BOOK REVIEWS

BERBERIAN, S. K., *Introduction to Hilbert Space* (Oxford University Press, 1961), 206 pp., 52s.

This little book has the unusual merit of being completely self-contained, nothing being assumed except the most elementary real analysis. It is written for the use of first and second year graduate students (U.S. pattern) but would be equally suitable for second or third year honours students in the U.K. All proofs are given in full detail and without appeal to hand waving, and the book contains many easy exercises, so that it can be recommended for unassisted reading by beginners. Naturally, the cost of this is that the book does not reach very far into the theory of operators in Hilbert Space, ending in fact with the spectral theorem for a compact normal operator. The spectral theorem for a bounded self-adjoint operator can now be proved so easily and explicitly that the reviewer believes that it could have been included without departing from the author's principles. However, this is an introduction, and as such it is excellent. Among the exercises are some, distinguished by a star, which are not exercises in the usual sense at all, but are statements of more advanced theorems. For example, one of these " exercises " is a statement of the Gelfand-Neumark theorem on the representation of B^*-algebras by operator algebras. The reader should perhaps be warned not to spend too large a part of his life in trying to prove these theorems for himself; but they are valuable in showing the reader something of the theories to which he is being introduced.

The professional mathematician will probably prefer to read a more condensed account, for example the relevant chapters of " Leçons d'analyse fontionelle " by F. Riesz and B. Sz.-Nagy, but for teaching purposes this book is ideal.

F. F. BONSALL

EGGLESTON, H. G., *Elementary Real Analysis* (Cambridge University Press, 1962), viii+282 pp., 37s. 6d.

The word " elementary " in the title of this book indicates that the course is self-contained and that advanced topics such as Lebesgue integration and Fourier series have been omitted. The choice of subjects in the theory of convergence and of functions of a real variable is wide, including for example series of complex terms, double series, functions of bounded variation and the Riemann-Stieltjes integral, and the treatment is remarkably full for a book whose basic length is only 205 pages. The last chapter gives a condensed account of the theory of functions of two real variables, with some results on repeated integrals, but double integrals are not included. An appendix deals with the properties of hemi-groups, groups and fields, with the definition of the integers by means of Peano's axioms, and with the definition of rational, real and complex numbers. In addition to the worked examples in the text, some 55 pages are devoted to hints on the solution of the exercises which follow each chapter.

This is a book which will be extremely useful to those who teach analysis, but in the reviewer's opinion the subject has been made unnecessarily difficult for students by an excessive use of mathematical symbols and jargon. Many statements in the book could be expressed more clearly, and a few more briefly, in plain English.

The use of English is not, however, the book's strongest point. The punctuation leaves something to be desired from time to time, particularly in the early chapters.

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Even the choice of words is occasionally careless. Thus on page 18 the author remarks that, in testing a sequence for convergence, we need to know the limit to which it converges. If we know the limit, there can surely be no question of testing the sequence for convergence; but even if we are trying to establish convergence we do not necessarily need to know the limit, as the author points out in his very next sentence. Again, after a student has read on page 20 that " it is nonsense to write $1/n \to 1/n^2$ as $n \to \infty$ " but that " we may be able to use the $\sim$ notation in such a case ", who is to blame him if he writes $1/n \sim 1/n^2$?

The author's use of symbols is sometimes misleading. For example, his class $IR(a, b)$ of functions improperly integrable over (a, b) apparently includes $R(a, b)$ as a sub-class, and the symbol $\forall$ is defined to mean " for all " but is used repeatedly to mean " for any ".

The printing and lay-out of the book are of the high standard always associated with the Cambridge University Press, though the consistent use of German letters to denote functions gives the text an unfamiliar appearance.

PHILIP HEYWOOD

ELSGOLC, L. E., *Calculus of Variations* (International series of monographs on Pure and Applied Mathematics, Vol. 19, Pergamon Press, Oxford, 1961), 178 pp., 30s.

The book begins with a preface to the " first Russian edition " but no mention is made of when and where this edition was published. The present English edition is printed in Poland; no information is given as to the identity of the translator into English.

The aim of the book is to provide students of engineering and technology with the opportunity of becoming familiar with the basic notions and standard procedures of the calculus of variations. The ground covered is the classical discussion associated with such names as Euler, Lagrange, Legendre, Jacobi, and includes a discussion of sufficient conditions based on the Weierstrassian field of extremals. The book finishes with a rather sketchy chapter on direct methods, such as Ritz's method.

The book is certainly not one for the Pure mathematician as there is very little pretence at rigour. Occasionally continuity or differentiability is mentioned, but at other times the problems are treated in a purely manipulative manner. On page 23 we find the statement " It is also assumed that the third derivative of the function $F(x, y, y')$ exists "; is " third derivative " a mistranslation of " partial derivatives of third order "? Again at the bottom of page 47 we read that the function $F(x, y, z, p, q)$ " is supposed to be differentiable ". Much more than differentiability of course is required. In any case the exact points at which existence of derivatives is required are not shown to the student.

Chapter I deals with fixed boundary problems, covering the usual ground and finishes with a rather inadequate discussion of the standard problem in parametric form. Chapter II deals with cases of variable boundary problems and contains a good account of applications to problems involving reflection and refraction. Chapter III deals with fields of extremals and in it sufficiency conditions are clearly displayed. Chapter IV deals with various problems of constrained extrema and includes a discussion of the isoperimetric problem. For the type of student for which the book is specially designed this chapter would seem to be too short and sketchy. This is true also of Chapter V which deals with direct methods. Perhaps the most useful part of the book is the large collection of examples. Besides many worked examples in the text there are problems at the end of each chapter and solutions to these are provided at the end of the book.

There is an attached errata slip giving five misprints but a first reading revealed a number of undetected errors which are either typographical or errors in translation.

R. P. GILLESPIE