depends fundamentally on the Yang–Mills equations but background from physics is not given since it is not needed explicitly for the mathematical development. The bulk of the book is about differential geometry and Chapter Six is probably the key: it is here that the crucial link between differential geometry and algebraic geometry is made. Calculations are best organised by using the Donaldson polynomials; these are introduced and their properties developed in the penultimate chapter. The final chapter is devoted entirely to making such calculations on specific examples. Considerable effort has gone into organising the material and many of the proofs are different from those in the literature. Choosing between the already large number of approaches was becoming something of a problem for anyone learning this material and the authors deserve our gratitude for providing such a clear and unified approach.

Donaldson's ideas are sure to have a considerable further impact on geometry and toplogy and this book will be the basic reference. It will undoubtedly inspire future generations of mathematicians to attack some interesting and difficult questions.

E. G. REES

KIRWAN, F., *Complex algebraic curves* (London Mathematical Society Student Texts 23, Cambridge University Press, Cambridge 1992) pp. viii+264, cloth 0 521 41251 X, £30, paper 0 521 42353 8, £13.95.

The stated aims of this book are to demonstrate to final-year undergraduates how basic ideas in pure mathematics which they have met previously are brought together in one of the great showpiece subjects of mathematics and, by adding some extra material for postgraduates, to exhibit the varied and useful nature of this core topic in algebraic geometry. In this it succeeds splendidly: the reader receives a broad and enlightening education on the many aspects of complex algebraic curves—the algebraic, the analytic, the topological—and on their rich history and wide range of application from the time of the Greeks up to the present day. The exposition has been given careful consideration; important ideas and results appear near the start of a chapter, with technical parts of proofs separated out and left to the end if necessary. Where needed, telling examples are worked out in detail, so that the reader can grasp the main point immediately. The many exercises provide a good grounding in the basic material, ranging as they do from concrete examples to theoretical questions (which can be quite testing) and applications in a wide variety of fields. The writing style is crystal clear and enthusiastic.

Chapter 1 provides motivation and historical background. Highlights include the connections between singularities and knots and links, and a discussion of the crucial role played by abelian and elliptic integrals (here, typically, applications to arc-length and to the motion of a simple pendulum are mentioned). Lots of examples of plane complex curves are given and sketched; in particular, the use made by the Greeks of certain curves in trisecting angles and in doubling the square is featured.

Chapter 2 introduces the reader to basic definitions and material. The relationship between algebraic and projective curves in the plane is given and the multiplicity at a point is defined. Chapter 3 builds on this by presenting the algebraic properties of curves (the book treats only plane curves). Highlights include Bézout's theorem (with applications) and a discussion of the additive group law on smooth cubic curves. Resultants are used to define intersection multiplicities and the calculation of these multiplicities is given algorithmically. The topological viewpoint is presented in Chapter 4 with a discussion of the degree–genus formula in the smooth case. Two treatments of this are presented; the first is in terms of deformation to a set of lines while the second involves cutting and gluing. This leads to a discussion of branched covers of the projective line and of the Riemann–Hurwitz formula. Complex analysis is added in Chapter 5 with the introduction of the central concept of a Riemann surface (the example of the projective line and of the complex torus being treated in detail). The Wierstrass $\wp$-function and its connection with cubic curves are treated comprehensively.

Chapter 6 introduces the crucial topic of differentials on a Riemann surface with the motivating question: 'does it make sense to differentiate a meromorphic function on a Riemann surface?'. Again the cases of the projective line and of the complex torus are treated in detail. The chapter builds towards Abel's theorem for complex tori and the Riemann–Roch formula. The latter is applied to give Chow's theorem for smooth plane curves and to prove the associativity of the addition law on smooth cubics.

More advanced material on singular curves is presented in the final chapter. First, the resolution of singularities is discussed. Then Newton polygons and the associated topic of Puiseux expansions are used to describe what a curve looks like near a singularity. Finally, the degree–genus formula for singular curves (Noether's formula) is proved using Puiseux expansions with the examples of cuspidal and nodal cubics being examined in detail. (These last two chapters would be very useful for anyone interested in the modern links between algebraic geometry and coding theory.) There are three appendices on algebra, complex analysis and topology.

There appear to be very few misprints; only the ones in 3.30 and in 5.23 are of any substance. The statement of 3.14 needs a little repair and there is a tiny formal difficulty (for example in 3.7) of resultants being viewed as having a non-zero degree when they might vanish. But these are minor cavils, set against the all-round excellence of this book.

L. O'CARROLL

BRECHTKEN-MANDERSCHEID, U., *Introduction to the calculus of variations*, translated by P. G. Engstrom (Chapman and Hall, London 1991), pp. viii + 200, cloth 0 412 36690 8, £27.95, paper 0 412 36700 9, £13.95.

This book gives a conventional first course on the calculus of variations. A brief introductory chapter, which mentions the standard examples, is followed by one on Euler's equation and the Erdmann corner conditions. Then come four chapters on necessary conditions, including those of Weierstrass, Legendre and Jacobi, and sufficient conditions. In the second half of the book the earlier ideas are extended to problems involving variable boundaries or parametric representations, problems depending on several functions or multiple integrals, and problems constrained by side conditions. The final chapter gives the direct approximation method due to Ritz.

From this it will be seen that the material is entirely appropriate. Most of the worked examples are from mathematics, with an emphasis on geometrical properties, a few are from physics, and the Ramsey growth model from economics represents applications outside the physical sciences. There are some exercises for the reader but not always very many.

It is most welcome to have such a careful account of the calculus of variations available in paperback format at a very reasonable price. If you want a thorough yet concise treatment you can hardly do better. But I did sometimes feel that the very painstaking style tended to obscure what was going on. For the good student this would be an excellent text, but the weaker student might master the subject more easily from a less conscientious treatment.

The text is beautifully printed, with large clear type making it a pleasure to read. There are a few apparent infelicities of translation, for example the description on page 6 of the area enclosed by a plane curve as "surface area" just two pages after a problem about genuine surface area.

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