May Term, 1930

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1

May Term. Lecture I. 4:45

Generality.

Very difficult, because many different kinds.

This doesn't mean a genuine concept, with species of it.

What's meant that we use expressions "all, some, every" in entirely different ways.

You can't therefore say one has one property, the other another.

But yet it's not a mere coincidence that we use the words in these ways.

They really have something in common – a structural quality.

It is like integers, & complex numbers.

In this case it is specially clear what the internal similarity consists in,

it is that the grammatical rules applying to one sort apply also in part to the other.

This is kind of similarity between space & time, brightness & pitch: e.g. that they form a series.

If we wrote a grammar of RED, BLUE, YELLOW, GREEN, & of C, C#, D, D# we should find some rules in common.

(1) Kind to which Russell's notation applies. 4:46

(x)ϕx, (∃x)ϕx.

i.e. where (1) can be regarded as logical product

(2) can be regarded as a logical sum.

Here the entities in question are what we denote by proper names.

These can be distinguished that in

"ξ exists", a proper name will make nonsense.

I.e. here x must take such values, that if you substitute them in "ξ exists", you get nonsense.

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This would be the sense of all in

All the primary colours are in this picture.

= ϕa . ϕb . ϕc . ϕd, & another proposition asserting that these are all is not wanted.

Contrast "All the people in this room have gowns",

here it isn't a logical product; because whichever you give, you must add "And there are no others".

I can't say "And there are no other primary colours".

This is comparatively rare.

(2) Oftener we have one which doesn't presuppose a totality.

"I met a man" can't = a disjunction.

"I met one of the people now alive" = a disjunction.

4:47 Take "There is a white circle in this square".

What sort of thing could be "a circle in a square", "a man in this room"?

We are regarding these as possible predicates: Frege said concepts were this.

We think of it as a substrate.

When we say "This is circular"; what is "this"? It must be something which could be recognised even, if it were not circular.

Giving names is only possible in very special cases.

Bring 2 pieces of chalk together; make them coalesce; & then separate them. Which is which?

If you could make 2 exactly similar chairs coalesce, you couldn't distinguish them.

"This" has ever so many different kinds of meaning, just as "pointing to" has.

I do the same thing, whether I point to a man or his colour; but here pointing to means something quite different.

If this were Russell's generality, there would have to be meaning in

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$\sim(\exists x)\sim\phi x$

= Everything is a circle in this square.

We must therefore find something which isn't necessarily a white circle in this square. 4:48

To find such a thing; you can take a point which is the centre of a circle: that need not have been.

This is a circle = Here is a circle.

There are a pair of coordinates such that they define a point which is a centre of a white circle in this square.

There is no totality of such pairs: an infinite totality isn't a totality.

In this case $(\exists x)\phi x$ isn't a logical sum.

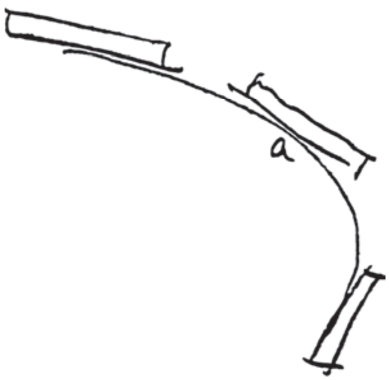
And that $(x)\phi x$ isn't a logical product

is made clear by

$\sim(\exists x)\sim\phi x$. This would mean "Every point is the centre of a circle" – which is unimaginable.[1]

Take a patch continually varying in colour from white to black.

Or take



when we say, it must have been at a intermediately.

[1] Moore later added: <because there are an infinite number of points?>

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4:49 The footrule rolled from here to there; & I saw it.

Does it follow from this that it must have been at a?

People say

> "If I see a continuous transition, I must see all the infinite shades of grey".

But what is it "to see a shade of grey"?

I must mean something which occupies a finite part, & therefore not an infinite number.

(2) Footrule.

If we talk of visual place; it doesn't follow that it was here.

I could see it there, only if it was there for a finite time.

In physical sense, it perhaps never was there: it might have skipped.

There is a sense in which we can say it was at all intermediate places.

And that is when we're talking of hypotheses.

A proposition can be verified or falsified, & is equivalent to a method of verifying or falsifying.

