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Abstracts of Australasian Ph.D. theses

Variational solution of certain nonlinear eigenvalue problems

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The "nonlinear eigenvalue problems" studied in this thesis are nonlinear in the eigenvalue but generally involve linear operators. One such problem, a standard formulation of the astrophysical problem of non-radial oscillations of stars, has been solved numerically by the Ritz method by several workers. However the problem is not covered by the classical theory of the Ritz method.

This thesis examines a class of nonlinear eigenvalue problems which contains the astrophysical problem as a special case. Various sufficient conditions are established for Ritz approximate (R.a.) solutions (solutions obtained by the Ritz method) of some of this class of problems to possess certain properties of the exact solutions and for R.a. solutions to converge to exact solutions. In several important respects these R.a. solutions are shown to be more sensitive to changes in coordinate functions than is the case for related linear eigenvalue problems. Theoretical results proved here are supplemented by results of carefully designed numerical experiments.

The problems studied here are converted into a form linear in the eigenvalue (but not by the classical method involving product spaces). R.a. solutions obtained from the linear and the nonlinear formulations are compared. New results are also proved concerning the use of the Ritz and several other numerical methods for linear eigenvalue problems in Hilbert space.

Fuller summaries of this work are contained in the author's papers [1 - 5]. However the thesis contains much material not in these papers,

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including a much more detailed discussion of results, an extensive review of the literature and extra numerical results. Moreover several theorems are proved in a slightly more general form than in the papers. For example, it is shown that Theorem 5 of [2] remains true when the special coordinate functions considered there are replaced by any coordinate functions satisfying Condition (c) of Theorem 4 of [5].

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