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Abstract

The problem of the effects of mutual collisions for the dynamics of interplanetary dust particles and grains is reviewed. Collisions are shown to give a rather characteristic dynamical signature, the importance of these effects depending mainly on the mean free collision time and the degree of inelasticity. Although a few attempts to look for collisional effects in the solar system have been made, rather much work remains to be done before the problem is fully understood.

Introduction

The dynamics of small interplanetary bodies, grains and dust - in short the meteoritic complex - depend on the type and strength of perturbing forces acting. For the smallest, charged dust particles the Lorentz force will be decisive. For particles in the micron to the millimeter size range radiation pressure with the associated Poynting-Robertson effect and solar wind pressure are important. For larger bodies planetary perturbations have to be taken into account. The dynamical effects of these forces have been discussed to a considerable extent in the literature [1]. Recent progress has particularly been made regarding the radiation pressure effects through the application of Mie scattering theory.

The types of forces mentioned above are examples of continuously acting forces. Of a quite different nature is the impulsive force

resulting from mutual collisions between members of the meteoritic complex. Such collisions have previously mostly been taken into account in order to explain the size spectrum of this complex [2]. Such studies concentrate on the fragmentational properties of high velocity impacts. The dynamical aspects of the collision process are grossly neglected by simply introducing the average impact velocity as a parameter in the model.

Since the dynamical effects of mutual collisions have been given little regard we shall here concentrate on this aspect, reviewing the model studies that have been done so far. Subsequently, the importance of the obtained results for the present day solar system will be discussed. This topic has only been discussed to a limited extent yet. Many questions remain unanswered. We do, however, hope to point out some of the characteristic dynamical signatures of mutual collisions so that the cause can be recognized when one is facing the effect.

The dynamical importance of mutual collisions was proposed by Alfvén [3], predicting that particles moving in neighbouring Kepler orbits around a central body would tend to be collisionally focused into a stream - a jetstream. This hypothesis constitutes a cornerstone of the cosmological theory of Alfvén and Arrhenius [4], jetstreams being considered the parent structure in which accretion of planets and satellites from smaller grains took place.

Insight into the physics of jetstreams can be gained by studying a simple model. Consider a circular jetstream with mass very much smaller than that of the central body so that selfgravitational effects are negligible. Let the mean free collision time in the stream be long compared to the orbital period. An individual grain in the stream will between collisions follow a Kepler orbit. Now compare the velocity components of this grain as it crosses the symmetry plane of the stream with the velocity $\sqrt{\mu/r}$ of an observer in a circular motion at the same distance. To lowest order in eccentricity e and inclination i the result is:

$$\begin{aligned}
 v_r &= \sqrt{\frac{\mu}{a}} e \sin E \\
 v_z &= \sqrt{\frac{\mu}{a}} i \cos (E - \omega) \\
 v_\varphi - \sqrt{\frac{\mu}{r}} &= \frac{1}{2} \sqrt{\frac{\mu}{a}} e \cos E .
 \end{aligned}
 \tag{1}$$

Here a is the semimajor axis, E the eccentric anomaly and w the argument of pericentrum. As is easily seen the contribution from each grain to the velocity spread in the stream in the azimuthal direction is systematically down by a factor 2 relative to the contribution in the radial direction. The velocity distribution in the stream is therefore necessarily strongly non-Maxwellian.

From the kinetic theory of gases it is well known that elastic collisions will tend to Maxwellize the velocity distribution. This means - as seen from (1) - that a balance between the average eccentricity and inclination of the individual orbits in the stream should be established. However, the deficiency of the velocity spread in the azimuthal direction compared to the radial direction can only be decreased by steadily increasing the average eccentricity. This means that more and more grains are put onto hyperbolic orbits through collisions.

The conclusion is therefore that the stream configuration can only be maintained in the presence of collisions with a sufficient degree of inelasticity. The energy lost through collisions can in principle be taken from two sources, from the potential energy of the stream resulting in a shrinking of the stream towards the central body, or from the thermal motions in the stream. If thermal motions constitute the main source of energy the mean eccentricity and inclination of the individual orbits in the stream must decrease with collisions. This is equivalent to a focusing of the stream into its plane of symmetry while the individual orbits at the same time are becoming more and more circular. In addition to this effect Alfvén also predicted the existence of a corresponding radial focusing in the sense that also the radial thickness of the stream should decrease with collisions under suitable conditions.

From this qualitative discussion of basic jetstream physics - which can also be repeated for eccentric jetstreams - we then proceed to the more quantitative model studies.

