BULL. AUSTRAL. MATH. SOC. 47H04, 47N10, 52A07, 90A14, 90D06, 90D13
VOL. 49 (1994) [47–54]

MAXIMAL ELEMENTS AND EQUILIBRIA FOR $\mathcal{U}$-MAJORISED PREFERENCES

KOK-KEONG TAN AND XIAN-ZHI YUAN

The purpose of this note is to give a general existence theorem for maximal elements for a new type of preference correspondences which are $\mathcal{U}$-majorised. As an application, an existence theorem of equilibria for a qualitative game is obtained in which the preferences are $\mathcal{U}$-majorised with an arbitrary (countable or uncountable) set of players and without compactness assumption on their domains in Hausdorff locally convex topological vector spaces.

1. INTRODUCTION

The existence of equilibria in an abstract economy with compact strategy sets in $\mathbf{R}^n$ was proved in a seminal paper of Debreu [7]. The theorem of Debreu extended the earlier work of Nash in game theory. Since then there have been many generalisations of Debreu's theorem by Arrow and Debreu [2], Borglin and Keiding [4], Ding, Kim and Tan [8, 9], Gale and Mas-Colell [11, 12], Mehta [19], Mehta and Tarafdar [20], Shafer and Sonnenschein [21], Sonnenschein [22], Tan and Yuan [23, 24], Tarafdar [25], Toussaint [26], Tulcea [27], Yannelis [28] and Yannelis and Prabhakar [29] and others. These papers generalise Debreu's theorem by considering preference correspondences that are not necessarily transitive or total, by allowing externalities in consumption and by assuming that the commodity space is not necessarily finite-dimensional. In these papers, the domain (and/or codomain) of the preference and constraint correspondences is assumed to be compact or paracompact.

Following the work of Sonnenschein [22], Gale and Mas-Colell [11] and Borglin and Keiding [4] on non-ordered preference relations, many theorems on the existence of maximal elements of preference relations which may not be transitive or complete, have been proved by Aliprantis and Brown [1], Bergstrom [3], Ding, Kim and Tan [8], Kim [15], Mehta [19], Mehta and Tarafdar [20], Tan and Yuan [23], Tarafdar [25], Toussiant [26], Tucela [27], Yannelis [28], Yannelis and Prabhakar [29], Walker [30] and others. However, their existence theorems of maximal elements deal with preference correspondences which have open lower sections or are majorised by correspondences with open

Received 28th January, 1993

lower sections. Since every correspondence with open lower sections must be lower semicontinuous, it is natural to study the existence of maximal elements of preference correspondences which are majorised by upper semicontinuous correspondences.

The objective of this note is to give some existence theorems for maximal elements and equilibria in qualitative games without the compactness (or paracompactness) assumption on the domain of the preferences which are majorised by upper semicontinuous correspondences instead of being majorised by correspondences which have open lower sections (for example, see [**3, 4, 8, 11, 12, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30**] et cetera). Our intention is to illustrate a certain technique that we think will be of use in various problems of mathematical economics. Many other results of the type proved here may be proved under more general conditions.

Now we give some notations. Let A be a set, we shall denote by 2^A the family of all subsets (including the empty subset $\emptyset$) of A. If A is a subset of a topological space X, we shall denote by $cl_X(A)$ the closure of A in X. If A is a subset of a vector space, we shall denote by coA the convex hull of A. If A is a non-empty subset of a topological vector space E and $S, T : A \to 2^E$ are correspondences, then coT, $T \cap S : A \to 2^E$ are correspondences defined by $(coT)(x) = coT(x)$ and $(T \cap S)(x) = T(x) \cap S(x)$ for each $x \in A$, respectively. If X and Y are topological spaces and $T : X \to 2^Y$ is a correspondence, then (1) T is said to be upper semicontinuous at $x \in X$ if for any open subset U of Y containing $T(x)$, the set $\{z \in X : T(z) \subset U\}$ is an open neighbourhood of x in X; (2) T is upper semicontinuous (on X) if T is upper semicontinuous at x for each $x \in X$; (3) the graph of T, denoted by Graph(T), is the set $\{(x, y) \in X \times Y : y \in T(x)\}$; (4) the correspondence $\overline{T} : X \to 2^Y$ is defined by $\overline{T}(x) = \{y \in Y : (x, y) \in cl_{X \times Y}\mathrm{Graph}(T)\}$ and (5) the correspondence $clT : X \to 2^Y$ is defined by $clT(x) = cl_Y(T(x))$ for each $x \in X$. It is easy to see that $clT(x) \subset \overline{T}(x)$ for each $x \in X$. We remark here that in defining upper semicontinuity of T at $x \in X$, we do not require that $T(x)$ be non-empty.