4:50 Hypotheses are not verifiable or falsifiable in the same sense.

Most of our sentences are hypotheses.

E.g. "There is a man sitting on the last bench but one".

Not[2] if we take it as pointing to the future, as we do.

"This is a piece of chalk" expresses a series of expectations, which essentially goes on for ever.

A hypothesis is a law by means of which propositions are constructed.

[2] After 'Not', Moore later added: <verifiable?>

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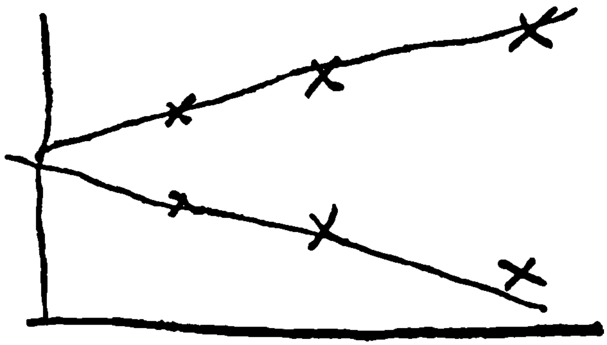
I can't verify that a lark has laid this egg, if I didn't see it do so.

What then do I mean by: A lark did lay it?



Suppose whenever we move upwards we see a spark.

What's meant by saying there was a flash, which we have never seen, & never shall?

We've abstracted from our experience, a rule pointing to the future, & <u>not</u> 4:51
verifiable because it points to the future ad <u>infinitum</u>.

It's a formal rule, by means of which we construct propositions: e.g. that in <u>this case</u> I shall get 2 flashes.

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2

4:52 May Term. II.

Generality. Kinds

(1) Correctly expressed by (x)ϕx, (∃x)ϕx, where only finite number of values.

(2) Where variable has an infinite possibility.

"I will divide this distance somewhere" ≠ an infinite logical sum, either here, or there.

But this isn't an infinite logical sum.

Many things we can't understand; but nothing in grammar is a thing we don't understand: grammar is complete.

This seems not to be so, because there is such a thing as mathematical discovery, & therefore it looks as if what is discovered is what we didn't yet know.

But there can be no incompleteness except within a space; there are no gaps in grammar: it is always complete.

Sheffer's discovery of stroke notation is a grammatical discovery. It's misleading to call this discovery.

Discovery proper can only be made where there's a system in which we can look for something.

Sheffer had no system by which to find p | q; nor have we.

Sheffer found a new space.

People ask: is p | p really what we call ~p?

4:53 The answer is: All that is wanted is a system of a given logical multiplicity; & Sheffer found such a system.

Before we couldn't have said: There's a gap there; there must be a system with only one logical constant.

Mathematical discovery is always unmethodical: you have no method for making the discovery.

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In logic & mathematics, it never is unimportant how you get there. If you get there, in a different way, you haven't got there.

Is every even number the sum of 2 primes?[3]

Has this sense?

(1) We have no method of answering this.

Therefore we can't look for the solution in the sense in which we can where we have.

What are mathematicians doing, when they work at it?

Suppose you'd forgotten how to multiply: would 224 × 37 have a meaning for you?

In logic & mathematics, you can't know the same thing in 2 independent ways.

Nor is there in case of sense-data.

In case of hypothesis, there are different evidences for same. Nowhere else.

What Pythagoras proved, is not what a man suspected from measuring 4:54
triangles.[4]

Measuring a triangle is an experiment.

Euclidean Geometry does not prophesy about results of an experiment.

What it says is: If I measure, & get 181, then, in Euclidean way of expression, I must say "I've measured wrong".[5]

[3] The statement that every even integer greater than 2 can be expressed as the sum of two primes is known as 'Goldbach's Conjecture'. It remains neither proved nor disproved.

[4] Pythagoras's theorem, perhaps the most famous theorem of Euclidean geometry, states that the square of the hypotenuse of a right-angled triangle (its longest side) is equal to the sum of the squares of the two other sides.

[5] Moore's notes give the angle as 191, but this is almost certainly a mistake. Lee's notes for this part of the lecture read '181°', not '191°' (Wittgenstein 1980, 17). See also 4:16.

3

4:55 May Term, III.