Analytical model studies

The model studies that have been performed so far have all restricted themselves to the simple situation of a stream of negligible total mass, consisting of grains of equal size, subject only to the

perturbing force due to mutual collisions and with a mean free collision time long compared to the typical orbital period. The Boltzmann equation:

$$\frac{\partial f}{\partial t} + \underline{v} \cdot \frac{\partial f}{\partial \underline{r}} + \underline{a} \cdot \frac{\partial f}{\partial \underline{v}} = I(f, f) \quad (2)$$

constitutes the natural framework for a quantitative discussion of this system. Here $f(\underline{r}, \underline{v}, t)$ is the distribution function of the stream particles in phase space, \underline{a} is the acceleration of an individual grain due to the gravitational field of the central body. The nonlinear Boltzmann collision operator can be given in the general form [5]:

$$I(f, f) = \int d\underline{g} \, d\Omega \frac{d\sigma}{d\Omega} |\underline{g}| \left\{ f(\underline{v} + \frac{1}{2} \underline{g}' - \frac{1}{2} \underline{g}) f(\underline{v} + \frac{1}{2} \underline{g}' + \frac{1}{2} \underline{g}) - f(\underline{v}) f(\underline{v} + \underline{g}) \right\}, \quad (3)$$

where \underline{g} and \underline{g}' are the relative velocity vectors of two colliding particles before and after the collision, Ω is the scattering solid angle and $d\sigma/d\Omega$ is the scattering cross section as a function of relative velocity. The degree of inelasticity is determined by the functional relationship between \underline{g} and \underline{g}' . For completely inelastic collisions $\underline{g}' = 0$ while $|\underline{g}'| = |\underline{g}|$ for elastic collisions. A more specific form of the collision operator for a particular collision model is given in [6].

Two types of expansion procedures for the Boltzmann equation have been attempted, both based on the assumption of a mean free collision time long compared to the orbital period. This author [6] makes use of a power series expansion in time of the distribution function. The collisional induced deviation $\delta f(\underline{r}, \underline{v}, t)$ of the distribution function from an initial state $f_0(\underline{r}, \underline{v})$, chosen as a function of \underline{r} and \underline{v} only through constants of motion of the two-body problem, is for small enough times t given by:

$$\delta f(\underline{r}, \underline{v}, t) = t \left(\frac{\partial f_0}{\partial t} \right) + \frac{1}{2} t^2 \left(\frac{\partial^2 f_0}{\partial t^2} \right) + \dots \quad (4)$$

The time derivatives are determined from the Boltzmann equation. It can now be shown that the collisional induced change in the distribution function gives rise to an additional mass flux in the stream:

$$\delta(n\underline{U}) \equiv \int d\underline{v} \, \underline{v} \, \delta f = -\frac{1}{2} t^2 \nabla \cdot \int d\underline{v} \, \underline{v} \, \underline{v} \, I(f_0, f_0). \quad (5)$$

By calculating the direction of the flux vector $\delta(n\mathbf{U})$ at different points of a cross-section of the stream an idea of the initial dispersal or focusing of the stream can be gained.

The obvious weakness of this approach, as clearly demonstrated by the numerical simulations to be described below, is that (5) might only portray initial transients in the stream due to "improper" starting conditions. It is, however, felt that combined with the experience gained from the numerical simulations, the method could give valuable information of the dependence of stream dynamics on the choice of specific collision models. Such a study has not yet been done.

The other expansion approach to the Boltzmann equation by Baxter and Thompson [5] directs itself to the question of the existence of the radial focusing mechanism, as predicted by Alfvén. These authors consider both three and two dimensional streams - the stream particles being constrained to move in the same plane in the latter case. Since their results are similar for the two cases, we restrict ourselves to the simpler two-dimensional case.

Circular jetstreams are considered. It is assumed that the distribution function depending on \underline{r} and \underline{v} only through angular momentum L and eccentricity e , $f(L, e^2)$, remains a slowly varying function of L . Making use of a Taylor series expansion of f in L the authors then derive an equation for the evolution of the angular momentum distribution:

$$h(L) \equiv \int dr d\varphi dv_r f(L, e^2(r, v_r, L)) , \quad (6)$$

which with the additional assumption $f(L, e^2) = h(L)\psi(e^2)$ takes the simple form:

$$\frac{\partial h}{\partial t} = D(L) \frac{\partial^2}{\partial L^2} h^2(L, t) . \quad (7)$$

