ON TURBULENT MIXING

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Several authors (myself included!) have suggested that turbulent mixing takes place in some, if not all, stars, and in particular that such mixing can explain the low solar neutrino flux. This turbulence is thought to be caused by differential rotation produced by braking due to angular momentum loss in a stellar wind, and/or to the effect of meridional circulation currents in redistributing angular momentum. Whilst such instabilities may exist even in the presence of a stabilizing distribution of chemical composition, they do not necessarily cause mixing. To be effective in mixing, the energy available to the instability be it differential rotation or any other mechanism, has to be sufficient to lift the helium rich matter in the interior of the star to the outer regions. This requires

$$\left(\frac{E_{rot}}{E_g}\right) > \left(\frac{\tau_{th}}{\tau_{nuc}}\right)^{\frac{1}{2}} \sim \frac{1}{50} \tag{1}$$

where E_{rot} is the kinetic energy in rotation, E_g the gravitational energy, τ_{th} the thermal time scale and τ_{nuc} the nuclear evolution time scale of the star.
The rate of change of the gravitational energy E_g due to the conversion of hydrogen to helium is of order

$$\frac{dE_g}{dt} = \frac{E_g}{\tau_{nuc}} \tag{2}$$

where τ_{nuc} is the nuclear time scale. For mixing we require

$$\frac{dE_{rot}}{dt} \geqslant \frac{dE_g}{dt} \tag{3}$$

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Now the energy in rotation is either decreasing due to a stellar wind, or replenished due to meridional circulation. In this latter case we have

$$\frac{dE_{rot}}{dt} = \frac{E_{rot}}{\tau_{rot}} , \quad \tau_{rot} = \tau_{th} \frac{E_{rot}}{E_g} \tag{4}$$

Inequality (3) then becomes

$$\left(\frac{E_{rot}}{E_g}\right)^2 > \frac{\tau_{th}}{\tau_{nuc}} .$$

The ^3He instability

There is however one source of energy that is able in principle to overcome the stabilization due to chemical composition gradients. This is the reservoir of ^3He that is built up away from the centre of the star. Gough and his coworkers have shown that this could drive an overstable g mode oscillation after some 3.10^8 years.
To demonstrate this, it is sufficient to note that in solar models some $0.03\ L_\odot$ is due to $H \rightarrow {}^3He$ reactions that do not complete burning to ^4He. Thus the rate of energy release from burning this ^3He by bringing it to higher temperature is of the same order. Thus mixing is energetically possible provided

$$0.03 L_\odot > \frac{dV_g}{dt} = \frac{V_g}{\tau_{nuc}} .$$

But $V_g/L_\odot = \tau_{th}$, hence this condition becomes

$$0.03 > \frac{\tau_{th}}{\tau_{nuc}} \sim 2 \cdot 10^{-4}$$

Thus mixing is indeed energetically possible.

But will it take place, and if so, how efficient will it be? One possibility could be that the oscillation sets down at finite amplitude, the ^3He produced at one layer being carried down to regions of higher temperature where it is burnt. Since ^3He burning varies with temperature like T^{20}, the oscillation will settle down with an amplitude of about $0.05\ H_T$ where H_T is the temperature scale height. This gives $\Delta r/R \sim 0.01$ and no effective mixing, just a mechanism for burning ^3He.
Alternatively, this oscillation could break down to subscale turbulence. The Reynolds number of the oscillation is very large. In this case the turbulence would diffuse ^3He to higher temperature regions at such a rate to ensure that the ^3He created at lower temperatures is destroyed at higher temperatures. Since the time scale of creation is about 3.10^8 years, and the distance scale for burning ^3He is 0.01 R, then we would expect a turbulent diffusion coefficient of

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$$\nu_t \sim \frac{(0.01\ R_\odot)^2}{3.10^8 ys} = 60.$$

Such a diffusion (corresponding to $R_e \sim 15$) will be effective in mixing the ^3He layer, but not effective enough to cause substantial mixing in the central regions.
But perhaps there is another way of converting the ^3He energy source into a more effective mixing mechanism!

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