THE EFFECT OF CONVECTION IN NONLINEAR ONE-ZONE STELLAR MODELS

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Abstract. We study the convective nonlinear one-zone models of the pulsating variable stars. In the small convective case, the solution shows chaotic behavior through the period doubling bifurcation as the temperature is lower although the temperature at which the chaos appears is lower than in the case of no convection. On the other hand, in the strong convective case, the solution only shows period-one limit cycle.

1. Introduction

The study of nonlinear simple one-zone models of the pulsating stars with the saturation effect of the kappa mechanism was performed by using numerical integration (Saitou et al., 1989). Their models were represented with the three ordinary differential equations and the solutions showed the chaotic behavior through the period-doubling bifurcation as the temperature is lower.

In this study, we examine the effect of the convection for the same one-zone models.

2. Model

We construct the pulsating stellar model from the Baker's (1966) one-zone stellar model and modify it adding Stellingwerf's (1986) equations for the convection.

The equations that describe our model are four ordinary differential equations as follows:

$$\frac{dx}{dt} = y, \tag{1}$$

$$\frac{dy}{dt} = (1+x)^2(1+z) - (1+x)^{-2}, \tag{2}$$

$$\begin{aligned}\frac{dz}{dt} = &-3\gamma y(1+x)^{-1}(1+z) \\ &-\varepsilon(1+x)^{-3}[(1-\eta)(1+x)^{\alpha}(1+z)^{\beta} + \eta(1+x)^{-1}(1+u)^3 - 1],\end{aligned} \tag{3}$$

$$\frac{du}{dt} = \zeta[(1+x)^{3/2}(1+z)^{1/2} - (1+u)], \tag{4}$$

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where t is the time in the unit of the free-fall time, x, y, z, and u respectively, the stellar radius, the radial velocity, the pressure, and the mean convective velocity in the unit of the equilibrium values, γ the adiabatic exponent, ε the nonadiabaticity or the ratio of the free-fall time to the thermal time, η the ratio of the convective luminosity to the total luminosity, and ζ the convective efficiency or the ratio of the free-fall time to a convective adjustment time.

We used the saturation effect of the kappa mechanism, and put α and β artificially as follows:

$$\alpha = a[(1+x)^3(1+z) - 1.2] + 21.6, \tag{5}$$

$$\beta = 3.6(1+x)^3(1+z)[(1+x)^3(1+z) - 0.2], \tag{6}$$

where a is a parameter to control the kappa mechanism. Also the parameter a correspond to the temperature of the shell of the model star and the temperature is high when the parameter a is large (see Saitou et al., 1989).

3. Calculation

The equations are solved by the Runge-Kutta method with the initial values of $x = 0$, $y = 0.001$, $z = 0$, and $u = 0$ at $t = 0$. The calculation is performed changing the parameters a, η, and ζ, and Table I shows the summary of the solutions. Figure 1 shows also the phase map and return maps, which are plotted the values of successive maxima, of a selected solution ($t = 500$ to 600).

TABLE I
Summary of Solution

η	a							
	20	15	12	10	7	5	3	1
0.0	1	4	c					
0.1		1	2	8	c			
0.2			1	1	2	c		
0.3							1	2
0.4							1	1
0.6							1	1

$\zeta = 1.0$
1: period 1, 2: period 2, 4: period 4,
8: period 8, c: chaos

4. Discussion

From Figure 1 and Table I, we fine that in the case of small convective luminosity, η, the solution shows chaotic behavior. However, the trajectory

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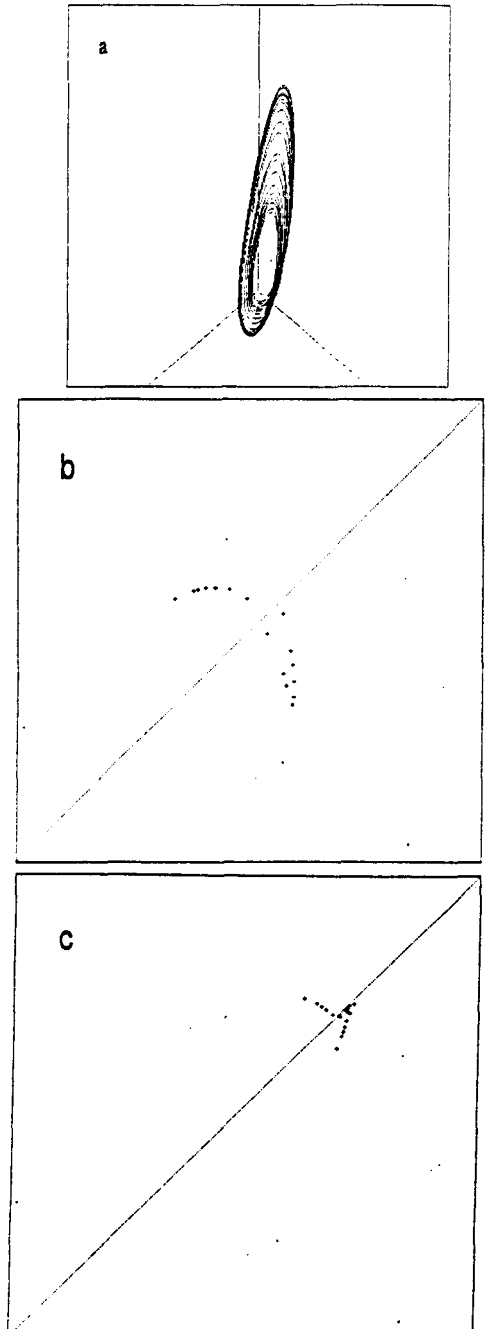

Fig. 1. The phase map and return maps of the solution for $a = 12.0$, $\eta = 0.04$, and $\zeta = 1.0$. (a) shows orbit in the x-y-z phase space. (b) and (c) show return maps plotted values of successive maxima for x and for y, respectively.

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of the larger convective luminosity is more regular and is especially only simple limit cycle at $\eta = 0.20$. This result can imply that the convective effect generally stabilizes the pulsation as well as the result of Stellingwerf (1986).

Table I shows that in the case of small convective luminosity the chaotic behavior is induced as the parameter a is small as well as in the case of no convection. However, the temperature at which the chaos appears is lower than no convection case.

5. Summary

We summarize our study for the convective one-zone model of the pulsating star as follows:

(1) Small convection induces the chaos as well as no convection but stabilizes the pulsation.

(2) Large convection products only the regular pulsation.

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