Sheffer's discovery.

Are any 2 different systems of signs different "spaces"? No.

If 2 systems are translatable into one another, they are the same.

Then Sheffer's is same as Russell's.
Sheffer didn't only introduce a definition.
He does define $p \mid q = \sim p \,.\, \sim q$.
But this isn't his discovery: Russell might have used this ~~definition~~ shorthand, without the discovery.
And Sheffer needn't have used shorthand.

His Discovery is $\left(\begin{array}{c} \sim p \,.\, \sim p = \sim p \\ \sim(\sim p \,.\, \sim q) \,.\, \sim(\sim p \,.\, \sim q) = p \vee q \end{array} \right.$

Frege uses and & not

Sheffer's discovery = $\begin{cases} \sim p \,.\, \sim p = \sim p \\ \sim(\sim p \,.\, \sim p) \,.\, \sim(\sim q \,.\, \sim q) = p \,.\, q \end{cases}$

Sheffer's discovery is to discover an aspect of the equations.

4:56 It's conceivable that Frege should have written everything in this way, & yet have said he had 2 primitive ideas.[6]

[6] Moore's summary notes: <Frege might have written everything with stroke, & yet said he had 2 primitive ideas> (10:71). In *Begriffsschrift*, Frege does have two primitive truth-functional connectives. However, they are not, as suggested above, 'and' and 'not', but rather 'not' and 'if-then'. See Frege 1879, §5-§7. Frege explicitly discusses this choice of connectives on page 20.

Perhaps to say Sheffer discovered a new "space" is misleading.

Point about impossibility of looking for space.

You can't find a connection in grammar, which is already there.

A "space" means everything of which you must be certain, in order to ask a question.

If I'm to look whether so & so is so, or not, that which I want to discover must be entirely describable beforehand.

You will find a ~~red~~ circle of this shade (shewing a colour) describes completely.

> ϕA contains a symbol which $(\exists x)\phi x$ doesn't, but the 2 descriptions are entirely alike.

Now "I'm trying to find a system with only one primitive idea" doesn't describe completely or exactly.

What makes it possible to describe is that description is logically all right.

Now you can try to find whether what is logically all right is physically so: 4:57

A generality which presupposes infinite possibilities is not therefore more complex.

When $(\exists x)\phi x$ is a logical sum, & $(x)\phi x$ a logical product, the complexity of ϕa
ϕb
ϕc
is already contained. And a disjunction of 4 numbers is more complex than one with 3.

If p entails q, then p asserts q.

Hence if p asserted an infinite disjunction, it would be infinitely complex.

Does an infinity follow from: On this band there's a continuous transition from white to black?

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You might think so, because you can say: This shade must be there; & so of all. But the shade occupies a finite stretch in the band.

So with a moving ball: It must have touched this end, & so an infinite number.

4:58 If we're talking of visual moving continuously, it never was anywhere: a thing only is in places where it stops.

With a hypothesis you can make up the proposition that it touched this, & that & so on ad infinitum.

In case of a figure in Euclid, the infinite possibility is represented by infinite possibility of visual space.

There's another sort of generality in logic, e.g. in

$$p \vee \sim p$$

Here's a real variable, & an infinity of special cases.

(1) This is not a logical product.

(2) We must have some idea of what can be substituted.

(3) No definition will help us.

The proof that angles of a triangle are 180, is a proof about space, not about this particular triangle.

And if it did prove about this triangle, it wouldn't prove about any other.

4:59 If there's an infinite possibility in your proof, this must be represented by an infinite possibility in your symbols: not by a description, nor by word "infinite".

Mathematical Induction

You seem to see: That can be done, without knowing how, in problem of "How many times must chalk be lifted to make this network?"

How is this? He has a symbolism already, but can't yet translate it into another.

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4

May Term, IV. 4:60

Generality.

Points, lines, planes: we can replace points, by lines, & in physics take events or colours.

Geometry & grammar: gives rules which allow us to use our symbols in a certain way.

E.g. suppose we apply it to a coloured plane: ~~e.g.~~ infinite Euclidean:

here lines are border lines between colours, & points are where borders intersect.

Now take "Any 2 points can be connected"

This = The proposition "they are connected" has sense.

And "any such proposition has sense" is grammar.

