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Some Comments on the Problem of Rotation and Mixing in Stars

Leon Mestel

Astronomy Centre, Univ. of Sussex, Falmer, Brighton, BN1 9QJ, England

In the 1950's, Sweet (1950) and Öpik (1951) rediscussed the resolution of the von Zeipel paradox in a uniformly rotating stellar radiative zone, showing that the time-scale of the Eddington (1929)–Vogt (1925) thermally-driven circulation would be of the order of the Kelvin-Helmholtz time τ_{KH}, divided by an appropriate value for the ratio q of centrifugal to gravitational acceleration. In a Cowling-type star, the streamlines emerge near the poles of the convective core, traverse the radiative envelope, and return to the core at the equator. In a rapid rotator, the time τ_c of circulation is still short compared with the time-scale τ_n for transmutation in the core of H into He, and it seemed that the mean molecular weight μ would remain nearly uniform, with the star evolving up and slightly to the left of the Main Sequence in the H-R Diagram; while in a moderate rotator, there could still be enough mixing to affect noticeably the expected evolution towards the Giant Domain. *Prima facie*, significant mixing would occur provided the value $q(r_c)$ of q at the convective core surface exceeded $\tau_{KH}/\tau_n \approx 10^{-3}$.

Such continuous mixing assumes that the circulation persists. In the absence of any torques, the advection of angular momentum by the circulation itself will alter the angular velocity field, yielding analogous instantaneous circulation fields with structures that in general differ greatly from the E-V-S-O P_2 form (e.g. Kippenhahn 1958). Already in his 1950 paper, Sweet noted that a weak magnetic field could stabilize the circulation against break-up into cellules, or indeed prevent the rotation field settling into a non-uniform state that allows local radiative equlibrium to be satisfied without advection of energy by circulation (Schwarzschild 1947; Roxburgh 1964). In the subsequent work on continuous mixing by the circulation, the need for a magnetic torque was quite explicit. Because the circulation speeds are so much less than the rotation speeds, the obvious requirement that the Alfvén speed greatly exceed the circulation speed is easily satisfied, yet with the poloidal magnetic forces negligible over the bulk of the star. However, in the low-density surface regions, the (inevitably non-spherical) magnetic contribution to hydrostatic equilibrium must be included (Mestel and Moss 1977): in a self-consistent treatment of the energy equation, the outer cellule found by Öpik may disappear, as will the singular velocities found by Baker and Kippenhahn (1959) for even slight deviations from uniform rotation. In a strictly axisymmetric, friction-free system, the mathematics requires just near *iso*rotation, in principle allowing differential rotation of individual field lines; but penetration of field lines into the outer regions of the convective core will effectively couple different field lines, as will even a modest departure from strict axisymmetry in the magnetic field (Mestel et al. 1988; Moss et al. 1990).

However, a persisting E-S 'Ω-circulation' in a Cowling-type star sets up a non-spherical μ-distribution, requiring a corresponding non-spherical perturbation to the temperature field. The effect on energy transport yields now a 'μ-current' field, which tends to oppose the Ω-current. In order that mixing persist, the above criterion is replaced *prima facie* by the much more stringent condition $q(r_c) > (\tau_{KH}/\tau_n)^{1/2} \approx 1/30$, a result confirmed by detailed analysis (Mestel 1953; Mestel and Moss 1986). The difficulty is that in a uniformly rotating Cowling model star, q increases between core and surface by almost the same factor 30: a star capable of steady E-S mixing would be rotating beyond the allowed surface centrifugal limit. (Even more stringent conditions hold in an evolved star with a burnt-out core and a shell energy source (Mestel 1957).) If nothing else intervenes, then one possibility is that a 'μ-barrier' is set up, deflecting the circulation at the core surface and preventing core-envelope mixing. Perhaps more plausibly, 'creeping paralysis' sets in: some partially processed gas flows out of the core, setting up a μ-distribution that generates a μ-current field equal and opposite to the E-S field, so that radiative equilibrium is set up and again continuous mixing is halted.

Further discussion on the general problem bifurcates at this point, again depending on whether or not magnetic torques are active. In the non-magnetic problem, one can indeed picture chemical evolution of the star as due to an ongoing competition between on the one hand the tendency of circulation both to advect nuclear-processed gas and to set up a non-uniform rotation field, and on the other hand the opposing effects of dynamical instabilities which will try and reduce shear and simultaneously iron out the non-uniform μ-distribution. But in a star which is even weakly magnetic, again the magnetic stresses are able easily to offset any advection of angular momentum, so also preventing the shear instabilities that can yield ongoing mixing.