The diffusion coefficient is given as:

$$D(L) = \frac{\pi}{4} L^4 \int dg_r dg_\varphi de^2 de'^2 \frac{\partial(r, v_r)}{\partial(e^2, e'^2)} \psi(e^2)\psi(e'^2) \sigma(\underline{g}) |\underline{g}| [(g_r^2 - g_\varphi^2)\beta(\underline{g}) - g_\varphi^2 \alpha(\underline{g})] . \quad (8)$$

Here $\alpha(\underline{g})$ and $\beta(\underline{g})$ are the average fractional energy loss and ave-

rage fractional energy deflected in a collision with relative velocity $\underline{g} = (g_r, g_\phi)$. These quantities are therefore measures of how much the relative velocity vector shrinks and twists in an average collision, respectively. Further, e and e' are the eccentricities of particles at (r, v_r, L) and $(r, v_r + g_r, L + r g_\phi)$ respectively, while ψ , σ and the Jacobian determinant are all positive quantities.

Since the angular momentum distribution (6) is closely related to particle density in the stream, (7) tells that if $D(L) < 0$ then the density at density maxima where $\partial^2 n^2 / \partial L^2 < 0$ will increase, leading to still more enhanced maxima. This phenomenon for which the term "negative diffusion" was coined would be the conjectured radial focusing. From (8) it is seen that an increasing inelasticity decreases $D(L)$ and therefore promotes such a focusing. The effect of energy deflection depends on the explicit form of $\beta(\underline{g})$ and scattering cross-section. One would normally expect a decreased energy deflection to promote radial focusing of the stream.

This conclusion is in qualitative agreement with the results of numerical simulations. The beauty of the result is, however, spoiled by the fact that the applied series expansion is valid only if the distribution function remains not only a slowly varying function of angular momentum but an even more slowly varying function of eccentricity. The latter requirement is clearly too restrictive.

Numerical simulation studies

Numerical simulations have by far given the best insight into stream dynamics yet [7]. Results obtained indicate that it is natural to divide the dynamical evolution of streams into two separate stages. The first of these seems to be almost independent of the particular choice of collision model. Starting from an arbitrarily prescribed initial state a rapid "thermalization" takes place during the first 1 - 2 mean free collision times. During this time the particle distribution adjusts itself such that an approximate balance between the mean square velocities in the radial and polar directions in the stream is set up. Since the mean square of these velocity components are determined by the distributions of eccentricities and inclinations, the width of these distributions adjust to each other during the first transient evolutionary stage.

This is demonstrated in figure 1 for four different simulations starting from the same initial state but with different degrees of inelasticity. The specific collision model employed will be described below. The width of the distributions of eccentricities and inclinations are represented by the average values of these quantities, $\langle e \rangle$ and $\langle i \rangle$. In the present case we clearly has a rather "unbalanced" initial state. This type of effect is also predicted from the time expanded Boltzmann equation [6].

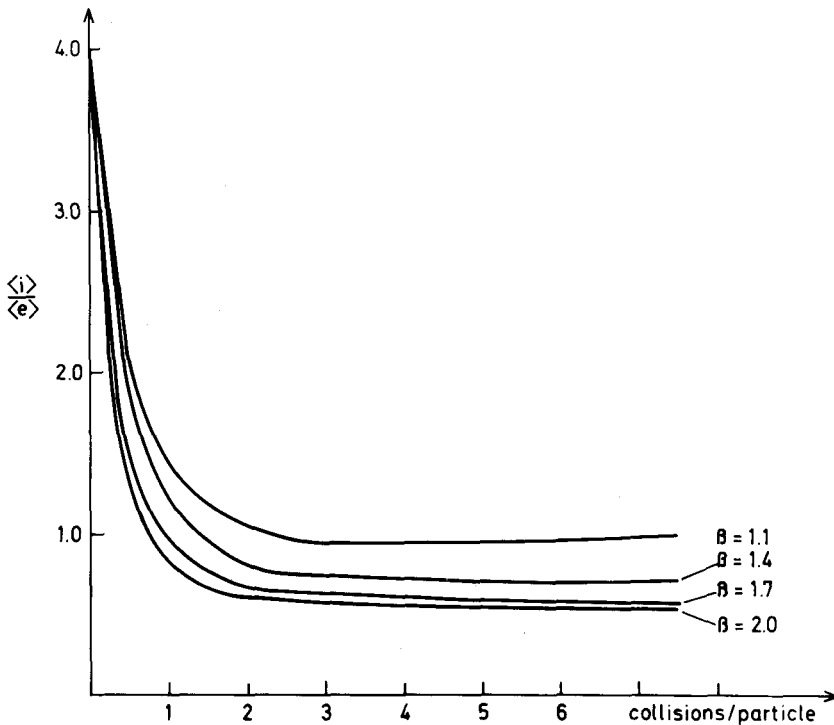


Figure 1. Evolution of the ratio of mean values of inclination and eccentricity for different degrees of inelasticity, starting from the same initial state.