Grammar is unjustifiable by means of language.

E.g. "This combination of symbols makes sense" can't be justified by a proposition about reality & the symbols & their relations.

Why not?

Grammar is, in a sense, a portrait of reality; but not like a picture of a man.

Now we have an old language by which we try to justify our new language.

We have to say: What the new language forbids, is really nonsense.

But what is forbidden in the new is also nonsense in the old.

Therefore grammar is in a sense autonomous; therefore can be looked at as 4:61
a game. Whence view that Mathematics is a game.

I will try to justify this.

Consider

"Any 2 points are connected by a straight line."

(not "all": but why not "all pairs of points"?)

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Now "I have a picture in which all the primary colours are" = one in which green, blue, red & yellow are.

There is no concept "primary colour": it is a pseudo-concept, or logical concept; & its name will be an index to (∃) in (∃x)ϕx.

In the case of "points" & "lines", it is more difficult, because here it appears as if these were concepts & things.

"Primary colour" is like "irrational number".

"All primary colours . . ." is a finite logical product.

But there's not a finite number of points or lines.

How does grammar express that a symbol is the symbol for a point?

You might say it says: If A is a name of a point, then certain things are allowed.

E.g. It is allowed to say "Two lines meet A". But then, how about "lines"?

4:62 It would mean that the variables determine one another.[7]

The explanation is that grammar is a set of rules applying to a certain set of symbols, like chess.

This gives a hint, as to the sort of generality in "Any 2 points".

It's like that of a rule of chess: E.g. "A pawn can be moved".

It may be objected that only finite number of pawns.

But we can imagine infinite games.

We can make rule: You can play this game with as many pawns[8] as you like: we can give permission to play with any finite number, not with an infinite number.

A rule here could be: A pawn may be moved in such & such a way; where there is no definite finite number of pawns.

This rule gives the rule according to which the rules for particular-sized chess-boards vary.

[7] Moore's summary notes: <"Point" & "line" seem to determine one another.> (10:71)

[8] The word looks more like 'primes', but this was probably a mistake on Moore's part.

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They solve a problem about the possibility of movement.

We could make use of a game of chess, as of geometry.

2 kinds of generality in geometry.

(1) that of rules, e.g. concerning movements of pawns; which is one in which only symbols are concerned, & is inductive[9]

(2) that of its application, which can't be expressed, because it depends on what world is like. 4:63

This latter is generality of arithmetic.

E.g. the rules 1, (1) + 1, ((1) + 1) + 1

This is why great difficulty in defining numbers: they apply to all sorts of different "things".

There are 2 sorts of "propositions" in mathematics, neither of them at all like what are usually called propositions.

(1) We prove one kind by a chain of equations.

This proceeds from axioms to other equations, by means of axioms.

$$(x + y)^2 = + 2xy + y^2$$

We can start with distributive & associative laws.[10]

(2) Proof by induction.

Take the series 1, (1) + 1, ((1) + 1) + 1, . . .

This can be written

$[1, \xi, \xi + 1]$ = First number, general number, & its next.

This has no meaning unless first number is given.

Def. $a + (b + 1) \stackrel{\text{def}}{=} (a + b) + 1$ D[11] 4:64

If b is 1, & a is a numeral, we get

[9] Moore later added: <(mathematical induction)>

[10] Moore later added: <(He intended to discuss Distributive as well as Associative, but began with latter & never got to Distributive)>

[11] Moore later added: <= Def.> just below this 'D'.

$$\underbrace{a + ((1) + 1)}_{\text{Known to be numeral}} = (a + 1) + 1$$

$$a + (((1) + 1) + 1) = (a + ((1) + 1)) + 1$$

& so on.

4:63v[12] What does this define? It defines the addition of any 2 numbers, given the meaning of + between 2 "1"s.

It is not a definition in the sense in which

$(1) + 1 = 2$ is so.

For it only says something about a & b, not about numbers.

It works as a series of definitions

$$a + ((1) + 1) = (a + 1) + 1$$

$$a + (((1) + 1) + 1) = \underbrace{(a + ((1) + 1)) + 1}$$ /by 1/

& so on

4:64 Prove.[13] $a + (b + c) = (a + b) + c$ A

Assume it for c, & prove it for c + 1.