Let X be a topological space, Y be a non-empty subset of a vector space E, $\theta : X \to E$ be a map and $\phi : X \to 2^Y$ be a correspondence. Then (1) ϕ is said to be of class $\mathcal{U}_\theta$ if (a) for each $x \in X$, $\theta(x) \notin \phi(x)$ and (b) ϕ is upper semicontinuous with closed and convex values in Y; (2) ϕ_x is a $\mathcal{U}_\theta$-majorant of ϕ at x if there is an open neighbourhood $N(x)$ of x in X and $\phi_x : N(x) \to 2^Y$ such that (a) for each $z \in N(x)$, $\phi(z) \subset \phi_x(z)$ and $\theta(z) \notin \phi_x(z)$ and (b) ϕ_x is upper semicontinuous with closed and convex values; (3) ϕ is said to be $\mathcal{U}_\theta$-majorised if for each $x \in X$ with $\phi(x) \neq \emptyset$, there exists a $\mathcal{U}_\theta$-majorant ϕ_x of ϕ at x. We remark that when $X = Y$ and $\theta = I_X$, the identity map on X, our notions of a $\mathcal{U}_\theta$-majorant of ϕ at x and a $\mathcal{U}_\theta$-majorised correspondence are generalisation of upper semicontinuous correspondences which are irreflexive (that is, $x \notin \phi(x)$ for all $x \in X$) and have closed convex values.

In this paper, we shall deal mainly with either the case (I) $X = Y$ and X is a non-empty convex subset of the topological vector space E and $\theta = I_X$, the identity map on X, or the case (II) $X = \prod_{i \in I} X_i$ and $\theta = \pi_j : X \to X_j$ is the projection of X onto X_j and $Y = X_j$ is a non-empty convex subset of a topological vector space. In both cases (I) and (II), we shall write $\mathcal{U}$ in place of $\mathcal{U}_\theta$.

Let I be a (possibly infinite) set cf players. For each $i \in I$, let its choice or strategy set X_i be a non-empty subset of a topological vector space. Let $X = \prod_{i \in I} X_i$. For each $i \in I$, let $P_i : X \to 2^{X_i}$ be a preference correspondence. Following the notion of Gale and Mas-Colell [**12**], the collection $\Gamma = (X_i, P_i)_{i \in I}$ will be called a qualitative game. A point $\widehat{x} \in X$ is said to be an equilibrium of the game Γ if $P_i(\widehat{x}) = \emptyset$ for all $i \in I$. For each $i \in I$, let A_i be a non-empty subset of X_i; if $i \in I$ is arbitrarily fixed, we define:

$$\prod_{\substack{j \neq i, \\ j \in I}} A_j \otimes A_i = \{x = (x_k)_{k \in I} \in X : x_k \in A_k \text{ for each } k \in I\}.$$

2. Existence of Maximal Elements

The following is Lemma 2.10 of [**24**]:

Lemma 2.1. *Let X and Y be two topological spaces, A be a closed (respectively, open) subset of X. Suppose $F_1 : X \to 2^Y$, $F_2 : A \to 2^Y$ are lower semicontinuous (respectively, upper semicontinuous) such that $F_2(x) \subset F_1(x)$ for all $x \in A$. Then the map $F : X \to 2^Y$ defined by*

$$F(x) = \begin{cases} F_1(x) & , \text{ if } x \notin A; \\ F_2(x) & , \text{ if } x \in A \end{cases}$$

is also lower semicontinuous (respectively, upper semicontinuous).

