PROBLEMS FOR SOLUTION

P31. Prove that if $p > 3$ is a prime $\equiv 3 \pmod 4$ and $\zeta = e^{2\pi i/p}$, then

$$\prod_r (1 + \zeta^r) = \left(\frac{2}{p}\right)$$

where r runs through the quadratic residues of p, and $\left(\frac{2}{p}\right)$ is the Legendre symbol of quadratic residuacity.

L.J. Mordell

P32. The equation

$$(1 + 2\cos\frac{\pi}{p})(1 + 2\cos\frac{\pi}{q}) = 1$$

is obviously satisfied by $p = q = 2$. Are there any other rational solutions with $p \geqslant q \geqslant 1$?

N.W. Johnson

P33. Let

$$R_n = R_n(x) = \sum_{r=0}^{n} \binom{n+r}{n-r} x^r .$$

Show that for $n > 0$,

$$R_{n+1}R_{n-1} - R_n^2 = x.$$

L. Lorch and L. Moser

P34. Determine all Riemann surfaces with a transitive group A of automorphisms (conformal self mappings). On these surfaces a not too narrow conformal geometry can be based.

H. Helfenstein

SOLUTIONS

P25. Let H be a complex in a finite additive group and let H contain 0 and at least one other element. Does there

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always exist a positive integer n such that the equation

$$(*) \qquad a_1x_1 + a_2x_2 + \ldots + a_nx_n = 0$$

has a non-trivial solution, $x_1 \in H$, $x_2 \in H$, ..., $x_n \in H$, for every n-tuplet of positive integers $a_1, a_2, \ldots, a_n$?

P. Scherk

Solution by Christine Ayoub. Let G have order m and let $n = m^2$. We prove that (*) has a non-trivial solution for every n-tuplet of positive integers $a_1, a_2, \ldots, a_n$.

Let x be any element of G different from 0. Since there are m^2 integers in the set $a_1, a_2, \ldots, a_n$, we can find a subset consisting of m integers, all congruent (mod m), say $a_{i_1}, a_{i_2}, \ldots, a_{i_m}$. Choose $x_j = 0$ for $j \neq j_k$ and $x_{i_k} = x$, $k = 1, 2, \ldots, m$. With these values substituted for x_j, the left hand side of (*) becomes $a_{i_1}x + \ldots + a_{i_m}x = ma_ix = 0$ since the order of x divides m, the order of the group. Thus (*) has a non-trivial solution.

P26. Show that the fundamental group of an orientable surface (2-dimensional manifold with a countable base) is isomorphic to a subgroup of Euclidean, spherical, or hyperbolic motions.

H. Helfenstein

Solution by the proposer. Although this is a topological statement the following function-theoretic proof may be of interest. According to Radó our surface is triangulable and can be provided in the large with a conformal structure compatible with its topological structure and becomes a Riemann surface R. The fundamental group of R is isomorphic to the group C of covering transformations of the universal covering surface R' of R. According to the conformal type of R' (sphere, plane, or unit disc) C is a group of spherical, Euclidean, or hyperbolic motions.

A solution to P22 was received from A. Makowski (Warsaw).

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