I.e. prove $a + (b + (c + 1)) = (a + b) + (c + 1)$

Use D & get

$$a + (b + (c + 1)) = a + ((b + c) + 1) \overset{D}{=} (a + (b + c)) + 1$$

$$\overset{A}{=} ((a + b) + c) + 1 \overset{D}{=} (a + b) + (c + 1).$$

Shorter Form.

$$a + (b + (c + 1)) \overset{D}{=} (a + (b + c)) + 1$$

$$\overset{A}{=} ((a + b) + c) + 1 \overset{D}{=} (a + b) + (c + 1).$$

12 Usually, Moore wrote his notes on the right-hand side of each opening, leaving the left-hand side blank for later comments, clarifications, or criticism. However, 4:63v, the page facing 4:64, contains material that reads like additional lecture notes. It is written in slightly thinner pencil strokes, suggesting that it may have been written later on. We have chosen to break it into two parts and insert them at the appropriate places in 4:64.

13 Moore later added: <Associative Law>

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2 striking features. 4:63v

(1) that in proving A we assume what we ought to prove

(2) that step 2[14] doesn't seem really to follow from definition,

Since (b + c) is not a numerical symbol.

It's justified, as we know, if c = 1.

Therefore real proof has to begin with c = 1

& what we really have is a whole series of ~~proofs~~ chains of equations, or rather a law which produces them.

Why can we use A? Because, if we substitute 1 for c, then 2nd transition is allowed by D. 4:64

We then get (a + (b + 2)) = ((a + b) + 2). 4:65

The proof really rests entirely on D, but it isn't simply a chain of equations.

The proof by induction means that a chain goes on, which isn't expressed by the chain.

The infinite law can't be expressed by an equation.

What relation has A to the proof?

A couldn't be the last of a chain of equations, because it is assumed as primitive.

Point is meaning of "true for all numbers".

[14] Moore later added a line connecting 'step 2' to '(a + (b + c)) + 1', the second formula in the shorter form of the proof.

5

4:66[15]

$$\overset{1}{a + (b + ((1) + 1))} \overset{D}{=} \overset{2}{a + ((b + 1) + 1)} \overset{D}{=} \overset{3}{(a + (b + 1)) + 1}$$

$$\overset{D}{=} \overset{4}{((a + b) + 1) + 1} \overset{D}{=} \overset{5}{(a + b) + ((1) + 1)}$$ [16]

This proves that 1 = 5, which is not what we want.

But we see what we do want; namely that we can similarly get a proof with 2, then with 3 & so on.
It may be called a spiral.

How does this prove Associative Law?
It's said: We know it holds for c = 1.
We know it holds for n + 1, if for 2.
∴ We know it holds for all numbers.
But the last is wrong.

We haven't proved it as algebraic formula; it is only a postulate as such, i.e. a rule of a game.
What we've proved is that the rule applies, if a, b & c are numbers.[17]

[15] The bottom half of the previous page is blank. In all the other notes for this year, a partially completed page is always followed by a new lecture, with a Roman numeral at the top. While Moore did not write a Roman numeral at the top of this page, this is most likely the point at which Lecture 5 began. However, the material after the page break is a continuation of the previous discussion, and according to King and Lee, both sets of topics were discussed on 19 May, the date of Lecture 4. See Wittgenstein 1980, xvi, 17–20.

[16] In each of the five formulas, the number '2' is written just above the first instance of '1', illustrating that the proof can be repeated for any number, not just for 1.

[17] Moore's summary notes: <As an algebraic formula, Associative Law is only a rule of a game & we haven't proved this; what we have proved is that it applies if a, b, c are numbers.> (10:07)

Suppose we have a chain

$$A \rightarrow B \rightarrow C \rightarrow E$$

This shews an internal relation between A & E.

If an equation is a grammatical rule, what can its importance be? 4:67

That I can replace 2 + 2 by 4, has no mathematical importance.

I could have written 2 + 2 = 5.

What gives it its importance is its demonstrability.

I.e. that, in virtue of A, $a + (b + 1) = (a + b) + 1$ D

it can be proved from

$(1) + 1 = 2$ def.

$(((1) + 1) + 1) + 1) = 4$ def.