The following result is essentially due to Hildenbrand [**13**, p.23–24] (see also [**16**, Theorem 7.3.10, p.86]):

Lemma 2.2. *Let X be a topological space and Y be a normal space. If $F, G : X \to 2^Y$ have closed values and are upper semicontinuous at $x \in X$, then $F \cap G$ is also upper semicontinuous at x.*

Proof: If $F(x) \cap G(x) \neq \emptyset$, the conclusion follows from [**13**, Proposition B.III.2, p.23–24] (also see [**16**, Theorem 7.3.10, p.86]). If $F(x) \cap G(x) = \emptyset$, since Y is normal, it is easy to see that there exists an open neighbourhood N of x in X such that $F(z) \cap G(z) = \emptyset$ for all $z \in N$; thus $F \cap G$ is also upper semicontinuous at x. $\square$

We remark here that in Lemma 2.2, we do not require $F(x) \cap G(x) \neq \emptyset$.

THEOREM 2.3. *Let X be a paracompact space and Y be a non-empty normal subset of a topological vector space E. Let $\theta : X \to E$ and $P : X \to 2^Y \setminus \{\emptyset\}$ be $\mathcal{U}$-majorised. Then there exists a correspondence $\Psi : X \to 2^Y \setminus \{\emptyset\}$ of class $\mathcal{U}$ such that $P(x) \subset \Psi(x)$ for each $x \in X$.*

PROOF: Since P is $\mathcal{U}$-majorised, for each $x \in X$, let $N(x)$ be an open neighbourhood of x in X and $\psi_x : N(x) \to 2^Y \setminus \{\emptyset\}$ be such that (1) for each $z \in N(x), P(z) \subset \psi_x(z)$ and $\theta(z) \notin \psi_x(z)$ and (2) ψ_x is upper semicontinuous with closed and convex values. Since X is paracompact and $X = \bigcup_{x \in X} N(x)$, by Theorem VIII.1.4 [**10**, p.162], the open covering $\{N(x)\}$ of X has an open precise neighbourhood-finite refinement $\{N'(x)\}$. For each $x \in X$, define $\psi'_x : X \to 2^Y \setminus \{\emptyset\}$ by

$$\psi'_x(z) = \begin{cases} \psi_x(z), & \text{if } z \in N'(x); \\ Y, & \text{if } z \notin N'(x), \end{cases}$$

then ψ'_x is also upper semicontinuous on X by Lemma 2.1 such that $P(z) \subset \psi'_x(z)$ for each $z \in X$.

Now define $\Psi : X \to 2^Y \setminus \{\emptyset\}$ by $\Psi(z) = \bigcap_{x \in X} \psi'_x(z)$ for each $z \in X$. Clearly, Ψ has closed and convex values and $P(z) \subset \Psi(z)$ for each $z \in X$. Let $z \in X$ be given, then $z \in N'(x)$ for some $x \in X$ so that $\psi'_x(z) = \psi_x(z)$ and hence $\Psi(z) \subset \psi_x(z)$; as $\theta(z) \notin \psi_x(z)$, we must also have that $\theta(z) \notin \Psi(z)$. Thus $\theta(z) \notin \Psi(z)$ for all $z \in X$.

Now we shall show that Ψ is upper semicontinuous. For any given $u \in X$, there exists an open neighbourhood M_u of u in X such that the set $\{x \in X : M_u \cap N(x) \neq \emptyset\}$ is finite, say $= \{x(u,1), \cdots, x(u,n(u))\}$. Thus we have that

$$\Psi(w) = \bigcap_{x \in X} \psi'_x(w) = \bigcap_{i=1}^{n(u)} \psi'_{x(u,i)}(w) \qquad \text{for all } w \in M_u.$$

For $i = 1, \cdots, n(u)$, since each $\psi'_{x(u,i)}$ is upper semicontionuous on X and hence on M_u with closed values and Y is normal, by Lemma 2.2, $\Psi : M_u \to 2^Y$ is also upper semicontinuous at u. Since M_u is open, $\Psi : X \to 2^Y$ is also upper semicontinuius at u. Hence Ψ is of class $\mathcal{U}$. ∎

We now prove the following theorem concerning the existence of a maximal element:

THEOREM 2.4. *Let X be a non-empty convex subset of a Hausdorff locally convex topological vector space and D be a non-empty compact subset of X. Let $P : X \to 2^D$ be $\mathcal{U}$-majorised (that is, $\mathcal{U}_{I_X}$-majorised). Then there exists a point $x \in coD$ such that $P(x) = \emptyset$.*

PROOF: Suppose the contrary, that is, for all $x \in coD$, $P(x) \neq \emptyset$. Then for each $x \in coD$, $P(x) \neq \emptyset$ and coD is also paracompact by Lemma 1 of [**9**, p.206] (see also

