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Abstract of Australasian PhD thesis

Elliptic partial differential equations with mixed boundary conditions

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In this thesis we consider the mixed boundary value problem

$$Au = f \quad \text{in} \quad \Omega ,$$
$$B_j^+ u = g_j^+ \quad \text{on} \quad \Gamma^+ , \quad j = 0, \ldots, m-1 ,$$
$$B_j^- u = g_j^- \quad \text{on} \quad \Gamma^- , \quad j = 0, \ldots, m-1 .$$

Ω is a bounded open subset of R^n whose boundary Γ is divided into two disjoint open sections Γ^+ and Γ^- by an $n - 2$ dimensional manifold ω in Γ. A is a properly elliptic partial differential operator on $\overline{\Omega}$ of order $2m$, and $B^\pm = \left(B_j^\pm\right)_{j=0}^{m-1}$ are normal boundary operators of orders $\left(m_j^\pm\right)_{j=0}^{m-1}$, satisfying the complementing condition with respect to A on $\overline{\Gamma^\pm}$ respectively. The coefficients of the operators, and Γ^+, Γ^-, and ω are all assumed arbitrarily smooth.

We find necessary and sufficient conditions on the coefficients of the operators in order that the problem be well-posed in Sobolev spaces. In fact, we construct an open subset I of the reals such that, if $D^s = \left\{u \in H^s(\overline{\Omega}) : Au = 0\right\}$ and $R_\pm^s = \prod_{j=0}^{m-1} H^{s-m_j^\pm-\frac{1}{2}}\left(\overline{\Gamma^\pm}\right)$, then, for $s \not\equiv \frac{1}{2} \pmod 1$, $(B^+, B^-) : D^s \to R_+^s \times R_-^s$ is Fredholm if and only if

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$s \in I$. Moreover, $I = \bigcap_{x \in \omega} I_x$ where the sets I_x are determined algebraically by the coefficients of the operators at x .

When A is a second order operator, the sets I_x are as follows. If $n \geq 4$, I_x is an open interval of length 1 . If $n = 3$, I_x is either an open interval of length 1 or is empty, depending on the coefficients of the operators at x . Finally, if $n = 2$, I_x is the set of reals which are not congruent (modulo 1) to some exceptional value e_x .

Because we assume $B^{\pm}$ to be normal, we are able to construct bounded sesquilinear forms $J^s[u, v]$ on a subspace of $H^s(\bar{\Omega}) \times H^{2m-s}(\bar{\Omega})$ which are related naturally to the maps $(B^+, B^-) : D^s \to R^s_+ \times R^s_-$. This relationship is proved by appealing to a five lemma which is proved and often used in the thesis. Via these forms, the problem of considering the Fredholm properties of $B^{\pm}$ is localised, so that for each $x \in \omega$ we obtain a similar problem for homogeneous operators with constant coefficients in $\mathbb{R}^n_+$. These operators act, not in Sobolev spaces, but in the closely related "spaces with homogeneous norms", and we develop some of the theory of these spaces. The problem is then readily converted to a Wiener-Hopf problem and solved. The thesis concludes with an interpolation theorem for the associated sesquilinear forms.