You can't question or affirm or negate the definitions: what is affirmable is the internal relation between $2 + 2 = 4$ & the definitions.

The definitions therefore are not like propositions; what is like a proposition is the assertion of internal relation.

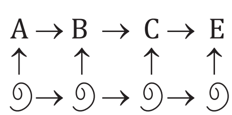
Proofs by induction.

We could prove in this way

$$(a + b)^2 = a^2 + 2ab + b^2$$

This would again be a spiral proof.

Thus taking



But the relation expressed by vertical arrows is quite different from that by sideways arrows. 4:68

Hence what we've really shewn is that we can substitute A for one spiral, B for another & so on.

The relation between A & its spiral is more like that between a name & its thing, then that between a conclusion & its proof.

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In ordinary mathematics there is something analogous to a proof & a question.

But an induction can be neither correct nor incorrect.

The relation between A & D, is not at all the same as that between A & B.

Neither D nor A need refer to numbers; &, if not so taken, then A doesn't follow from D. There's no algebraic proof of A by D.

~~What the proof shews: is that A could be given a numerical meaning, if D already has one.~~

The spirals aren't questionable, or negatable or assertible, any more than definitions are.[18]

4:69 This is connected with:

Does Law of Excluded Middle apply to mathematical propositions? e.g. about infinity.

Brouwer suggests it doesn't hold.[19]

That there's an alternative to being true or false; viz. undecidable.

Suppose you say

Is $\pi' = \pi$?

We can't tell.[20]

To such a case no law of logic applies.

A question is essentially something which could be answered.

It's nonsense to say there are 777's in the infinite development.

If we found them in 10 years; we should have answered the question for a 10 years' development.

[18] For further discussion of inductive proof, spirals, and related matters, see *Philosophical Remarks* (Wittgenstein 1964) §163 ff., *The Big Typescript* (Wittgenstein 2005) §126 ff., and 6b:14.

[19] See Brouwer 1908 and Brouwer 1923.

[20] Moore later added: <π' = the number which, if there are 3 consecutive 7's in π, has 3 consecutive 5's in the place in which π has 3 7's.>

The impossibility of developing π, is not a physical impossibility; which must be of something I can describe.

With irrationals we introduce a new mathematical world, though correlated or comparable with rationals.

= we can define a > & < between irrationals, which has same multiplicity as > & < between rationals.

There's a logical similarity between rules for rationals & irrationals.

If Smith found a proof that no 3 sevens, then he would have found a new 4:70
irrational number, $\pi' = \pi$.

If he only found the 3 sevens, he would have found no new irrational.

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6

4:71 May Term VI.

Does Mathematics consist of Tautologies?

How did the idea of "tautology" arise? From thinking about: What is subject-matter of Logic?

Frege thought "logische Gegenstände",[21] Russell "logical constants".

Russell felt that "or" & "not" were not like ordinary names.

I was helped by Frege, who, in his first work, had <u>explained</u> (not defined) "or" etc. in terms of <u>true</u> & <u>false</u>.[22]

He gave a list something like

T	T
F	T
T	F
F	F

What Frege said was that p . q was true, if p, & q.

This means only to give a <u>property</u> of p . q: not a complete description.

But "or" "not" etc. seem to have no other properties: this list gives the <u>essence</u> of "and" etc.: it could stand <u>as a symbol for</u> "and".

$$p \, . \, q \quad = \quad \begin{array}{c|c|c} p & q & \\ \hline T & T & T \\ F & T & F \\ T & F & F \\ F & F & F \end{array}$$

4:72 The advantage of this symbol is that it shews <u>quantity</u> of sense: that some propositions say <u>more</u> about reality than others.

E.g. it says <u>more</u> than p or q (~(~p . ~q))

[21] German for 'logical objects'. [22] See Frege 1879, §7, 17–20.

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One gives more freedom, in the sense in which an infinitely long string gives complete freedom.

This symbol also shews that one combination, gives every freedom

viz. T
T
T
T

This, though obviously a particular case, is yet not a proposition at all – says nothing.

Thus it was clear Logic consists of tautologies – says nothing.

But what, then, is value of Logic?

Wittgenstein thought at first that generalised tautologies did say something: but soon saw that they say as little as the others.