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[**17**, p.49]). Now applying Theorem 2.3, there exists a correspondence $\Psi : coD \to 2^D$ of class $\mathcal{U}$ such that for each $x \in coD$, $P(x) \subset \Psi(x)$. Since Ψ is upper semicontinuous with non-empty closed and convex values, by a fixed point Theorem of Himmelberg [**14**, Theorem 2, p.206], there exists $x \in coD$ such that $x \in \Psi(x)$. This contradicts that Ψ is of class $\mathcal{U}$. Hence the conclusion must holds. $\square$

We note that Theorem 2.4 is closely related, though not comparable, to those existence theorems of maximal elements in [**1, 3, 8, 11, 12, 15, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30**, et cetera].

3. Existence of Equilibria in Locally Convex spaces

In this section, we shall give some applications of Theorem 2.4. First we have the following:

Theorem 3.1. *Let X be a non-empty convex subset of a Hausdorff locally convex topological vector space and D be a non-empty compact subset of X. Let $P : X \to 2^D$ be $\mathcal{U}$-majorised and $A : X \to 2^D$ be upper semicontinuous with closed and convex values. Then there exists a point $\widehat{x} \in coD$ such that either $\widehat{x} \in A(\widehat{x})$ and $P(\widehat{x}) = \emptyset$ or $\widehat{x} \notin A(\widehat{x})$ and $A(\widehat{x}) \cap P(\widehat{x}) = \emptyset$.*

Proof: Let $F = \{x \in X : x \in A(x)\}$. We first note that F is closed in X since A is upper semicontinuous with closed values. Define $\phi : X \to 2^D$ by

$$\phi(x) = \begin{cases} P(x), & \text{if } x \in F, \\ A(x) \cap P(x), & \text{if } x \notin F. \end{cases}$$

If $x \notin F$ and $A(x) \cap P(x) \neq \emptyset$, then $X \setminus F$ is an open neighbourhood of x in X and since P is $\mathcal{U}$-majorised, there exist an open neighbourhood $N(x)$ of x in X and a mapping $\psi_x : N(x) \to 2^D$ such that (1) for each $z \in N(x)$, $P(z) \subset \psi_x(z)$ and $z \notin \psi_x(z)$ and (2) ψ_x is upper semicontinuous with closed and convex values. Without loss of generality, we may assume that $N(x) \subset X \setminus F$. We now define the mapping $\Psi_x : X \to 2^D$ by $\Psi_x(z) = A(z) \cap \psi_x(z)$ for each $z \in N(x)$. Then (1) again by Lemma 2.2 (note that D is compact Hausdorff so that D is normal), Ψ_x is upper semicontinuous with closed and convex values and (2) for each $z \in N(x)$, $z \notin \Psi_x(z)$. Thus Ψ_x is a $\mathcal{U}$-majorant of ϕ at x.

Now suppose that $x \in F$ and $P(x) \neq \emptyset$; then by assumption there exist an open neighbourhood $N(x)$ of x in X and $\psi_x : N(x) \to 2^D$ such that (a) $P(z) \subset \psi_x(z)$ and $z \notin \psi_x(z)$ for each $z \in N(x)$ and (b) ψ_x is upper semicontinuous with closed and convex values. Define $\psi'_x : N(x) \to 2^D$ by

$$\psi'_x(z) = \begin{cases} \psi_x(z), & \text{if } z \in N(x) \cap F, \\ A(x) \cap \psi_x(z), & \text{if } z \in N(x) \setminus F, \end{cases}$$

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then (i) for each $z \in N(x)$, it is easy to see that $\phi(z) \subset \psi'_x(z)$ and $z \notin \psi'_x(z)$, (ii) the mapping $A \cap \psi_x : N(x) \setminus F \to 2^D$ defined by $(A \cap \psi_x)(z) = A(z) \cap \psi_x(z)$ for each $z \in N(x) \setminus F$ is upper semicontinuous with closed and convex values by Lemma 2.2. It follows that the mapping ψ'_x is also upper semicontinuous with closed and convex values by Lemma 2.1 since $N(x) \setminus F$ is open in $N(x)$. This shows that ψ'_x is a $\mathcal{U}$-majorant of ϕ at x. Therefore ϕ is $\mathcal{U}$-majorised. By Theorem 2.4, there exists a point $\widehat{x} \in coD \subset X$ such that $\phi(x) = \emptyset$. By the definition of ϕ, either $P(\widehat{x}) = \emptyset$ and $\widehat{x} \in A(\widehat{x})$ or $A(\widehat{x}) \cap P(\widehat{x}) = \emptyset$ and $\widehat{x} \notin A(\widehat{x})$. ∎