It is perfectly proper and indeed desirable that some workers should study in depth the strictly non-magnetic problem; but equally I feel it is incumbent on them to emphasize what severe upper limits on the internal magnetic field strengths are then implicitly imposed (e.g. Mestel and Weiss 1987).

In the first paper on μ-currents (Mestel 1953), it was noted that the whole process of mixing or non-mixing by the thermally-driven E-S currents in a stable radiative zone is very delicate, and one needs always to look for competing effects which can reduce or nullify the μ-current choke over the leisurely time-scale of main-sequence evolution. We have seen how a modest magnetic field can maintain nearly uniform rotation, preventing the shear instabilities that can cause mixing. There is however another, rather more subtle dynamical effect of a large-scale magnetic field that can cause mixing between the convective core and the radiative envelope. An obliquely rotating magnetic star – with the magnetic dipole non-aligned with the angular momentum axis – has some properties in common with a top (Spitzer 1958; Mestel 1999). The magnetic perturbations to the density cause an Eulerian nutation about the magnetic axis, and the consequent changes in the distribution of the centrifugal perturbations in turn require internal motions to maintain hydrostatic equilibrium: the reduced symmetry of the oblique rotator enforces coupling between the poloidal and toroidal dynamics (Mestel and Takhar 1972; Mestel et al. 1981; Nittman and Wood 1981). These dynamically-forced, oscillatory 'ξ−motions' have the period of the Eulerian nutation, much longer than the free oscillation periods of a star, but possibly shorter than the Kelvin-Helmholtz or the nuclear time-scales. In a rapid rotator the motions will have a large amplitude. One can imagine the E-S currents attempting steadily to set up a μ-distribution that yields radiative equilibrium and zero circulation, while the ξ-motions continually destroy the μ-distribution by the periodic dragging of matter into the convective core, where it is rapidly homogenized. Dissipation of the ξ-motions, e.g. by Ohmic diffusion in the low-temperature surface regions, will act to destroy the obliquity of the basic field, and so ultimately to kill off the ξ-motions. But as long as the motions persist, we can expect that some degree of mixing beyond the expected limit of convective overshoot does occur in rapidly rotating stars. This could be significant if observation forces on us to accept that such non-turbulent mixing does occur, especially since, at least in the case of the Sun, helioseismology appears to be confirming that near uniform rotation is the preferred state for the radiative interior.

References

Baker, N. & Kippenhahn, R. 1959, ZfAp, 48, 140

Eddington, A.S. 1929, MNRAS, 90, 54.

Kippenhahn, R. 1958, ZfAp, 46, 26

Mestel, L. 1953, MNRAS, 113, 716

Mestel, L. 1957, ApJ, 126, 550

Mestel, L. 1999, *Stellar Magnetism* (Oxford: Oxford University Press)

Mestel, L. & Moss, D.L. 1977, MNRAS, 178, 27

Mestel, L. & Moss, D.L. 1986, MNRAS, 221, 22

Mestel, L. & Takhar, H.S. 1972, MNRAS, 156, 419

Mestel, L. & Weiss, N.O. 1987, MNRAS, 226, 123

Mestel, L., Moss, D.L. & Tayler, R.J. 1988, MNRAS, 231, 873

Mestel, L., Nittman, J., Wood, W.P. & Wright, G.A.E. 1981, MNRAS, 195, 979

Moss, D.L., Mestel, L. & Tayler, R.J. 1990, MNRAS, 245, 550

Nittman, J. & Wood, W.P. 1981, MNRAS, 196, 491

Öpik, E.J. 1951, MNRAS, 111, 278

Roxburgh, I.W. 1964, MNRAS, 128, 157 and 237

Schwarzschild, M. 1947, ApJ, 106, 427

Spitzer Jr., L. 1958, In *Electromagnetic Phenomena in Cosmical Physics*, ed. B. Lehnert (Cambridge: Cambridge University Press), p. 169

Sweet, P.A. 1950, MNRAS, 110, 548

Vogt, H. 1925, AN, 223, 229