The value lies in the fact that a particular combination of propositions does say nothing. Compare with a zero method of measurement: that the pointer remains at 0 shews something.

That $((p \supset q) \,.\, p) \supset q$ says nothing shews something about structure.

Mathematical propositions are not tautologies: why?

Ramsey thought he could shew that equations are tautologies.[23] His theory of identity consists of 2 parts 4:73

(1) functions in ~~identity~~ extension

(2) equations are tautologies.

One can talk of (2) without discussing (1).

He tried to find a function of 2 variables, such that if you substituted different values you got a contradiction, if same a tautology.

It's easy to find function doing latter;

e.g. $\phi x \supset \phi y$.

Let's ask whether this sort of function can define $x = y$?

[23] See part four of Ramsey's 'The Foundations of Mathematics' (1925) (Ramsey 1990, 212–16) and the letters written by Wittgenstein and Ramsey on identity in 1927 (Wittgenstein 2012, 158–61).

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No: because if you look at $\phi(\hat{x}) \supset \phi(\hat{y})$, you have no idea whether it is a tautology or not.

You can only see that it becomes one, if you already know that you have substituted same value. It can't help you to know whether a = b.

Objection to theory that a = b is a symbolic convention.

Ramsey referred to

I have n pairs of shoes, & $n^2 - 2n - 3 = 0$.[24]

saying that whole proposition wasn't partly about symbols.[25]

Answer. $(\exists n)((\hat{x}(\phi x) \in n) . (n^2 - 2n - 3 = 0))$.

Here you can replace equation by its root.

The proposition has no sense; unless you know solution or how to solve it.

4:74 Like "I have n shoes, & n is a number I've written on some blackboard in the world", which has no meaning.

It gives an incomplete proposition, & completes it by a symbolic rule; & says something only if you know how to use the rule.

Equations are merely symbolic conventions, & get significance only by being members of a calculus.[26]

What's the relation between this calculus, & a calculus of tautologies?

Take addition of cardinal numbers.

Russell wrote an addition theorem as a tautology as follows:

Use as definitions

$$(\exists x,y)\phi x . \phi y \stackrel{\text{def}}{=} (\exists(1+1)x)\phi x$$

$$(\exists x,y)\phi x . \phi y . \sim(\exists x,y,z)\phi x . \phi y . \phi z \stackrel{\text{def}}{=} (Ex,y)\phi x . \phi y$$

Then what Russell does is to say 2 + 3 = 5

$$((E2x)\phi x . (E3x)\psi x . \sim(\exists x)/\text{Ind.}/\phi x . \psi x) \supset (E(2+3)x)\phi x \vee \psi x.$$

[24] This sentence was originally 'I have x pairs of shoes, & $x^2 + 2x + 3 = 0$' but it was modified during the lecture.

[25] See Ramsey 1990, 180–3.

[26] Moore's summary notes: <Ramsey said that a = b is not a symbolic convention. But all equations are symbolic conventions, & get significance only by being members of a calculus.> (10:07)

What corresponds to theorem of addition is not this proposition, but that it is a tautology, if it is.

Take, as sample

((E1x)ϕx . (E1x)ψx . Ind.) ⊃ (Ex,y)((ϕx ∨ ψx) . (ϕy ∨ ψy)).

This is a tautology: but that it is depends entirely on number of symbols with the E's. 4:75

I use a calculus of numbers applied to calculus of tautologies.

E.g. replace 2, 3, 5 by 34, 57, 91.

The tautology can't therefore replace arithmetical calculus.

Objection to: "Calculus is a game".

Is it a game with ink & paper? No.

But also: Subject-matter of chess isn't pieces of wood.

What's characteristic of chess is logical multiplicity of its rules.

The confusion made in this argument is that rules of chess are about bits of wood.

Suppose you looked at 2 people playing chess as a natural phenomenon.

If there were, I could use the rules as hypotheses about pieces of wood of a particular shape.

The rules of calculus, & hypotheses would be of an entirely different nature.

In the calculus I have rules for the usage of so & so; not hypotheses about so & so.

Frege inferred that mathematics deals not with symbols, but with what is symbolised: that the symbols have meaning. 4:76

What is essential to the rules is the logical multiplicity which all the different possible symbols have in common.

But the game isn't a symbol for this.

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