Journal of the International Neuropsychological Society (2000), **6**, 365.

LETTER TO THE EDITOR

Reliable Change formula query: A statistician's comments

IAN S. ABRAMSON
Department of Mathematics, University of California, San Diego

Dr. Hinton-Bayre has correctly noted a discrepancy here between the Jacobson and Temkin procedures. His comparison of prediction bounds both confirms this and suggests too that the differences are minor. There is a large-sample explanation: suppose $\begin{pmatrix} x \\ x' \end{pmatrix}$ denotes a vector of before and after readings on a random variable, following a shift model with fixed variance: its expectation is $\begin{pmatrix} \mu \\ \mu + \delta \end{pmatrix}$, and its covariance matrix is $\sigma^2 \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$. This is a general formulation and a consequence of the standard random coefficient model (with or without practice effects) that implicitly underlies the change-score problem. Normality need not even be assumed at this point. ρ is the correlation coefficient between x and x'.

The difference $x' - x$ then has mean δ, and its standard deviation is readily shown to be $\sqrt{2\sigma^2(1-\rho)}$. The question is how to estimate this from paired-sample data. The Temkin approach is to ignore the form of the expression in the parameters ρ and σ^2, and just empirically take the sample variance of the observed differences. The Jacobson approach observes the form and plugs in estimates of σ^2 and ρ: ρ by $r_{xx'}$ (Hinton-Bayre's symbol—it is just a sample correlation), and σ^2 (strangely perhaps) by s_1^2, the sample variance of the initial scores. I say strangely, because there is no reason not to pool the variation in the before and after readings, and get a more efficient (i.e., less wasteful) estimator which would be denoted by $S^2_{pooled} = \frac{1}{2}(S_1^2 + S_2^2)$ with a smaller (indeed, essentially minimal) standard error for estimating σ^2.

Now it can be algebraically verified that the Temkin proposal is exactly equivalent to Hinton-Bayre's formula with the term s_1^2 replaced by S^2_{pooled}. Both are consistent, and for large samples they will be close. To this extent the Temkin and Jacobson procedures are close, but to the extent that S^2_{pooled} is better than s_1^2, the Temkin procedure is better.

Reprint requests to: I.S. Abramson, Department of Mathematics, University of California, San Diego, 9500 Gilman Drive, MC 0112, La Jolla, CA 92093. E-mail: iabramson@ucsd.edu

https://doi.org/10.1017/S1355617700633131 Published online by Cambridge University Press