A more detailed analysis shows that not only the width but also the shape of the distribution of inclinations and eccentricities are adjusted during the first stage. This is demonstrated in figure 2 for a circular jetstream. Shown are particle number histograms as a function of eccentricity at three different times for a particular simulation. Starting from a uniform distribution of eccentricities in the interval $e = 0.1$ to 0.3 the distribution can be approximated quite

well with a Rayleigh distribution after only one mean free collision time. The same conclusion applies to the distribution of inclinations.

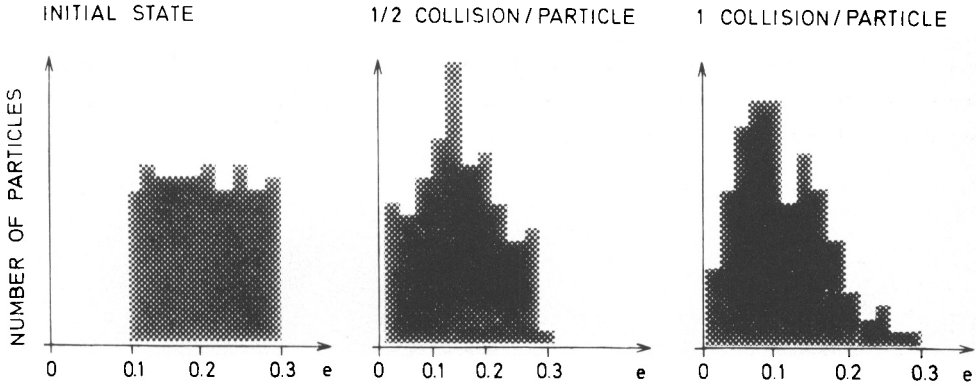


Figure 2. Evolution of the distribution of eccentricities.

A simple physical explanation of this fact can be given along the following lines. Consider a particle with specified orbital elements. In the first approximation the motion of this particle can be considered as a circular motion superimposed two harmonic oscillations, one in the radial direction with amplitude ae and one in the polar direction with amplitude ai . A large number of such oscillators interact via interparticle collisions. One would expect this system of interacting oscillators to evolve towards a state a equilibrium with Rayleigh distributed oscillator amplitudes. This would again give rise to Rayleigh distributed eccentricities and inclinations:

$$F(e) \sim e \exp(-\alpha e^2) \quad (9)$$

and

$$F(i) \sim i \exp(-\delta i^2) . \quad (10)$$

The result applies to circular jetstreams, that is for streams having an isotropic distribution of individual pericentrum vectors \underline{p} , $\langle \underline{p} \rangle = 0$. For elliptic jetstreams, $\langle \underline{p} \rangle = \underline{\eta} \neq 0$, the distribution of eccentricities takes the alternative form:

$$F(e) \sim e I_0(2\alpha\eta e) \exp(-\alpha(e^2 + \eta^2)) , \quad (11)$$

where $I_0(z)$ is the modified Bessel function of zero order. This is a Rice distribution. It is known from the theory of random signals to be the resulting amplitude distribution if a narrow-band gaussian noise is superimposed on a deterministic sinusoidal signal. Certain physical similarities exist between this system and our elliptic stream. For small degrees of anisotropy, $|\langle \underline{p} \rangle| / \langle |\underline{p}| \rangle \ll 1$, the Rice distribution reduces to a Rayleigh distribution. In the opposite case, $|\langle \underline{p} \rangle| / \langle |\underline{p}| \rangle \approx 1$, a narrow gaussian distribution centered at $e = \eta$ results.

The properties of jetstream dynamics discussed so far seems to be essentially independent of the choice of collision model. For the subsequent and more slowly evolving stage this is not so. Here the degree of inelasticity and the amount of energy deflection in a typical collision seems to be of decisive importance. It is therefore necessary at this point to describe in some detail the different collision models that have been used. Both two and three-dimensional simulations were performed. The latter ones were all done with what will be called the β -model. It is described in terms of one parameter β by which help the degree of inelasticity can be varied. The pre- and post-collisional relative velocities \underline{g} and \underline{g}' are related by:

$$\underline{g}' = \underline{g} - \beta \underline{g} \cdot \underline{k} \underline{k}, \quad (12)$$

where \underline{k} is the unit impact vector, parallel to the line connecting the centres of two colliding particles at impact. The component of the relative velocity normal to \underline{k} is left unchanged while the parallel component is reversed and diminished, β taking values in the range (1, 2). Elastic collisions correspond to $\beta = 2$. For $\beta = 1$ half the kinetic energy in the centre of mass system will be lost in an average collision.

