and $C''_i \cdot C''_i \geqslant C'_i \cdot C'_i$ for all i. (Note that in [2], we did not define preadmissible configurations for Case II symplectic fillings of links of tetrahedral, octahedral or icosahedral singularities of Type $(3, 1)$, however a definition similar to the other cases can be given.)

Algorithm 1 Enumerate symplectic fillings of the link of a quotient surface singularity of Type $(3,1)$

procedure ListExceptionalCollections(Q) ▷ *Given the data* $Q=(d,-c_1,\dots,-c_k)$ *of a compactifying divisor, list the set of all possible collections* $S=(e_1,\dots,e_k)$ *that may arise as the data of a collection of exceptional curves*

start with the empty collection $S_0=(0,\dots,0)$
increment e_1 in steps of 1 until it reaches c_1-1
then set e_1 to be 0 and increment e_2 by 1
then increment e_1 in steps of 1 again until it reaches c_1-1 again
then set e_1 to be 0 again and increment e_2 by 1 again
repeat this process until e_2 reaches c_2-1
then set e_2 to be 0 and increment e_3 by 1
repeat in this way until S becomes $(c_1-1,\dots,c_k-1)$

end procedure

procedure BlowDownCaseI(Q,S) ▷ *Given the data* Q *of a compactifying divisor* K *of a Case I filling of the link of a quotient surface singularity of Type* $(3,1)$ *and exceptional curves data* S*, returns the data of the configuration* K' *obtained after a collection of exceptional curves represented by* S has been blown down

$Q \leftarrow (d,-c_1+e_1,\dots,-c_k+e_k)$
return Q

end procedure

procedure AdmissibilityTestCaseI(Q) ▷ *Check whether* $Q=(d,-c_1,\dots,-c_k)$ *is the data of a preadmissible configuration of a Case I filling of the link of a quotient surface singularity of Type* $(3,1)$

```
if k = 0 then
    if d < 9 then
        return 1
    else
        return 0
    end if
end if
let 1 ⩽ s ⩽ k be the smallest index, if it exists, such that c_s = 1
if no such s exists then
    return 0
end if
```

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Algorithm 1 Continued

let Q' denote the data of the configuration after C_s is blown down
return AdmissibilityTestCaseI(Q')
end procedure
procedure EnumerateFillingsCaseI(Q) ▷ *Enumerate Case I symplectic fillings of the link of the quotient surface singularity of Type* $(3,1)$ *associated to the data* $Q=(d,-c_1,\dots,-c_k)$
for all possible collections $S=(e_1,\dots,e_k)$ **do**
$Q' \leftarrow$ BlowDownCaseI(Q,S)
if AdmissibilityTestCaseI(Q') **then**
list Q together with S as a filling
end if
end for
end procedure
procedure BlowDownCaseII(Q,i,j,S) ▷ *Given the data* Q *of a compactifying divisor* K *of a Case II filling of the link of a quotient surface singularity of Type* $(3,1)$*, integers* i *and* j*, and exceptional curves data* S*, returns the data of the configuration* K' *obtained after* E *and* F *and a collection of exceptional curves represented by* S *have been blown down*
$c_i \leftarrow c_i - 1$
$c_j \leftarrow c_j - 1$
$d \leftarrow d+1$
$Q \leftarrow (d,-c_1+e_1,\dots,-c_k+e_k)$
return Q
end procedure
procedure AdmissibilityTestCaseII(Q,i,j) ▷ *Check whether* $Q=(d,-c_1,\dots,-c_k),i,j$ *is the data of a preadmissible configuration of a Case II filling of the link of a quotient surface singularity of Type* $(3,1)$
if $k=1$ **then**
if $d<9$ and c_1=0 **then**
return 1
else
return 0
end if
end if
let $1\leqslant s\leqslant k$ be the smallest index, if it exists, such that $c_s=1$ and $s\neq i$ and either $s\neq j$ or $s=k$

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Algorithm 1 Continued

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    if no such s exists then
        return 0
    end if
    let Q' denote the data of the configuration K' = A' ∪ B' ∪ D' ∪ C'_1 ∪
    ··· ∪ C'_{k−1} obtained by blowing down C_s
    define i' and j' by the condition that D' intersects the string
    C'_1, ..., C'_{k−1} at C'_1 and at C'_j and that B' intersects the string at C'_i
    return AdmissibilityTestCaseII(Q', i', j')
end procedure
procedure EnumerateFillingsCaseII(Q) ▷ Enumerate Case II sym-
plectic fillings of the link of the quotient surface singularity of Type (3, 1)
associated to the data Q = (d, −c_1, ..., −c_k)
    for all pairs (i, j) with 1 ⩽ i ⩽ j ⩽ k do
        for all possible collections S = (e_1, ..., e_k) do
            Q' ← BlowDownCaseII(Q, i, j, S)
            if AdmissibilityTestCaseII(Q', i, j) then
                list Q together with i, j and S as a filling
            end if
        end for
    end for
end procedure
```

In the algorithm above, we assume that the compactifying divisor K associated with the data Q sits in some rational surface R. Collections of exceptional curves $\{E_\alpha\}$ in R that are disjoint from $D \cup A \cup B$ and such that $E_\alpha \cdot \bigcup C_i = 1$ for each α will be recorded by the data $S = (e_1, \ldots, e_k)$, where e_i denotes the number of exceptional curves intersecting C_i. Thus to list all possible Case I fillings of the singularity link associated to Q, it is sufficient to enumerate all possible k-tuples S such that on blowing down a collection of exceptional curves represented by S, K is transformed into a preadmissible configuration. Similarly, to list all possible Case II fillings of the singularity link associated to Q, it is sufficient to list all possible numerical data (i, j) and S such that on blowing down E and F and a collection of exceptional curves represented by S, K is transformed into a preadmissible configuration.

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Supplementary material

Supplementary material is available at http://dx.doi.org/10.1017/nmj.2016.42.

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Mohan Bhupal
Department of Mathematics
Middle East Technical University
06800 Ankara
Turkey
bhupal@metu.edu.tr

Kaoru Ono
Research Institute for Mathematical Sciences
Kyoto University
Kyoto 606-8502
Japan
ono@kurims.kyoto-u.ac.jp