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Bounds for solutions of diophantine equations

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In this thesis, I consider two independent problems on bounds for solutions of diophantine equations in several variables. Part I concerns a pair of simultaneous diagonal quadratic equations, and Part II a single non-diagonal cubic equation. In each case, I show that, under suitable hypotheses, the equations have non-trivial integer solutions whose size is bounded in terms of the coefficients in the equations. The main theorems are as follows:

PART I. *Consider the equations*

$$(1)\qquad \begin{aligned} f(\mathrm{x}) &= \lambda_1 x_1^2 + \ldots + \lambda_n x_n^2 = 0 , \\ g(\mathrm{x}) &= \mu_1 x_1^2 + \ldots + \mu_n x_n^2 = 0 , \end{aligned}$$

where the λ_i, μ_i *are integers, and let*

$$(2)\qquad \begin{aligned} \Lambda &= \max_{1 \le i \le n} \left(|\lambda_i|, |\mu_i|\right) , \\ \Theta &= \prod_{i,j} \Delta_{ij} , \end{aligned}$$

where the product is over all pairs i, j *such that*

$$\Delta_{ij} = \lambda_i \mu_j - \lambda_j \mu_i \neq 0 .$$

Also let v *be the number of subscripts in the largest set* $\{i_1, \ldots, i_v\}$

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with $\Delta_{i_j i_k} = 0 \quad (1 \le j \le v,\ 1 \le k \le v)$.

Suppose that, for all real a, b *(not both* 0 *), the form* $af + bg$ *is indefinite and explicitly contains at least* 5 *variables (which implies* $v \le n-5$ *), and that*

$$n = 9\ , \quad v \le 2 \quad (= n - 7)\ .$$

Then

(a) *if* (f, g) *is "normalised" in the sense that* Θ *in* (2) *cannot be reduced by operations of a specified kind on the pair* (f, g) *, then the system* (1) *has an integer solution* $\mathbf{x}$ *satisfying*

$$0 < |\mathbf{x}| \ll \Lambda^{139}\ ;$$

(b) *without the normalisation hypothesis,* (1) *has an integer solution* $\mathbf{x}$ *satisfying*

$$0 < |\mathbf{x}| \ll \Lambda^{144}\Theta^{279} \quad \left(\ll \Lambda^{40320}\right)\ .$$

(Here, and below, $|\mathbf{x}| = \max|x_i|$ and $\ll$ denotes an inequality with an unspecified constant factor.)

PART II. *Consider the equation*

$$C(\mathbf{x}) = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} c_{ijk}x_i x_j x_k = 0\ , \tag{3}$$

where the c_{ijk} *are integers, and let*

$$M = \max|c_{ijk}|\ .$$

If $n = 17$ *and* $C(\mathbf{x})$ *is non-singular, then* (3) *has an integer solution* $\mathbf{x}$ *satisfying*

$$0 < |\mathbf{x}| \ll M^{\psi}\ ,$$

where $\psi = 8 \times 10^8$.

The existence of non-trivial solutions to the equations (1) and (3) has previously been proved by Cook [1] and Davenport [2], [3]. Earlier work on bounds for solutions of such diophantine equations has been

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essentially restricted to a single diagonal equation (see, for example, [4]). The method used in this thesis, as in all the work just mentioned, is an adaptation of the Hardy-Littlewood circle method. The major difficulties to be overcome in seeking bounds for solutions of simultaneous diagonal equations (Part I) and of a non-diagonal equation (Part II), rather than for a single diagonal equation, arise in connection with the so-called singular series and singular integral.

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