The two-dimensional simulations were performed with different collision models. In addition to the β -model results for the snowflake model will also be reviewed. In this model the components of the post-collisional relative velocity parallel and perpendicular to the pre-collisional relative velocity is given by:

$$\begin{aligned} g'_{\parallel} &= g(1 - C \cos \theta) \\ g'_{\perp} &= B g \sin \theta \cos \theta, \end{aligned} \quad (13)$$

where θ is the angle between \underline{g} and \underline{k} . By varying the parameters

C and B the amount of energy lost and deflected in collisions can be varied, respectively.

The simulations indicate that the evolution of the distributions of eccentricities and inclinations during the second stage depends mainly on the degree of inelasticity. When the inelasticity is sufficient, that is, when 30 - 40 per cent of the kinetic energy in the centre of mass system of two colliding particles is lost in an average collision, the widths of these distributions decrease with time. This means that the orbits of the individual particles will evolve towards circular orbits in the symmetry plane of the stream. If this requirement is not fulfilled the opposite evolution takes place. The average eccentricity and inclination then increase with time, more and more particles being put onto hyperbolic orbits and literally being "kicked off" from the central body. This is demonstrated in figure 3 which refers to the same set of simulations as discussed in figure 1. The β -values 1.1, 1.4, 1.7 and 2.0 correspond to average energy losses of 50, 40, 25 and zero per cent respectively. The initial increase in eccentricity is due to the "thermalization process" described above. After only 5 elastic collisions per particle an appreciable fraction of these particles belong to the tail of the distribution of eccentricities extending beyond $e = 1$ and are therefore "kicked off". It is important to note that any amount of energy loss will not bring about a focusing of the stream in its plane of symmetry. The inelasticity has to exceed a certain limit before the inherent tendency of elastic collisions to Maxwellize the velocity distribution is overcome. This limit is not exceeded for $\beta = 1.7$. The

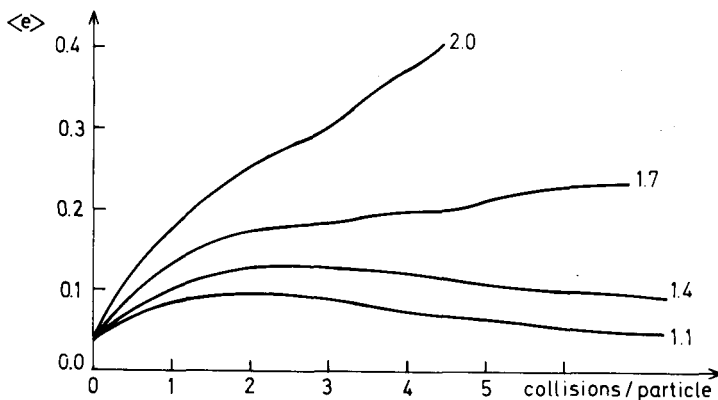


Figure 3. Evolution of the mean value of eccentricity for different degrees of inelasticity.

energy lost in this case is taken from the potential energy, the average value of $1/a$ increasing with time.

The final effect to be discussed is that of the radial focusing of the stream. This effect seems to depend critically on the average amount of energy deflected. The smaller the deflection of the relative velocity vector in an average collision, the better are the chances that a radial focusing will take place. With the β -model no radial focusing was observed in the three or two-dimensional simulations. The two-dimensional simulations with the snowflake model do, however, show this effect under favourable conditions, reductions in the radial width of the stream with a factor 2 to 3 having been observed. This does, however, require a rather small amount of energy deflected in the average collision. This is demonstrated in figure 4, where the spread of the