The following is an equilibrium existence theorem of a qualitative game:

THEOREM 3.2. *Let $\Gamma = (X_i, P_i)_{i \in I}$ be a qualitative game such that for each $i \in I$,*

(a) *X_i is a non-empty convex subset of a Hausdorff locally convex topological vector space E_i and D_i is a non-empty compact subset of X_i;*
(b) *the set $E^i = \{x \in X : P_i(x) \neq \emptyset\}$ is open in X;*
(c) *$P_i : E^i \to 2^{D_i}$ is $\mathcal{U}$-majorised;*
(d) *there exists a non-empty compact and convex subset F_i of D_i such that $F_i \cap P_i(x) \neq \emptyset$ for each $x \in E^i$.*

Then there exists a point $x \in X$ such that $P_i(x_i) = \emptyset$ for all $i \in I$.

PROOF: Since D_i is a non-empty compact subset of X_i for each $i \in I$, the set $D = \prod_{i \in I} D_i$ is also a non-empty compact subset of X. Now for each $x \in X$, let $I(x) = \{i \in I : P_i(x) \neq \emptyset\}$. Define a correspondence $P : X \to 2^D$ by

$$P(x) = \begin{cases} \bigcap_{i \in I(x)} P'_i(x), & \text{if } I(x) \neq \emptyset, \\ \emptyset, & \text{if } I(x) = \emptyset, \end{cases}$$

where $P'_i(x) = \prod_{\substack{j \neq i, \\ j \in I}} F_j \otimes P_i(x)$ for each $x \in X$.

Then for each $x \in X$ with $I(x) \neq \emptyset$, $P(x) \neq \emptyset$. Let $x \in X$ be such that $P(x) \neq \emptyset$. Fix an $i \in I(x)$. By assumption (c), there exist an open neighbourhood $N(x)$ of x in E^i and $\phi_i : N(x) \to 2^{D_i}$ such that (i) for each $z \in N(x)$, $P_i(z) \subset \phi_i(z)$ and $\pi_i(z) \notin \phi_i(z)$ and (ii) ϕ_i is upper semicontinuous with closed and convex values. Note that by (b), $N(x)$ is also an open neighbourhood of x in X and for each $z \in N(x)$, $P_i(z) \neq \emptyset$ so that $i \in I(z)$ for each $z \in N(x)$. Now we define $\Phi_x : N(x) \to 2^D$ by $\Phi_x(z) = \prod_{\substack{j \neq i, \\ j \in I}} F_j \otimes \phi_i(z)$ for each $z \in N(x)$. We observe that (1) for each $z \in N(x)$, $P(z) \subset P'_i(z) \subset \Phi_x(z)$ and $z \notin \Phi_x(z)$; (2) Φ_x has closed and convex values and (3) since $\prod_{\substack{j \neq i, \\ j \in I}} F_j$ and $\phi_x(z)$ are compact for each $z \in N(x)$, it is easy to see that Φ_x is also upper semicontinuous. Therefore, Φ_x is an $\mathcal{U}$-majorant of P at x. Thus P is $\mathcal{U}$-majorised. Now by Theorem 2.4, there exists a point $x \in coD \subset X$ such that $P(x) = \emptyset$ which implies that $P_i(x) = \emptyset$ for all $i \in I$. $\square$

For the existence of equilibria of abstract economies (or generalised games) in which preferences are not $\mathcal{U}$-majorised in topological vector spaces or locally convex topological vector spaces, we refer to [**1**, **2**, **3**, **4**, **5**, **6**, **7**, **8**, **9**, **11**, **12**, **15**, **18**, **19**, **20**, **21**, **22**, **23**, **24**, **25**, **26**, **27**, **28**, **29**, **30**], and the references wherein.

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Department of Mathematics, Statistics
and Computing Science
Dalhousie University
Halifax, Nova Scotia
Canada B3H 3J5
kktan@cs.dal.ca and yuan@cs.dal.ca