Canad. J. Math. Vol. **69** (4), 2017 pp. 851-853 http://dx.doi.org/10.4153/CJM-2017-004-3 © Canadian Mathematical Society 2017



Erratum: Translation Groupoids and Orbifold Cohomology

Dorette Pronk and Laura Scull

Abstract. We correct an error in the proof of a lemma in *Translation groupoids and orbifold cohomology.* Canad. J. Math 62(2010), no. 3, 614–645. This error was pointed out to the authors by Li Du of the Georg-August-Universität at Göttingen, who also suggested the outline for the corrected proof.

This note contains a correction of an error in the proof of a lemma in [1]. This error was pointed out to the authors by Li Du of the Georg-August-Universität at Göttingen, who also suggested the outline for the following corrected proof.

The lemma in question reads as follows.

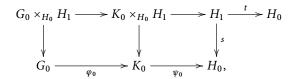
Lemma 8.1 ([1]) The class of essential equivalences between Lie groupoids satisfies the 3-for-2 property, i.e., if we have homomorphisms $\mathfrak{G} \xrightarrow{\varphi} \mathfrak{K} \xrightarrow{\psi} \mathfrak{H}$ such that two out of $\{\varphi, \psi, \varphi \circ \psi\}$ are essential equivalences, then so is the third.

The given proof of this lemma is incorrect in the case where $\psi \circ \varphi$ and ψ are essential equivalences. There the following is stated:

It is a standard property of fibre products that if any two out of (A), (B), and the whole square are fibre products, so is the third.

This is incorrect in general; in particular, when φ and $\psi \circ \varphi$ are merely fully faithful it is not necessary that ψ is also, and counter-examples can be created. Below is a corrected proof of the case in question.

Proof We consider the case where φ and $\psi \circ \varphi$ are essential equivalences. Since $\psi \circ \varphi$ is essentially surjective, the map $G_0 \times_{H_0} H_1 \rightarrow H_0$ is a surjective submersion. This map factors as the top arrow in the diagram



and we see that this implies that the composite of the last two maps, $K_0 \times_{H_0} H_1 \rightarrow H_0$, is a surjective submersion.

Received by the editors January 9, 2017.

Published electronically April 26, 2017.

AMS subject classification: 57S15.

Keywords: orbifold, equivariant homotopy theory, translation groupoid, bicategory of fractions.

Next we consider the diagram

$$\begin{array}{c|c} G_1 & \xrightarrow{\varphi_1} & K_1 & \xrightarrow{\psi_1} & H_1 \\ \hline (s,t) & & & & & \\ (s,t) & & & & & \\ G_0 \times G_0 & \xrightarrow{\varphi_0 \times \varphi_0} & K_0 \times K_0 & \xrightarrow{\psi_0 \times \psi_0} & H_0 \times H_0. \end{array}$$

Since φ and $\psi \circ \varphi$ are essential equivalences, the left square (A) and the entire rectangle are both pullbacks. We want to show that the right square has to be a pullback as well. As indicated by the discussion above, the fact that φ is essentially surjective is an important ingredient. In fact, we would like to assume that φ_0 is actually surjective.

If φ_0 is not surjective, then consider the weak pullback groupoid

$$G' = G \times_K^w K \xrightarrow{\varphi'} K$$
$$\pi \bigg|_{\chi} \cong \bigg|_{1_K}$$
$$G \xrightarrow{\varphi} K.$$

Since φ is an essential equivalence, so is φ' . In addition, π is also an essential equivalence, because it is a weak pullback of an identity arrow (which is obviously an essential equivalence).

So we replace (A) by a new square (A'), which is again a pullback:

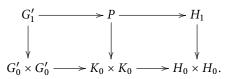
$$\begin{array}{c|c} G_1' & \xrightarrow{\varphi_1'} & K_1 & \xrightarrow{\psi_1} & H_1 \\ (s,t) & & & & & & \\ (s,t) & & & & & & \\ G_0' \times G_0' & \xrightarrow{\varphi_0' \times \varphi_0'} & K_0 \times K_0 & \xrightarrow{\psi_0 \times \psi_0} & H_0 \times H_0. \end{array}$$

Furthermore, the entire rectangle is again a pullback: note that $\psi \circ \varphi' \cong (\psi \circ \varphi) \circ \pi$. The latter is an essential equivalence as a composite of essential equivalences, and hence so is the former, because it is isomorphic to an essential equivalence. We also have that the map $\varphi': G'_0 = G_0 \times_{K_0} K_1 \times_{K_0} K_0 \to K_0$, defined by $(x, k, t(k)) \mapsto t(k)$, is surjective, since φ is essentially surjective.

Now consider the pullback

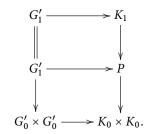
852

Since the map $s\pi_1$ is a surjective submersion, this pullback is a smooth manifold, and we get a smooth map $K_1 \rightarrow P = K_0 \times_{H_0,s} H_1 \times_{t,H_0} K_0$. Next consider the diagram



We know that the right square is a pullback, and therefore the left square is a pullback if and only if the whole rectangle is a pullback. But the whole rectangle is a pullback as we just observed, and so the left square is a pullback.

So now consider



The bottom square is a pullback according to the previous argument, and we know that the whole rectangle is a pullback, since $\varphi': G' \to K$ is fully faithful. Therefore, the top square is also a pullback.

Now the bottom map is a surjective submersion (it is surjective as argued above and it is a submersion because the groupoids are étale), and therefore the pullback map $G'_1 \rightarrow P$ is also a surjective submersion. Then looking at the top square, we see that the pullback of the map $K_1 \rightarrow P$ is the identity map, and hence a diffeomorphism. Therefore the original map must also have been a diffeomorphism, so $K_1 \cong P$ and the original square (B) is a pullback, as required.

References

 D. Pronk and L. Scull, *Translation groupoids and orbifold cohomology*. Canad. J. Math 62(2010), no. 3, 614–645. http://dx.doi.org/10.4153/CJM-2010-024-1

Department of Mathematics and Statistics, Dalhousie University, Halifax, NS B3H3J5 e-mail: pronk@mathstat.dal.ca

Department of Mathematics, Fort Lewis College, Durango, Colorado 81301-3999, USA e-mail: scull_l@fortlewis.edu