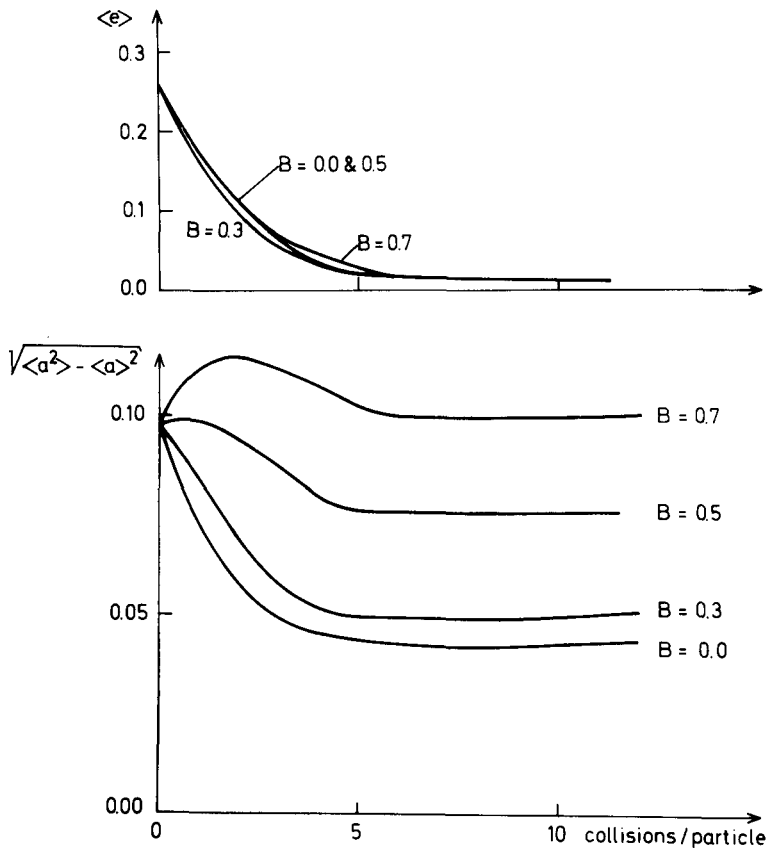


Figure 4. Radial focusing for different amounts of energy deflection, starting from the same initial state.

individual semi-major axis has been taken as a measure of the radial width of the stream.

The radial focusing seems to require that a certain minimum width condition for the stream be satisfied. There thus seems to exist a maximal allowed radial density gradient in the stream for a given spread of eccentricities. This effect is demonstrated in figure 5 which refers to a set of two-dimensional simulations, keeping the collision model and the initial distribution of eccentricities unchanged, but varying the initial radial width of the stream.

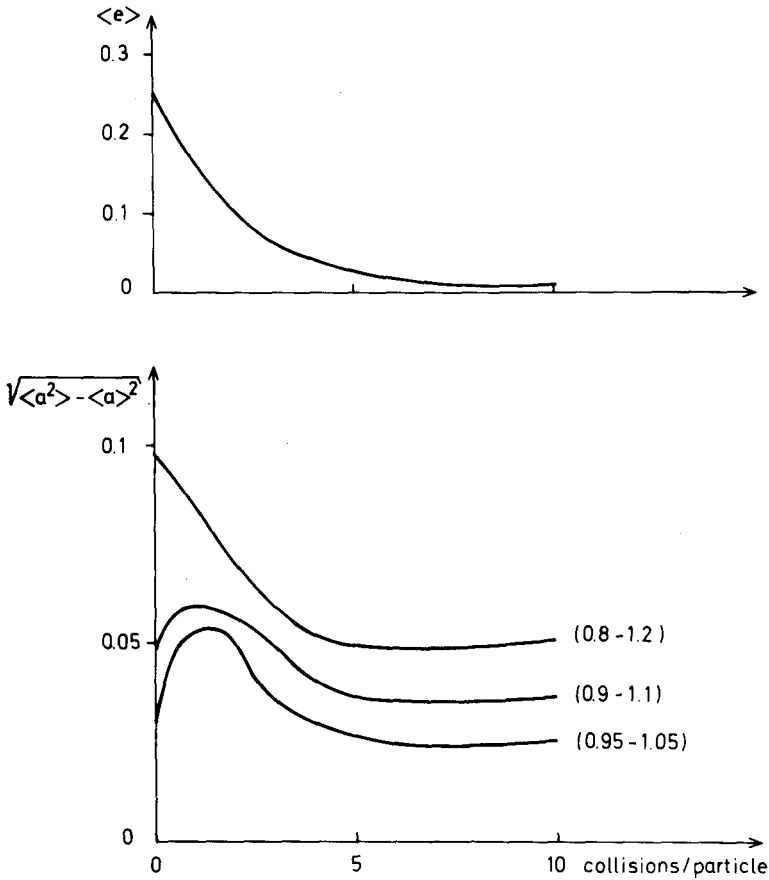


Figure 5. Radial focusing in streams of different initial radial thickness.

Finally, any radial focusing requires a sufficient spread of eccentricities. It is through this spread that the necessary coupling

between particles in the stream is at all possible. When the average eccentricity has decreased to almost zero no more effect is seen in figures 4 and 5.

Dynamical effects of collisions in the present solar system

The studies reviewed above are all restricted to rather idealized stream models. Only streams of same sized spherical particles have been considered. Thus possible characteristic effects due to a broad particle size spectrum are not known. Particle spin has not been taken into account.

The model studies have mainly considered number conserving collisions. Fragmentational collisions are, however, expected to be of importance - at least in other connections [2] - and possibly also accretional collisions. The latter ones will necessarily be completely inelastic. Laboratory studies on hypervelocity impacts indicate that about one half of the kinetic energy is spent on crushing and heating the rock [8]. This would bring fragmentational collisions in line with the most inelastic collisions studied in the numerical simulations. Another facet of fragmentational collisions is that dispersing a given mass in smaller particles will increase the collision frequency and therefore tend to enhance the dynamical importance of collisions.

Contributions to the dynamics of the meteoritic complex come from different sources as noted in the introduction. A simple way of estimating the relative importance of these sources is to compare their different characteristic time scales. For collisions the relevant time scale is clearly the mean free collision time.

With these introductory remarks in mind we next turn to a discussion of possible dynamical effects of collisions in our present day solar system. This discussion will necessarily have to be rather sketchy and partly speculative.

It is expected that the thermalization effect of collisions would be one of their most characteristic dynamical signatures. A search for such characteristics can be made in the visual asteroid population. The ratio of the mean values of inclination and eccentricity of the numbered asteroids turns out to be $\langle i \rangle / \langle e \rangle \approx 1$. Further, in figure 6 the number of these asteroids as functions of inclination and eccentricity

city are plotted together with the Rayleigh distributions adjusted to the corresponding mean value. Inclinations are here taken relative to the ecliptic plane. A more correct procedure would require inclinations relative to the symmetry plane of the asteroid population. Even if the actual distributions do not fit the Rayleigh distributions to within the 95 per cent fractile of a χ^2 -test, similarities are clearly seen. It is tempting to speculate that we are here observing an effect of collisions in the asteroid population.

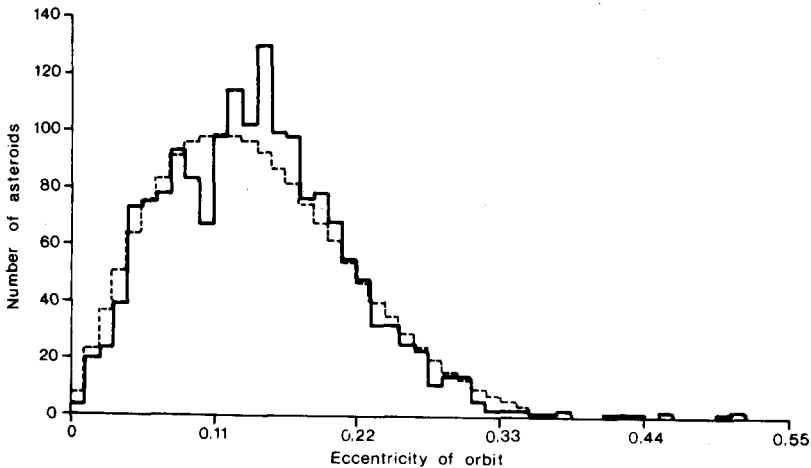
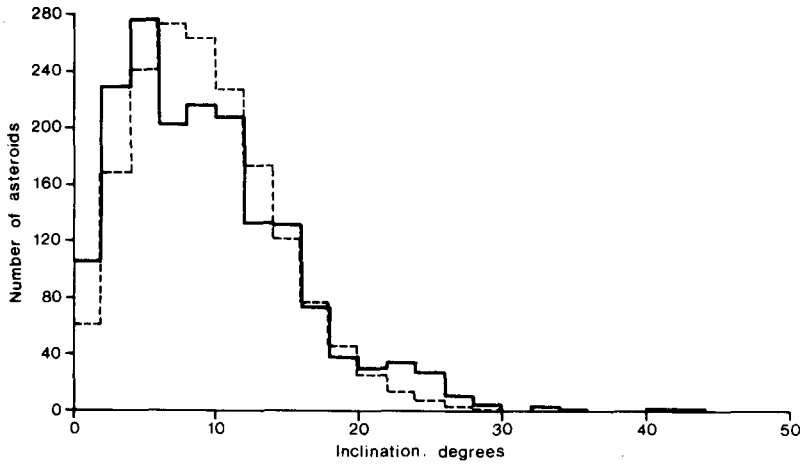


Figure 6. Distributions of inclinations and eccentricities for the numbered main belt asteroids, together with the corresponding Rayleigh distributions.

A characteristic property of the Poynting–Robertson effect for the interplanetary dust population [1] is that the individual orbital planes remain unchanged while the average eccentricity decreases. This process would eventually lead to a collisionally "unstable" dust population. If the mean free collision time in this population is comparable to Poynting–Robertson lifetimes, a thermalization process is expected to take place whereby a finite average eccentricity is maintained at the expense of a decreasing average inclination. Thus, since the Poynting–Robertson effect only constitutes one of at least two competing processes in the real system, conclusions from existing Poynting–Robertson effect calculations should be taken with caution. Whether any decrease in average inclination with decreasing solar distance has been observed is not known to this author.

The simulation results have also been applied to the Saturnian ring problem [9]. Whatever the origin of the rings their particles must at present or in the past have suffered mutual collisions. From the mere existence of the rings it could then be concluded that these particles should have a rather high degree of inelasticity. Secondly, ring lifetime arguments put an upper limit to the possible ring thickness. This follows because increased thickness leads to increased collision frequency and thereby increased rate of energy dissipation. This question can be studied using the results from the numerical simulations. From a given ring thickness the average inclination of the individual particle orbits in the ring is determined. This in turn determines the corresponding average eccentricity. With Rayleigh distributed inclinations and eccentricities we thus see that the ring thickness uniquely determines the velocity distribution in the ring. In this way the energy dissipation rate can be estimated. The conclusion is reached that the ring thickness is at least an order of magnitude less than the 1 – 2 km thickness often inferred from optical observations [10].

The applications discussed so far all refer to what was previously denoted as circular jetstreams. We, however, also observe elliptic stream configurations in the solar system, such as meteor streams and asteroidal jetstreams. The latter ones [3], [11] are represented by clusterings of visual asteroids in similar orbits. Although a rigid analysis of the statistical significance of such clusterings has not yet been attempted, the probability of a random appearance at our times of the major asteroidal jetstreams seems sufficiently small that a physical mechanism able to hold the streams together against the secular perturba-

tions due to Jupiter, should be sought. According to Alfvén's hypothesis, this mechanism is the collisional focusing effect, the visual jetstream asteroids being taken as some kind of Brownian particles in a background collisionally dominated stream of subvisual asteroids. Attempts of looking for the collisional thermalization process in any of these proposed elliptic streams have not been made. Estimates of mean free collision times obviously do not exist. Upper limits to such times set by optical brightness constraints also seems to be lacking.

For meteor streams Mendis [12] has made an attempt to estimate the relative importance of different perturbing forces. Planetary perturbations will give rise to a secular variation of the individual particle orbits in the stream. This will give rise to slowly changing and revolving orbits with typical times for a complete revolution of the order of 10^5 years. This time in itself is of little interest since a revolution of the whole meteor stream is of no relevance in the present connection. Mendis does, however, estimate the timescale for differential perturbations to bring individual grains out of the main bulk of the stream to be of the same order of magnitude. The timescale for the orbit of a $100 \mu\text{m}$ dust grain to shrink by an amount comparable to the radial thickness of typical meteor streams by the Poynting-Robertson effect is also found to be of the same order of magnitude. On the other hand Mendis comes out with 10^4 years as a typical mean free collision time for a meteor stream of mass 10^{18} g, thickness 10^7 km, semi-major axis 3 a.u., consisting of particles of average size $100 \mu\text{m}$ and having an average internal velocity of about 100 m/s. This is certainly an indication that we are in the right ballpark and that important collisional contributions to the dynamical evolution of meteor streams are expected. More detailed studies of this aspect are needed.

A slight warning should be raised at this point. Discussions based entirely on the concept of mean free collision time will not always suffice to settle the problem. Thus, to establish the collisional effects on planetary induced longitudinal focusing in meteor streams [13], a more complete study would be necessary.

Conclusion

So far model studies of the dynamical importance of mutual collisions have only treated rather idealized situations. Dynamical effects of fragmentational collisions and a broad particle size spectrum have

not been dealt with. The interaction between a stream and a background gas or sporadic meteor population has not been given any detailed discussion. Nevertheless, studies performed have already revealed characteristic dynamical signatures expected from mutual collisions.

Only limited effort has been made to apply these results to the real solar system. Evaluations of mean free collision times and expected degree of inelasticity are required. It is hoped that such questions will be given consideration in the future and that the effects of collisions on the dynamics of our solar system will no more simply be overlooked but ascribed their proper weights.

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